Open Pit-Underground Mining Options and Transitions Planning: A Mathematical Programming Framework for Optimal Resource Extraction Evaluation

by

Bright Oppong Afum

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Thesis Examiners/Examinateurs de thèse:

Dr. Eugene Ben-Awuah
(Supervisor/Directeur(trice) de thèse)

Dr. Marie Hélène Fillion
(Committee member/Membre du comité)

Dr. Martin Hudyma
(Committee member/Membre du comité)

Dr. Mohamed Dia
(Committee member/Membre du comité)

Dr. Mustafa Kumral
(External Examiner/Examinateur externe)

Dr. Kalpdrum Passi
(Internal Examiner/Examinateur interne)

Approved for the Faculty of Graduate Studies
Approuvé pour la Faculté des études supérieures
Dr. Lace Marie Brogden
Madame Lace Marie Brogden
Acting Dean, Faculty of Graduate Studies
Doyenne intérimaire, Faculté des études supérieures

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ABSTRACT

Near-surface mineral deposits that extend to great depths are amenable to both open pit mining and/or underground mining. The strategic planning of such mineral deposits often leads to several variations of open pit-underground (OP-UG) mining option(s) and transitions including (a) independent open pit (OP) mining, (b) independent underground (UG) mining, (c) simultaneous open pit and underground (OPUG) mining, (d) sequential OPUG mining, and (e) combinations of simultaneous and sequential OPUG mining. Notable limitations to recent developments in the OP-UG mining options and transitions optimization problem includes one or more of the following: a) lack of rigorous mining optimization approach, b) lack of solution optimality assessment, c) lack of geotechnical consideration for the mining options and transition zones, d) lack of consideration of exhaustive variables for essential UG mining complexes, and e) non-comprehensiveness and inefficiency of the implementation models.

The main research objectives are 1) propose an optimization technique for OP-UG mining options and transitions planning, and 2) develop, implement and verify a theoretical optimization framework based on Mixed Integer Linear Programming (MILP) model to determine: a) the most suitable mining option(s) to exploit an orebody; b) the position of the required crown pillar, time and order of development of primary and secondary accesses and main ventilation opening, and the schedule of geotechnical support of the secondary development accesses and stopes if UG mining option is considered; and c) the ore and waste extraction schedules that maximizes the net present value (NPV) of the mining project.

MATLAB programming platform was chosen for the MILP formulation implementation and a large-scale optimization solver, IBM ILOG CPLEX, was used for this research. The MILP formulation was tested and implemented with an experimental copper dataset and two real gold deposit case studies. The first case study verified the appropriateness of the optimization technique and strategies used in the MILP framework for open pit-underground mining options and transitions planning. The second and third case studies are implemented with stockpile management and multiple essential underground infrastructures to enhance practicality and rigor of the MILP model. The third case study was additionally evaluated with industry standard software, Whittle, and the results compared to that from the MILP model.

The MILP model scheduled the deposit with combined sequential and simultaneous OPUG mining over 8 years mine life while Whittle scheduled the deposit for OP mining over a mine life of 20 years. The NPV generated by the MILP model was \$ 4.01 billion while the NPV generated by Whittle Milawa NPV algorithm was \$ 2.31 billion, representing about 42.4% loss in financial benefits. The stripping ratio from Whittle OP mining was 2.79 compared to 0.34 from MILP model for the OPUG mining. Analysis of the results showed that, the MILP model significantly avoids the mining of excessive waste to uncover mineralized material by switching from OP to UG mining option. This MILP framework implementation for extraction of deep-seated near-surface deposits demonstrate potential value to a mining project at the prefeasibility stage when the global mining options decisions are guided by a rigorous optimization process. The MILP framework do not evaluate the impact of varying crown pillar dimensions on the mining options.

Keywords

optimal resource extraction evaluation; net present value; mixed integer linear programming; mining options and transitions; optimization; open pit-underground mine planning

CO-AUTHORSHIP STATEMENT

The author of this thesis is the primary author for all manuscripts published out of this work. Associate Professor Eugene Ben-Awuah is the supervisor of the author's PhD and is included as co-author for each published manuscript. Five peer reviewed manuscripts have been produced directly from this research work. Two have been published in peer reviewed journals, two have been submitted to other peer review journals, and one is being prepared for submission for publication. The concepts and formulations of this research were shared with the research and mining professionals' community at an international conference in Poland (Application of Computers and Operations Research in the Mineral Industry – APCOM) in June 2019, research seminar at the Mining Optimization Laboratory of the University of Alberta, Laurentian University community through the Bharti Engineering School seminar and the annual Research Week festival, and mining professionals in Canada (IAMGOLD Corporation). The list of peer reviewed publications resulting from this PhD work are as follows:

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LIST OF ABBREVIATIONS

ASR	Allowable Stripping Ratio
CAPEX	Capital Expenditure
CEE	Competitive Economic Evaluation
CMO	Competitive Mining Option
COMET	Concurrent Object Modeling and Architectural Design Method
CPU	Central Processing Unit
DCF	Discounted Cash Flow
EBV	Economic Block Value
IP	Integer Programming
IRR	Internal Rate of Return
IV	Incremental Value
LP	Linear Programming
MILP	Mixed Integer Linear Programming
MMG	Minerals and Metals Group
MP	Mathematical Programming
MPM	Mathematical Programming Model
NPV	Net Present Value
NSCS	Non-trivial Strongly Connected Subgraph
OP	Open Pit
OPUG	Open Pit and Underground
OP-UG	Open Pit-Underground
OSR	Overall Stripping Ratio
PCPSP	Precedence Constrained Production Scheduling Problems
QCP	Quadratically-Constrained Program
QP	Quadratic Program
ROM	Run-of-Mine
SMU	Selective Mining Unit
SOCP	Second-Order Cone Programs
SOS	Special Ordered Set
SUMOTO	Surface-Underground Mining Options and Transitions Optimization
UG	Underground
VCR	Vertical Crater Retreat

CHAPTER 1

INTRODUCTION

1.1 Background

When a mineral deposit is discovered, several technical studies are rigorously conducted to ascertain the economic viability of the deposit. At the prefeasibility and feasibility stages of a mining project, many decisions are made before the commencement of the project. The correctness and accuracy of these decisions provide confidence to both mine planners and investors. One key decision is the suitable mining option(s) required to exploit the mineral deposit. The decision on an optimal extraction option becomes complicated when the deposit extends from the surface to great depth. A deep-seated ore body showing significant outcrops could be exploited using different mining options and an important exercise is the determination of the optimal mining option(s). Mining option is defined as the extraction method required to exploit a mineral deposit based on the prevailing technical, economical, safety, environmental and social conditions. The mining option could either be surface mining methods, underground mining methods, or both (Bakhtavar, et al., 2009a). The term mining option has also been used to refer to the initiatives or choices undertaken in the extractive industry to expand, change, defer, abandon, or adopt strategies for a mining method(s), ore extraction sequence, contracting mining services, and sometimes investment opportunities based on changing socio-economic, technological, technical, environmental or market conditions (Afum & Ben-Awuah, 2017).

Optimization studies for each specific mining option, especially surface or open pit mining is common. However, optimization studies that sort to find the suitable mining option for any deposit amenable to both open pit and underground extraction is uncommon. The optimal mining option for extracting a near-surface deposit may include: (a) independent open pit (OP) mining; (b) independent underground (UG) mining; (c) simultaneous open pit and underground (OPUG) mining; (d) sequential OPUG mining; and e) combinations of sequential and simultaneous OPUG mining. Exploiting a deposit solely by an option where all operations are exposed to the atmosphere is classified as independent OP mining option. For an independent UG mining option, the extraction of the deposit is solely conducted within the earth crust. Simultaneous OPUG mining option refers to the concurrent extraction of a deposit by both OP and UG mining options. On the other hand, in sequential OPUG mining option, the deposit is first exploited by OP mining option and after closure of the OP mining operations, UG mining option follows or vice versa. Combinations of sequential and simultaneous OPUG mining options refer generally to the extraction of a deposit by sequential OPUG mining wherein simultaneous OPUG mining occurs during the transition.

The choice of the most economic mining option(s) can be implemented through a global optimization approach. Mining options and transitions optimization studies are becoming popular in the mining industry. They could serve as essential tools for the evaluation of current discoveries of complex mineral deposits or for existing operations seeking to maximize resource recovery. Recent research works to solve the mining options and transitions problem has evolved from the determination of transition depth to the evaluation of extraction sequence in the presence of crown pillar positioning optimization. Reviews on the techniques for solving the mining options and transitions optimization problem have been studied and detailed by Afum & Ben-Awuah (2017), and Bakhtavar (2015b). The complexity of underground mining dictates that a more sophisticated optimization framework is required to solve the surface-underground mining options and transitions problem for global optimality. A typical approach is a multi-objective optimization formulation that is fully controlled by "all" the essential constraints to obtain a globally feasible solution to the surface-underground mining options and transitions problem. The multi-objective modeling framework provides a platform for trade-offs between competing objectives, helping the mine planner to choose preferred global solution (Ben-Awuah, et al., 2012; Foroughi, et al., 2019).

The principal research objective is to develop a rigorous optimization framework and methodology to evaluate the financial benefit and resource recovery ratio for a given orebody that could be exploited by both open pit and underground mining options. This thesis documents a mixed integer linear programming (MILP) framework for optimal resource extraction evaluation in open pit-underground mining options and transitions planning. The model selects the most suitable mining option for extracting a deposit that is potentially amenable to either or both open pit and underground mining. The mining option selection is made in the presence of a suitable crown pillar, capital development (shaft/decline) and operational developments (level, ore drives and crosscuts). In the case when the model selects UG mining option as the preferred suitable extraction option or part of the option, the crown pillar position and schedules for the capital and operational developments are further determined. Integrating three-dimensional (3D) crown pillar positioning into the optimization process allows the OP and UG mining options the fair opportunity to economically compete for selection. The geomechanical properties of the crown pillar are not directly integrated in the mathematical programming framework. This implies that once the crown pillar location that provides the best economics has been identified, additional geomechanical studies will be required during detailed planning at the feasibility stage. Evaluating deposits in the presence of several technical complexities and extraction scenarios for a suitable optimal mining option adds significant value to a mining project.

1.2 Problem definition

The two principal objectives of mine planning are the development of the most economic mineral exploitation strategy that maximizes the investment returns and the achievement of a maximum resource recovery (Bohnet, 2011). Optimization of the resource extraction strategy however depends greatly on the type of mining operations. When the mineral deposit appears as an outcrop and further extends to great depths, such deposits can be exploited by OP mining or UG mining or both. Several forms exist when the deposit is amenable to both OP and UG mining options – sequential and/or simultaneous. Sequential exploitation describes the extraction process where OP mining is proceeded by UG mining or vice versa while simultaneous exploitation indicates that both OP mining and UG mining are occurring at the same time. Current optimization models for strategic evaluation of such mineral deposits primarily focus on the depth of the transition point between OP mining and UG mining. An optimization algorithm or model that comprehensively and simultaneously determines an optimized open pit mine, transition interface, and underground mine for such ore bodies will be of major benefit to the mining industry.

Shortfalls to previous research on mining options optimization have been reviewed and opportunities for further studies identified. Notable limitations to recent developments in the OP-UG mining options and transitions optimization problem includes one or more of the following: a) lack of rigorous mining optimization approach, b) lack of solution optimality assessment, c) lack of geotechnical consideration for the mining options and transition zones,

d) lack of consideration of exhaustive variables for essential UG mining complexes, and e) non-comprehensiveness and inefficiency of the implementation models.

The depth and economic outline of the OP mine, crown pillar location, and the UG mine impact the net present value (NPV) of the mineral extraction process. The strategic schedules for a typical OP-UG mining operation require that both OP and UG mining options compete for the same mineral reserve during optimization (Ben-Awuah, et al., 2016; Afum & Ben-Awuah, 2019). The research problem presented here involves the development of a mathematical formulation based on a MILP model to globally determine the: (a) optimal pit limit outline if any; (b) extraction schedule of the *K* mining blocks within the optimal pit limit outline over *T* different periods in the case of OP mining, (c) optimal UG mining boundary if any; (d) extraction schedule of the *K* mining blocks within the UG mining boundary over *T* different periods in the case of UG mining; (e) development schedules of the primary and secondary accesses within the UG mining boundary over *T* different periods when UG mining option is preferred; (f) rock support schedules of the operational development openings and stopes within the UG mining boundary over *T* different periods when UG mining is a preferred option; and (g) extracted schedule of the *K* mining blocks within the combined optimal pit outline and economic UG mining boundary over *T* different periods when OPUG mining is the preferred option. The developed mathematical framework maximizes the mining project's NPV in the presence of physical, technical, and economic constraints. The physical constraints relate to the type of mining methods being deployed, interactions between the mining options, and the mineral extraction sequence relations defined in each mining option. The technical constraints control for the mining, processing, and development capacities, the quality of the ore material extracted for processing, and UG rock support considerations. Economic constraints integrate selling price of the commodity, mining and processing recoveries, mining and development costs, and discount rate into the formulation. Figure 1-1 is a schematic representation of the mining options and transitions planning problem.

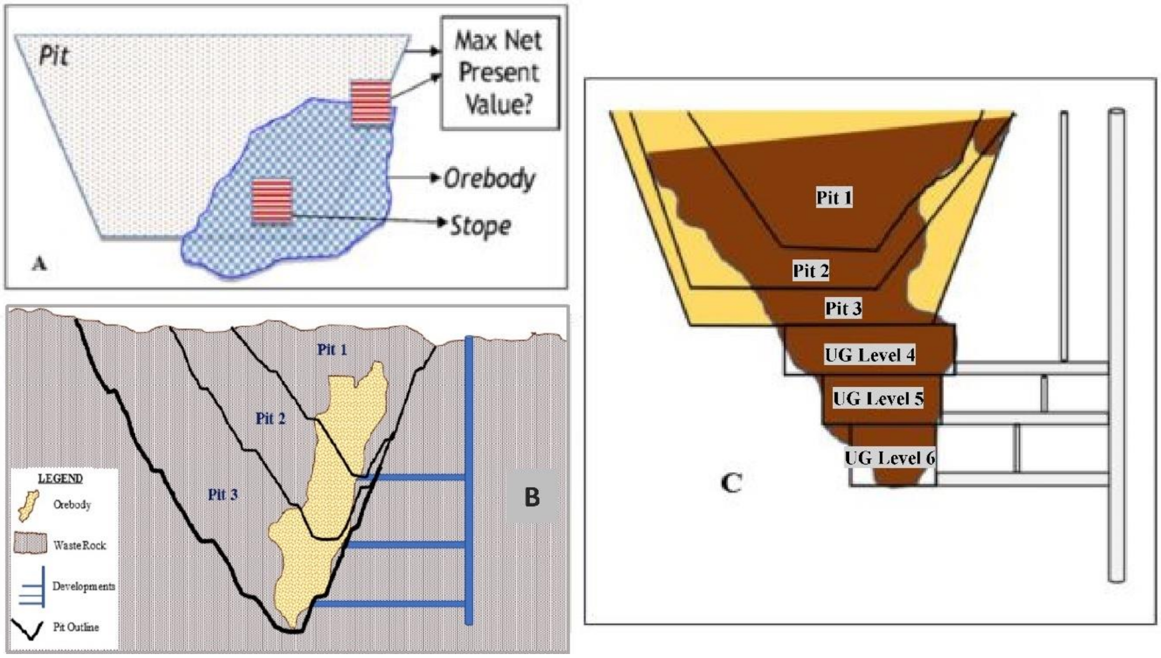


Figure 1-1: Schematic representation of the open pit-underground (OP-UG) mining options and transitions planning problem. (A) illustrates evaluation of an orebody to generate maximum net present value (NPV) depending on how each mining block is extracted; either through open pit (OP) mining, open stope extraction, or both. (B,C) demonstrate the extraction of a mineral resource by OP, underground (UG), or both open pit and underground (OPUG) mining for optimum resource development planning. [A – Ben-Awuah, et al. (2016); B – Bakhtavar, et al. (2010); C - (Afum, et al., 2019a)]

1.3 Assumptions

It is assumed that the selective mining units (SMUs) for mining (OP mining) is equivalent to the stope sizes for underground mining. In the proposed mixed integer linear programming (MILP) framework, the SMUs are represented by mining blocks in relation to surface mining and stopes for underground mining. Following after Mousavi & Sellers (2019), the concept of dynamic cut-off grade in which no predefined cut-off grade is specified for each mining option is employed in this framework. The location of each SMU is represented by the coordinates of the centroid. It is assumed that a crown pillar is required for the exploitation of the ore body by underground mining.

Other than the assumed rock strength properties of the crown pillar, and estimated cost of rock support and reinforcement of the underground openings, no other geotechnical properties were included in this study. Technical and economic data used in the implementation of the mathematical programming framework were either computed using the Costmine database (CostMine, 2016) or adopted from existing mining operations. The size of the crown pillar is assumed to be one vertical length of a stope or bench; thus, one unknown bench or level in the block model will represent the crown pillar. This crown pillar thickness assumption requires that once the crown pillar location has been identified, additional geomechanical studies is conducted to validate it thickness during detailed planning at the feasibility stage. It is important to note that geotechnical methodologies for sizing of crown pillars are well known (Kumar, et al., 2017), and therefore detailed geotechnical studies should be the basis of pillar design at the feasibility stage of the mine. Similarly, the effect of water on siting of the crown pillar, ore extraction in the stope, and cost of UG water management is not considered in this research. For underground mining, mineralized material extraction is achieved by naturally supported mining methods (room and pillar, sublevel stoping, open stoping, vertical crater retreat (VCR) and vein mining); artificially supported mining methods (stull stoping, square set, cut and fill, shrinkage, and resuing); and unsupported mining methods (block caving, sublevel caving, and top slicing). The ore extraction sequence is by either retreating or advancement methods, or both.

1.4 Summary of literature review

Surface mining is known to be relatively highly productive, very economic, and safer for workers compared to underground mining for most suitable deposits. However, recent evolution in environmental regulations and societal expectations may result in the development of small, high-grade deposits by shallow open pits (OP) or in the establishment of high-grade underground (UG) mines in place of extensive OP operations (Nelson, 2011). Optimizing the extraction of a mineral deposit in the presence of both surface mining methods and UG mining methods result in the most economic decision generated by identifying the best mining option for the deposit. Some studies have been conducted to solve the surface-underground mining options and transitions optimization problem. These studies have focused on determining the transition depth and the resulting production schedules for the OP and UG mining operations using simplified optimization frameworks. These models do not address the multi-objective optimization nature of the surface-underground mining options and transitions problem, and do not formulate the problem with a complete description of the practical mining environment. Specifically, the existing models do not incorporate essential developmental infrastructure such as primary and secondary mine access, ventilation requirement, and geotechnical support and reinforcement in the optimization framework. Results from these models often lead to localized optimal solutions or biased solutions that are usually impractical to implement in the mining environment (Nilsson, 1982, 1992; Luxford, 1997; Bakhtavar, et al., 2008, 2009a, 2009b; Bakhtavar, et al., 2012; Bakhtavar, 2013; Dagdelen & Traore, 2014; Bakhtavar, 2015b, 2015a; Chung, et al., 2015; King, et al., 2017; Orlin, 2017; MacNeil & Dimitrakopoulos, 2017; Whittle, et al., 2018). According to Tuck (2011), there are two main types of lateral and vertical openings in UG mining categorized into primary and secondary developments. Primary developments consist of the construction of permanent openings that must stand throughout the mine life, and include shafts, declines, adits, and tunnels. However, secondary developments entail the construction of temporary openings such as ore drives, crosscuts, and waste drives in the UG mine mostly required for a particular production unit. These definitions for primary and secondary developments are adopted throughout this research.

Existing optimization algorithms used in attempting the mining options problem include the Lerchs-Grossman algorithm, Seymour algorithm, floating cone technique, network flows, dynamic programming, neural network, theory of graphs, and mathematical formulations (Ordin & Vasil'ev, 2014). Some authors have studied the surface-underground mining options and transitions optimization problem with available commercial software packages including Surpac Vision, Datamine's NPV Scheduler, Whittle Four-X, Geovia MineSched, integrated 3D CAD systems of Datamine, Vulcan, MineScape, MineSight, Isatis, XPAC, Mineable Reserve Optimizer (MRO), Blasor pit optimization tool, COMET cut-off grade and schedule optimizer, and Datamine Studio 3 (Achireko, 1998; Opoku & Musingwini, 2013; Roberts, et al., 2013; Dagdelen & Traore, 2014; De Carli & De Lemos, 2015). The techniques used by these authors are not generic but mostly scenario based and often lead to localized optimization solutions.

Although Bakhtavar, et al. (2009b) employed a heuristic algorithm to compare economic block values computed for both open pit and underground mining on a depth flow basis to solve the surface-underground mining options and transitions optimization (SUMOTO) problem, results from the heuristic algorithm do not offer a measure of optimality as is the case in mathematical programming optimization. Notable authors that used mathematical programming to solve the mining transition problem limit their model to the determination of transition depth and block extraction sequence for the open pit and underground mining operations (Luxford, 1997; Bakhtavar, et al., 2008, 2009a; Roberts, et al., 2009; Bakhtavar, et al., 2012; Bakhtavar, 2013; Roberts, et al., 2013; Dagdelen & Traore, 2014; Ordin &

Vasil'ev, 2014; Bakhtavar, 2015b, 2015a; Ben-Awuah, et al., 2015; Ben-Awuah, et al., 2016; Chung, et al., 2016; King, et al., 2017; Whittle, et al., 2018). Similarly, other authors have developed stochastic mathematical programming models to solve the surface-underground mining options and transitions optimization problem. They focused on determination of the transition depth in 2D environment, and do not incorporate other essential underground mining constraints such as primary and secondary development, ventilation shaft development, and geotechnical requirements for the development openings and stopes in the optimization framework (Opoku & Musingwini, 2013; Bakhtavar, et al., 2017; MacNeil & Dimitrakopoulos, 2017). This is because the optimization of underground mines is computationally complex (Huang, et al., 2020b) and integrating it with open pit mining makes it more challenging (Nhleko, et al., 2018).

The positioning of the required crown pillar thickness in the SUMOTO problem is key to the operations of such mines. Some authors pre-selected the depth of the crown pillar (transition depth) before evaluating portions above the crown pillar for open pit mining and portions below the crown pillar for underground mining (Ordin & Vasil'ev, 2014; Ben-Awuah, et al., 2015; De Carli & De Lemos, 2015; Ben-Awuah, et al., 2016; King, et al., 2017). This may lead to suboptimal solutions and will require evaluating multiple crown pillar locations in a scenario-based approach. A few authors have attempted to incorporate the positioning of the crown pillar in the optimization process (Bakhtavar, et al., 2012; Bakhtavar, et al., 2017; Whittle, et al., 2018; Afum & Ben-Awuah, 2019; Afum, et al., 2019a). Their models were good improvements over previous works but were missing some constraints like ventilation requirement and rock strength properties required for practical implementation. The transition from OP to UG mining is a complicated geomechanical process which requires the consideration of rock mass properties (Fengshan, et al., 2012; Yardimci, et al., 2016).

Bakhtavar (2015b) reviewed the combined open pit with underground mining methods for the past decade and noticed that the transition problem has been implemented in either simultaneous or non-simultaneous modes. He asserts that non-simultaneous mode of combined mining is more acceptable because large-scale underground caving methods with high productivity and low costs can be used. However, in simultaneous mode, horizontal and vertical slices underhand cut and fill with cemented backfill is more feasible to be used with

OP mining. Afum, et al. (2019a) implemented a mathematical programming model that allows the optimization approach to decide whether the mineral deposit should be exploited with either simultaneous, non-simultaneous, sequential or any of these combinations thereof.

Most existing models in general do not include the requirements of essential underground mining infrastructure such main access to the underground mine (shaft or decline or adit development), ventilation development, operational development (levels, ore and waste drives, crosscuts), and necessary vertical development (ore passes, raises). Equally, these existing models do not incorporate rock strength properties in the SUMOTO problem. Although these essential infrastructure and geotechnical characteristics of the rock formation are significant to underground mining operations, their added complexities make it difficult to be included in the SUMOTO models. According to Bullock (2011b), mine planning is an iterative process that requires looking at many options and determining which, in the long run, provides the optimum results. Using such iterative process could lead to some inferior or sub-optimal solution(s) that do not constitute the global optimal solution.

The strength of the application of Mathematical Programming Models (MPMs) for mine optimization problems is employed in this research. Mathematical programming models are known to be rigorous and their solutions have a measure of optimality. The application of mathematical optimization to OP mining limits started with the implementation of graph theory commonly referred to as the LG algorithm (Lerchs & Grossman, 1965). Subsequently, the optimization problem was also modelled as a maximum flow problem (Picard, 1976). Some benefits of MPMs include (Martinich, 1997):

1. Rigorous – mathematical programming models are precise, and their structure describes the thought process of the modeler in terms of the decision variables (objective functions), and the decision environment (constraints).
2. Objectivity – mathematical programming models are objective since all assumptions and definitions are clearly stated. Even though these models may reflect the experience and preference of the modeler, any biases can be identified by observers.
3. Tractability – mathematical programming models allow large and complex problems to be solved in their reduced form by employing the significant interrelationships among the variables constituting the problem. Thus, complicated decision-making problems are relatively approximated and simplified.

4. Model solution – mathematical programming models make problems amenable to mathematical and computer solution techniques.
5. Sensitivity or parametric analysis - mathematical programming models make it relatively easy to find the optimal solution for a specific model and its variations due to repeatability.

A mathematical programming models (MPM) based on MILP optimization framework for evaluating the mining option(s) for a deposit has been developed, implemented, and tested on a gold deposit case study. The MILP framework is based on the Competitive Economic Evaluation (CEE) approach introduced by Afum & Ben-Awuah (2017). According to Afum, et al. (2019a), the CEE optimization technique allows the optimizer to select the most suitable mining option(s) and extraction strategy for the deposit. The mining options evaluated are independent OP, independent UG, simultaneous OPUG, sequential OPUG, and combinations of simultaneous and sequential OPUG. The proposed MILP framework incorporates the required UG mine ventilation development, rock support and reinforcement of the operational development (level, ore and waste drives, crosscuts) and stopes. The UG mine ventilation development often incorporates a series of bored raises and lateral drive development, and the construction of ventilation controls. Ventilation controls are a range of objects such as regulators, including doors and walls which need considerable time and money to create. The term support generally refers to the various types of rock support used to protect underground workers and may include steel mesh, shotcrete, fibrecrete, and a variety of types of steel straps. Reinforcement on the other hand refers to the various types of rock reinforcement to help prevent rock movement and may include a variety of types of rockbolts, cablebolts, rebar, and dowel. Cablebolting in stope development can particularly be very costly and may introduce considerable time delays. Most cablebolts need 30 days for the portland cement grout to properly set for the cablebolts to be fully functional.

In summary, the features and strengths of the proposed MILP framework are that: (a) the CEE optimization approach is unbiased; (b) the crown pillar positioning is incorporated into the optimization process and not predetermined; (c) the production schedule or extraction strategy of the selected mining option is time-dependent; (d) the construction of essential

UG mining infrastructure (main ventilation, capital and operational development requirements) are incorporated in the formulation; and (e) the rock support required to reinforce the operational development openings and stopes are considered in the optimization model.

1.5 Research objectives

The primary objective of this research is to develop a theoretical framework for selecting the optimal mining option(s), defining the mining transition depth, and evaluating the extraction strategy for developing any given orebody amenable to OP and/or UG mining using mathematical programming (MP) optimization framework. The various mining options to be considered in the optimization framework are: (a) independent OP mining, (b) independent UG mining with crown pillar, (c) simultaneous OPUG mining with crown pillar, (d) sequential OPUG mining with crown pillar, and (e) combinations of simultaneous and sequential OPUG mining with crown pillar. The objectives of the study will focus on the following:

1. Propose and develop an optimization process for OP-UG mining option(s) and transition planning.
2. Develop an optimization framework that maximizes the NPV of a mining options evaluation project while generating a strategic extraction schedule for OP and/or UG materials.
3. Incorporate the following decision complexities into the optimization framework:
   a. three-dimensional (3-D) crown pillar positioning,
   b. UG primary access development,
   c. UG operational development,
   d. ore quality or grade blending constraints,
   e. stockpile management,
   f. mineral extraction sequence,
   g. UG geotechnical considerations, and
   h. UG ventilation requirements.
4. Develop computer codes/tools to implement the formulated models for practical mining options projects.

1.6 Context and scope of work

There are essentially two main ore extraction systems: surface mining methods and underground mining methods. Open pit mining technique is a branch of surface mining methods considered as the most applied mining method. Open pit mining has advantages over underground mining methods in terms of mining recovery, production capacity, mechanization, grade blending and cut-off grades control, flexibility of operation, ore loss and dilution control, and safety of workers. However, underground mining is highly acceptable than open pit mining in terms of environmental and social perceptions. It leaves a lesser mining footprint compared to open pit mining. In some cases, due to the orebody configuration, the mine economics may support both or one of these mining methods.

Transitioning from OP to UG mining or vice versa is sometimes considered a strategy when maximizing the economic project value and the resource recovery factor. In this case, appropriately optimizing the interaction between OP and UG mining operations is an essential contribution to the project's success. From a practical point of view, planning for the transition requires a long lead-time as the implications on the ultimate pit and the underground boundary design can be significantly (Roberts, et al., 2009; Finch, 2012; Hassan, et al., 2012; Newman, et al., 2013; Ben-Awuah, et al., 2016; King, et al., 2017). Therefore, determination of the transition depth and extraction strategy should be meticulously examined before implementation. This meticulous approach ensures the economic benefits and associated risks are not missed during the planning stage. Transitioning from OP to UG mining or vice versa involves considerable alterations of parameters and decision variables in the system of mining Based on the production capacities, mining equipment together with logistics and transportation arrangements could also be impacted. The rock stability and ground control requirements could also change. Without detailed strategic planning, many challenges can arise influencing the success of the mining project.

This research focusses on the development of a mathematical programming framework and methodology for appraising the impact of applying the several forms of mining options to exploit a mineral deposit. The various forms of mining options include independent OP mining, independent UG mining, sequential OP and UG mining, and simultaneous OP and UG mining while considering the impact of primary access and operational developments, 3-D crown pillar positioning, stockpile management, geotechnical requirements of the operational development and stope mining, *in-situ* ore quality or grade blending constraints and ventilation requirements. The research will investigate the strategy of extracting an orebody using:

1. Independent OP mining;
2. Independent UG mining with crown pillar;
3. Simultaneous OPUG mining with crown pillar;
4. Sequential OPUG mining with crown pillar; and
5. Combinations of simultaneous and sequential OPUG mining with crown pillar.

It is assumed that a crown pillar is required for the exploitation of the orebody by underground mining. In situations where the required crown pillar has economic value higher than an artificial crown pillar, this assumption will impact profitability of the project. For UG mining, stoping is achieved by naturally supporting mining methods (room and pillar, sublevel stoping, open stoping, vertical crater retreat (VCR) and vein mining), and caving methods (sublevel caving, block caving, and top slicing). The sequence of underground ore extraction could be either retreating or advancement methods or both. Future selling prices of the commodity and costs of mining the mineral deposit is held constant. Sensitivity analyses are therefore key components in the evaluation methodology of this research to ensure re-optimized production schedules aligns with current industrial practices. Similarly, the geological block model is assumed to be deterministic.

The collection of drilling, sampling, and geotechnical data for constructing the geologic block model is outside the scope of this study. Determination of technical and economic parameters including the required crown pillar size, stope size definition, nature and cost of rock support and reinforcement, rock mass characterization, price of commodity, and mining cost used as inputs during the implementation and testing of the model are also outside the scope of this research. Effort is made to use industry technical and economic data for implementation and verification of the developed model to obtain more realistic and practical results. It is important to state that, detailed geomechanical study of rock formation may influence the design and location of the crown pillar (also sometimes referred to as the surface crown pillar).

1.7 Research methodology

To achieve the research objectives, the main challenges that need to be addressed include complexity vs practicality; multiple conflicting objectives; OPUG mining options and interface dynamics; and geotechnical considerations. A kriged block model was developed using GEOVIA GEMS (Dassault, 2020b) and used as input for the optimization model. MATLAB 2018a (Mathworks, 2018) was used as the programming platform to define the formulated framework. IBM ILOG CPLEX Optimization Studio (ILOG, 2015) which is based on the branch and cut optimization algorithm (Holmstrom, et al., 2009) is integrated into MATLAB to solve the formulated MILP problem.

The model was implemented and verified with three experimental case studies of copper and gold resources. The MILP model framework was used to determine the mining option(s) required to exploit the deposit, the transition depth, and the integrated strategic annual production schedules for the case studies. The long-term schedules for developing the primary access, secondary accesses, ventilation openings, and geotechnical rock support and reinforcement for the openings and stopes were generated. The third case study was implemented in Whittle and the results compared with that from the MILP model. The research experimentations compared the selected mining option, annual stripping ratio, production rates, average mineral grade, and NPV of the mining project.

The breakdown of the research tasks completed for the realization of the study objectives are enumerated as follows:

1. Propose and develop an optimization process for open pit-underground mining option(s) and transition planning.
2. Propose and develop a theoretical framework for a MILP model for determining the optimal mining option(s) required to exploit a mineral resource taking into consideration open pit and underground production schedules, 3-D crown pillar positioning (transition depth), mineral extraction sequence, and underground primary access and operational developments, and ventilation development requirement.
3. Integrate stockpile management and geotechnical considerations in terms of rock support and reinforcement in the MILP optimization.
4. Test, calibrate and verify the MILP frameworks with synthetic dataset.

5. Analyze the results obtained using the synthetic datasets and discuss the practicality of the results to further improve the theoretical formulations.
6. Implement the MILP formulations for a real mineral deposit to define the life of mine strategic OPUG production schedules for a mining options evaluation.
7. Assess the impact of applying the MILP formulation and the mining options evaluation workflow based on the determined NPV and practicality of the strategic production schedule generated.
8. Compare the performance of the MILP framework with an industry standard optimization tool.
9. Document the research methodology, workflow and parameter calibration for the model.

A summary of the research methodology is presented in Figure 1-2. This is a schematic representation of the mining options and transitions optimization workflow for evaluating a deep-seated but near-surface deposit amenable to both OP and UG mining. A block model of the given orebody is fed into the formulated MILP framework. The economic and technical parameters required to evaluate the orebody mining options are introduced. The MILP model interrogates the orebody to determine the most suitable mining option, life of mine, and ore extraction strategy by utilizing available processing plant, waste dump, OP stockpile and UG stockpile capacities. When UG mining is part of the preferred mining option, the model also determines the position of the crown pillar, and the schedules for capital development (primary access, ventilation raises and accesses), secondary development (levels, drives, crosscuts), and geotechnical rock support and reinforcement delays in the operational development openings and stopes.

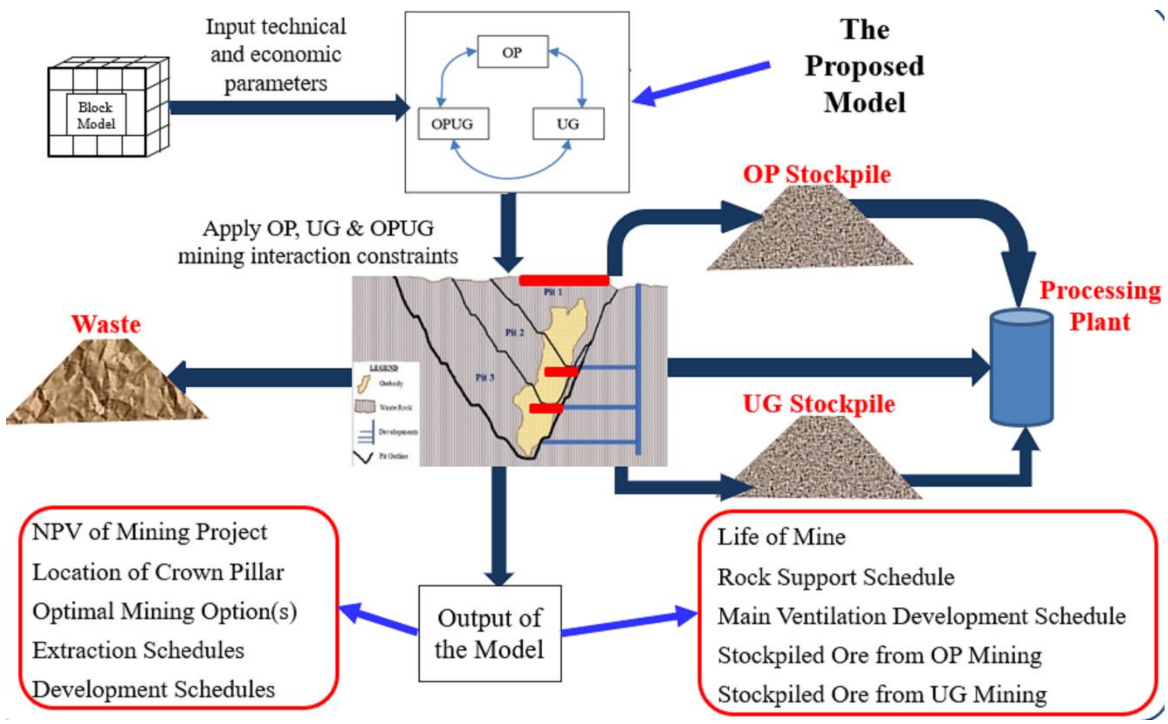


Figure 1-2: Schematic representation of the workflow for evaluating deep-seated deposits amenable to OPUG mining.

1.8 Scientific contributions and industrial significance of the research

The main contributions of this research are the proposed optimization methodology referred to as Competitive Economic Evaluation (CEE) and the development of an optimization framework based on Mixed Integer Linear Programming (MILP) for OP-UG mining options and transitions planning. The MILP framework integrates an unmined 3D crown pillar, primary and secondary developments, ventilation requirements and grade blending constraints in the optimization of the mining options and transitions planning problem. The research further introduces geotechnical requirements and stockpile management in the MILP model for improved practical implementation. The NPV of the mining project is maximized after optimization and the generated mining option(s) schedules satisfy all the specified technical and economic constraints.

The industrial significance of this work includes the introduction of an optimization workflow and a MILP formulation that seeks to enable the mining industry to generate a strategic production schedule for mineral resources amenable to OPUG mining. The simultaneous optimization of mining, processing, ore quality (grade blending), stockpiling, unmined 3-D crown pillar, primary access and operational development, ventilation development, and rock support and reinforcement requirements are desirable for the strategic planning of a mine at the prefeasibility stage. Additional information obtained from the strategic mining options evaluation results complement the selection process of an appropriate stoping method for a typical underground mining operation.

1.9 Organization of the thesis

This thesis is made up of five (5) chapters. Chapter 1 introduces the PhD research. It defines the problem, highlights key assumptions, summarizes the literature review, and presents the objectives and scope of the research, the general methodology, and the contributions of this research.

Chapter 2 provides a summary of the literature review undertaken for this research. Previous optimization methodologies and models on the OP-UG mining options and transitions planning problem were discussed, and the limitations and gaps in this research area have been highlighted. A section of this chapter is dedicated to the discussion of optimization models and their strengths in handling real world problems.

Chapter 3 introduces the proposed optimization process and the mathematical formulation for evaluating mineral resources amenable to OPUG mining options and their transitioning complexes. The unbiased Competitive Economic Evaluation (CEE) optimization process was highlighted. The multi-objective function and mining constraints in relations to the mixed integer linear programming (MILP) model have been documented.

Chapter 4 presents the implementation and verification of the multi-objective MILP model. The model was applied to a large-scale gold deposit to evaluate the rigorousness of the OP-UG mining options and transitions optimization model. Application of the model to other deposits have been published in peer reviewed journals and presented in the Appendices. The sensitivity of selected input parameters in model were evaluated. A comparative study of the developed model and an industrial optimization tool, GEOVIA Whittle, was discussed in this chapter.

Chapter 5 presents the summary and conclusions of the thesis. The contributions of the PhD research are also documented in this chapter. Recommendations for future work on OP-UG mining options and transitions optimization studies are highlighted.

CHAPTER 2
LITERATURE REVIEW

2.1 Background

This chapter presents a summary of mining options classification and mathematical programming models (MPMs), and reviews relevant literature on algorithms and models for open pit-underground (OP-UG) mining options and transitions optimization. The chapter also discusses relevant techniques and methods of mathematical programming, previous works on OP-UG mining options and transitions optimization, and the comprehensiveness and efficiency of existing models in being deployed for complex deposits. Gaps in literature and opportunities that form the rationale for this PhD research and its implementation in the mining industry are further identified. Finally, the chapter highlights the limitations of current models and algorithms for the OP-UG mining options and transitions optimization problem and the application of mathematical programming framework based on mixed integer linear programming (MILP) for rigorous mineral resource extraction evaluation.

2.2 Classification of mining methods (mining options)

Mining is defined as the process of exploiting a valuable mineral resource naturally occurring in the earth crust (Caro, et al., 2007; Newman, et al., 2010). The extraction of mineral resources from the earth crust is classified broadly into two; surface mining and underground (UG) mining. In surface mining, all the extraction operations are exposed to the atmosphere while in UG mining, all the operations are done in the bosom of the earth crust. The main objective of a mineral project development is the maximization of investment returns; the "*golden rule*" of mining or the investor's "*law of conservation*" (Bohnet, 2011). Therefore, adopting the best mining option that maximizes the project's value is a requirement to the establishment of a successful mine. Planning a surface mine is often simpler compared to an underground mine because there are broad similarities between different variations of surface mining as opposed to the variations of underground mining. Thus, planning an underground mine is necessarily complicated by the availability of many different types and variations of mining systems (Bullock, 2011b). These surface and underground mining variations are also generally referred to as classes of mining methods. The classification of surface mining methods and underground mining methods are respectively illustrated in Figure 2-1 and Figure 2-2.

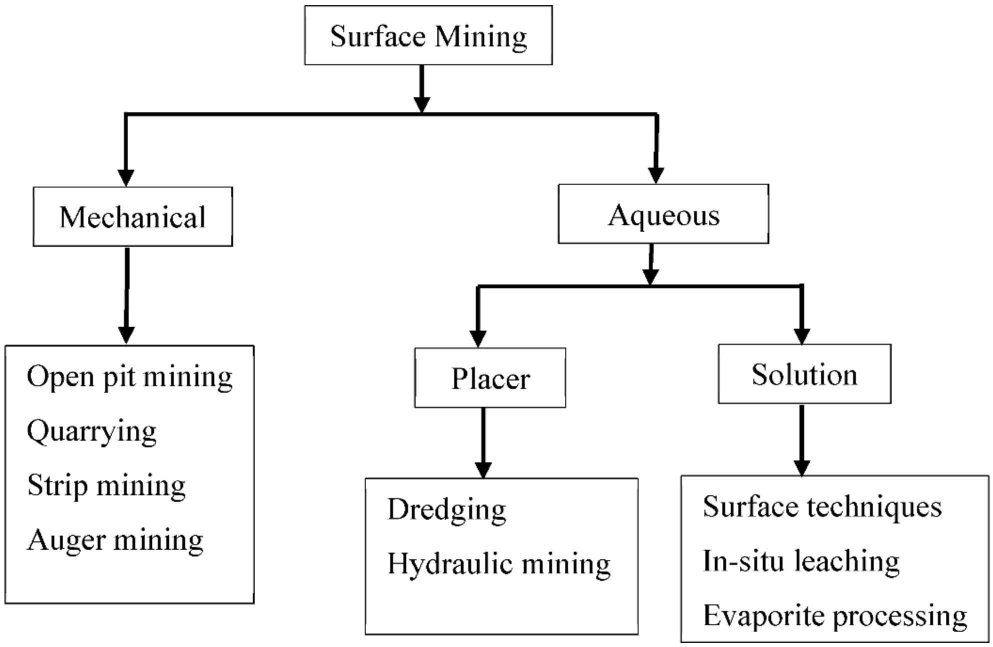


Figure 2-1: Classification of surface mining methods (Adler & Thompson, 2011)

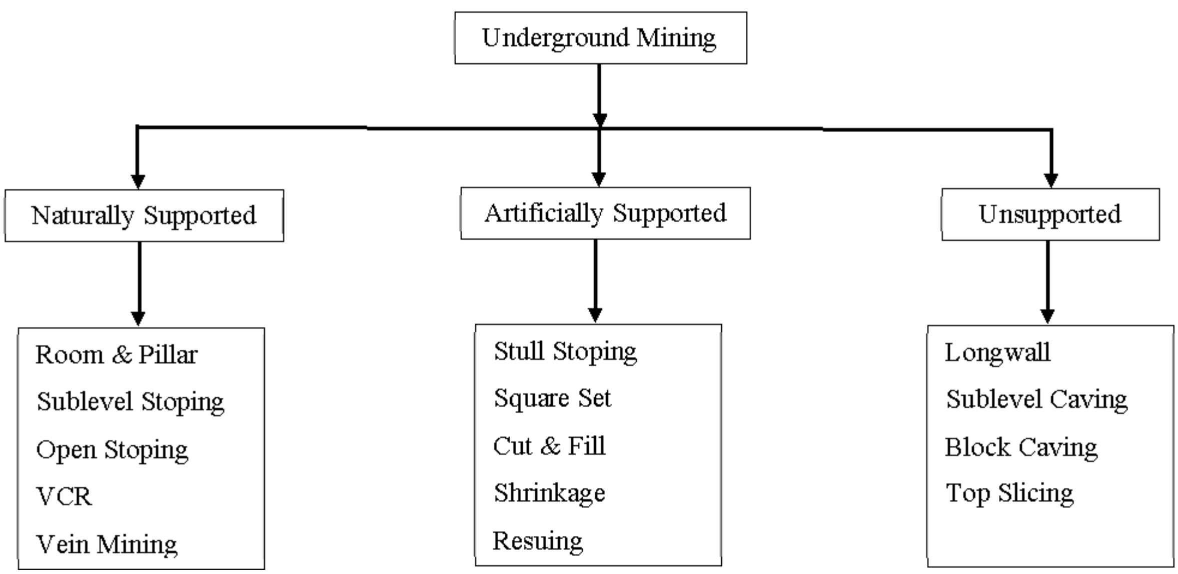


Figure 2-2: Classification of underground mining methods (Adler & Thompson, 2011)

Surface mining methods are broadly classified into mechanical and aqueous extraction methods (Figure 2-1). Mechanical surface mining methods include open pit mining, quarrying, strip mining and auger mining while aqueous surface mining methods include placer mining and solution mining. Placer mining includes dredging and hydraulic mining while solution mining includes surface techniques such as *in-situ* leaching and evaporite processing. Based on the rock formation strength, UG mining methods are broadly classified into naturally supported methods, artificially supported methods, and unsupported methods (Figure 2-2). Naturally supported mining methods include room and pillar, sublevel stoping, open stoping, vertical crater retreat (VCR), and vein mining. Artificially supported mining methods include stull stoping, square set, cut and fill, shrinkage, and resuing while unsupported methods include longwall, sublevel caving, block caving, and top slicing. According to Nelson (2011), some of the factors that must be considered when choosing between surface or underground mining methods include:

1. Extent, shape, and depth of the deposit;
2. Geological formation and geomechanical conditions;
3. Productivities and equipment capacities;
4. Availability of skilled labor;
5. Capital and operating costs requirements;
6. Ore processing recoveries and revenues;
7. Safety and injuries;
8. Environmental impacts, during and after mining;
9. Reclamation and restoration requirements and costs; and
10. Societal and cultural requirements.

2.3 Mineral projects evaluation

When a mineral deposit is discovered, several evaluations are conducted towards the project's viability. The evaluation methods used are broadly grouped into two: positive evaluation methods and normative evaluation methods (Torries, 1998). Positive evaluation methods assess the quantity and quality of the mineral project while normative evaluation methods assess the social and ethical values of the mineral project. Positive evaluation methods deal with investigations related to the geology of the formation, technology required to develop the deposit, investment decisions such as net present value (NPV), internal rate of return (IRR), and options valuations, and financial evaluation to show how funds will be raised and repaid in future. This research focuses on positive evaluation methods for mineral projects with the exception of how funds are raised and repaid for the project.

A mineral property could be described as being at early-stage or advanced-stage exploration, development, defunct, dormant, or production stage (SAMVAL, 2016). Three types of studies are undertaken according to the stage of life of the mineral project under evaluation. These studies are scoping study, prefeasibility study and feasibility study (VALMIN, 2015; CIMVAL, 2019). Scoping study is a preliminary assessment of the technical and economic viability of a mineral property while a prefeasibility study is a comprehensive study of the viability of the mineral project subject to operational constraints at which the preferred UG mining method or OP mining arrangement is established, including an effective mineral processing method.. Feasibility study, on the other hand, is a comprehensive study on the design and cost of the selected mining option for developing the mineral project. Usually, the confidence level associated with a prefeasibility study is lower than a feasibility study while the confidence level for a scoping study is also lower than a prefeasibility study.

The outcome of a prefeasibility study on a mineral property is a mineral reserve that is profitable. Thus, the mineral resource is technically and economically evaluated and if it is profitable, it becomes a mineral reserve otherwise it remains a mineral resource until prevailing factors (commonly referred to as modifying factors) become favorable (CIMVAL, 2019). The details of the evaluation studies for a mineral project depends mainly on the stage of life of the mine and the prevailing regulatory requirements of the region. These detailed evaluation considerations may include geological and geostatistical modeling, geotechnical investigation, mining optimization studies, cost-benefit analysis, equipment selection, rock transportation studies, rock stability and slope requirement assessment, crown pillar location investigation, blasting and fragmentation studies, environmental baseline studies, environmental management and impact studies, and mine closure and reclamation studies. During prefeasibility studies, typical technical and economic considerations include geological, geostatistical and mining optimization investigations. These investigations primarily define the spatial grade distribution of the deposit, and uncovers the size, shape, depth (extent), and orientation of the deposit, and further validate the profitability associated with the mining strategy.

Identifying the preferred mining option (whether OP or UG or both) during prefeasibility studies on a mineral property involves strategic optimization analysis. These analysis ensure much value is attained during the implementation of the resource development plan. When the mineral resource is closer to the earth surface (sometimes with significant outcrop), OP mining evaluation studies are outrightly conducted. When the mineral resource is deeply buried in the earth crust (with no significant outcrop or presence near the earth surface), UG mining evaluation studies are conducted. However, when the mineral deposit is significantly closer to the earth surface and also deeply buried in the earth crust, the deposit becomes amenable to both OP and UG mining. In such cases, the portion of the orebody near the earth surface is evaluated to be exploited by OP mining method to produce early revenue, while the deeper portion is either outrightly evaluated for UG mining or the evaluation is deferred for later years in the future (Opoku & Musingwini, 2013). In general, OP mining methods are characterized by relatively low mining and operating costs, high stripping ratio, and extended time in accessing the mineral ore (Koushavand, et al., 2014; Ben-Awuah, et al., 2016) while UG mining is characterized by high mining and operating costs, high grade ore, and earlier times in retrieving the ore (Finch, 2012; Pourrahimian, et al., 2013; Terblanche & Bley, 2015; Huang, et al., 2020b, 2020a).

In most cases, for mineral deposits amenable to OPUG mining, the UG mining evaluation is not undertaken during prefeasibility studies but conducted in later years when the OP mine stripping ratio increases towards the critical limit. The effect of this traditional evaluation approach is an increased overall mining cost and a potential loss of financial benefits (Elkington, 2013; Breed, 2016). Similary, where UG mining commences at the onset of the mineral project, and later converts or transitions to OP mining, significant financial loss can occur if the global mining strategy was not defined for the entire mineral deposit during prefeasibility studies. Historically, some mining companies that transitioned from OP to UG or vice versa include Telfer, Golden Grove, and Sunrise Dam in Australia (Mawby & Rankin, 2013); Grasberg mine in Indonesia (Freeport-McMoRan, 2016); Akwaaba and Paboase mines of Kinross Chirano Gold Mine in Ghana (Afum, et al., 2019b). In addition, Lac Des Iles mine located in Canada and Newmont Ahafo mine in Ghana both operate OPUG mining operations.

The extraction evaluation of such mineral resources amenable to OPUG mining is referred to in this research as mining options optimization. Mining options optimization has been used by other researchers and professionals to refer to the initiatives or choices undertaken in the extractive industry to expand, change, defer, abandon or adopt strategies for a mining method(s) and sometimes investment opportunities; based on changing economics, technology or market conditions (Shinobe, 1997; Bakhtavar, et al., 2008, 2009a; Marketwired, 2016). The importance of rigorously assessing the economics for a mineral deposit extraction, before deciding on the mining option to adopt is therefore essential to mineral resource planning. This may ensure important decisions of canceling a major OP pushback and transitioning to UG mining or vice versa is known at the onset of the mining project (Bakhtavar, et al., 2009b; Elkington, 2013).

2.4 Evaluation techniques for mining options and transitions planning

The outcome of an evaluation study for a mineral deposit amenable to OPUG mining includes the optimal mining option, strategic extraction plan, and a transition depth or location. The variations of the mining option are independent OP mining, independent UG mining, concurrent OP and UG mining, OP mining followed by UG mining, and UG mining followed by OP mining. The strategic extraction plan includes the sequences of rock extraction and the determination of life of mine and transition depth. The transition depth defines the location or position of the crown pillar. The extraction strategy when OPUG mining is prefered could either be sequential mining or parallel mining or both (Finch, 2012). Respectively, other researchers used the terms simultaneous or non-simultaneous or combined OPUG mining to refer to these same mining options (Bakhtavar, et al., 2017).

Sequential mining is when the mineral deposit is continuously extracted by an independent OP mining method(s) until the pit limit, followed by UG mining method(s), while parallel mining is when the mineral deposit is simultaneously or concurrently extracted by OP and UG mining in the same period or time. Transitioning is the main challenge for OPUG mining projects due to the complexity and implications of where and when to position the crown pillar (or identify the transition depth) (Opoku & Musingwini, 2013). Over the years, five fundamental approaches have been used to determine the transition point or location of the crown pillar (Chen, et al., 2001; Chen, et al., 2003; Finch, 2012; Whittle, et al., 2018). These techniques are (1) biggest economic pit, (2) incremental undiscounted cash flow, (3) automated scenario, (4) stripping ratio, and (5) opportunity cost analyses.

The biggest economic pit approach is a technique that evaluates the discovered mineral resource for OP mining and any portion of the deposit that falls outside the optimal pit limit is later evaluated for UG mining. This technique is a traditional method that is simple and widely used in assessing deposits that show significant features of being exploited by OPUG mining. The technique asssesses and compares the marginal cost of stripping uneconomic rock material to the marginal revenue from processing one unit of associated mineralized rock material. When the marginal difference is positive, the optimal pit outline enlarges but a negative difference terminates the pit (Finch, 2012).

Incremental cash flow approach is an evaluation technique that assesses the mineral resource by comparing the marginal OP profit per depth to the marginal UG profit. When the marginal OP profit is equal to or less than the marginal UG profit, OP mining terminates and the associated depth then acts as the transition point. This transition depth is typically shallower compared to the biggest economic pit technique (Finch, 2012). This method assumes that UG mining profits do not depend on the depth of operation, therefore, there will be a point where the marginal profits from UG mining operation will exceed that from OP mining operation.

The automated scenario analysis approach accounts for discounting unlike the incremental undiscounted cash flow approach. It is based on the premise that, per an equivalent unit of throughput, UG mines are characterized by high cut-off grades and therefore have higher cash flow compared to OP mines for the same throughput. The approach is implemented by compiling schedules for OP and UG mining and comparing the computed NPV for each potential transition point. Thus, a set of transition points are evaluated and the OPUG mining arrangements that offers the highest NPV is selected for further analysis and design (Finch, 2012). This method is time consuming and complex.

The stripping ratio analysis features the use of allowable stripping ratio (ASR) planned by mine management for the OP mine and overall stripping ratio (OSR) computed per depth for the OP mine to determine the transition depth (Chen, et al., 2001; Chen, et al., 2003). The stripping ratio is expressed by this relation with emphasis on exploitation cost of 1 tonne of ore in UG mining and in OP mining, as well as, removal cost of waste in relation to 1 tonne of ore extracted by OP mining. As OP mining deepens, the stripping ratio usually increases, increasing the overall mining cost. An overall stripping ratio (OSR) is calculated and used to determine the breakeven point of the OP mine relative to its depth. The OP mine transitions to UG mine when OSR is equal to the ASR established by management of the mining project.

The opportunity cost technique is an extension of the LG algorithm that optimizes the OP ultimal pit while considering the value of the next best alternative UG mining option. This approach also employs the strength of the undiscounted cashflow technique and assumes that rock materal in the transition zone will be mined by UG method if not mined by OP method (Whittle, et al., 2018). The methodology ensures that a minimum opportunity cost is achieved for the selected optimal mining option at the expense of the unselected mining option.

An evaluation technique that leverages the advantages of these fundamental approaches were recently introduced by Afum & Ben-Awuah (2019). This approach is referred to as Competitive Economic Evaluation (CEE) technique. The CEE process evaluates each block of the mineral deposit and economically decides: (a) blocks suitable for OP mining, (b) blocks suitable for UG mining, (c) unmined blocks and (d) unmined crown pillar simultaneously. The CEE optimisation strategy is an unbiased approach that provides fair opportunity to each mining block for selection by a mining option.

2.5 Notable research on mining options and transitions planning optimization

Historically, a cash flow and NPV based algorithm was introduced in 1982 to examine the OP-UG transition interface (Nilsson, 1982). Subsequently, in 1992, Nilsson reviewed the previous 1982 algorithm and produced a new algorithm that considers the transition depth as a critical input for evaluating deposits amenable to OP and UG extraction (Nilsson, 1992). An algorithm for determining the depth of transition was thereafter introduced by Camus (1992). This algorithm was presented based on the economic block values for OP and UG extraction methods. The technique involves the implementation of the OP extraction algorithm taking into consideration an alternate cost due to UG exploitation. In 1997, a model was developed by Shinobe (1997) that enables the mine operator to determine the optimum time of conversion based on discounted cash flow (DCF) techniques, and cost estimation equations according to O'Hara & Suboleski (1992). The model assumed that the UG resources were confined, and their extraction is technically feasible. Whittle Programming Pty developed an applied approach referred to as quantified operational scenarios for interfacing OP-UG mining methods in the OP to UG transition problem (Tulp, 1998).

In 2001 and 2003, an evaluation technique based on allowable stripping ratio (ASR) with a mathematical form for the objective function was introduced by Chen, et al. (2001) and Chen, et al. (2003). Volumes of ore and waste within the final pit limit were assumed to be a function of depth for determining the transition point. A heuristic algorithm based on economic block values of a 2-dimensional block model that compares the total value when using OP mining for extracting a particular level to UG mining of the same level was developed for the mining options problem (Bakhtavar, et al., 2008). The algorithm was based on the fact that typical ore deposits showing significant outcrops can be potentially exploited by OP mining followed by UG mining. Thus, mining is completed at the initial levels by OP extraction methods while transitioning to UG mining as the operation deepens from the middle levels.

Until 2009, only a few available algorithms could solve the optimal transition depth problem with some limitations. Bakhtavar, et al. (2009a) developed a model for solving the transition depth problem by modifying Nilsson's algorithm. The model generated two different mining schedules for OPUG mining. Each mining method is employed to extract mining blocks on the same level in series. The incremental NPVs of the extraction for each level blocks are compared and if the incremental NPV of the OP mine is larger than that of the UG mine, the algorithm transcends by adding the next series of level blocks to the previously optimized mining schedule; and the incremental NPV of the OPUG mine is compared again. Evaluation results from the first level to the last level is monitored to identify the optimal transition depth (level) for the establishment of a crown pillar. The remaining portions of ore below the crown pillar were evaluated and extracted utilizing UG stoping method(s).

Another heuristic model based on the economic block values of OPUG extraction was developed by Bakhtavar, et al. (2012). This new model was an improvement over the previous model by factoring in the NPV achieved through the mining process. Thus, for any level of the block model, the computed NPV from OP mining operation is compared to the NPV derived from UG mining operation for the same level. Although the model solves the transition problem using some technical and economic parameters, it does not consider mining and processing capacities (equipment requirements), and uncertainties in the geological and geotechnical characteristics of the orebody. Uncertainties of ore grades were considered in subsequent mathematical programming frameworks for the transition problem (Bakhtavar, et al., 2017; MacNeil & Dimitrakopoulos, 2017). However, the implementation of these existing models have always assumed the mining options and transitions planning scheme to be a stepwise process and hence implements their solution strategy as OP mining followed by UG mining either in parallel (simultaneous) modes or sequential (non-simultaneous) modes (Ben-Awuah, et al., 2016; Bakhtavar, et al., 2017; King, et al., 2017; MacNeil & Dimitrakopoulos, 2017; Whittle, et al., 2018). The challenge to this assumption also forms the basis of this research. Table 2-1 shows a matrix comparison of notable research on the OP-UG mining options optimization problem in the last decade.

As seen in Table 2-1, previous research on the mining options and transitions problems mainly focuses on the determination of transition depth before optimizing the production schedule for the mining arrangement. Formulating a model that allows the optimizer to select the most suitable extraction strategy, including independent OP, independent UG, simultaneous OPUG, sequential OPUG, or combinations of simultaneous and sequential OPUG, to exploit any deposit under consideration will be a major addition to the mining industry. It is important to note that in general previous research employed NPV and feasible production schedule as major performance indicators for the developed models and algorithms.

Table 2-1: Notable research on the OP-UG mining options optimization problem for the past decades

	Name of Author(s)	Whittle et al.	MacNeil & Dimitrakopoulos	King et al.	Ben-Awuah et al.	De Carli & de Lemos	Ordin & Vasil'ev	Dagdelen & Traore	Roberts et al.	Opoku & Musingwini	Bakhtavar et al.
	Year	2018	2017	2016	2016	2015	2014	2014	2013	2013	2012
	Research Focus	Transition depth & production schedule	Transition Depth - Instances	Transition Depth - Instances	Assessment of Transition Problem	Transition Depth - Instances	Transition Depth - Dynamic	Transition Depth	Assessment of Transition Problem	Transition Depth - Dynamic	Transition Depth
Approach Used	Model / Algorithm	Modification of maximum graph closure method	Stochastic Integer Model	MILP	MILP		Dynamic Programming - lag, trend, nonlinear	MILP - OptiMine® used to optimize the transition problem	MILP - OP & UG		(0-1) Integer Programming
	Software Application				Evaluator	Studio 3 & NPV Scheduler		OptiMine, Whittle, Studio 5D and EPS	Blasor for OP optimization; COMET for OP schedule; Evaluator for UG optimization	Whittle for OP optimization; XPAC for OP schedule; Datamine's MRO for UG optimization	
Outputs / Performance / Transition Indicators	NPV	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	IRR				Yes	Yes					
	Mining Recovery					Yes					
	Avg. ROM Grade								Yes	Yes	
	Metal Price to Cost Ratio									Yes	
	Production Rate						Yes		Yes	Yes	
	Stripping Ratio					Yes	Yes			Yes	
	Mine Life					Yes	Yes	Yes	Yes		
	Revenue						Yes				
	Mining Cost								Yes		
	Production Schedule		OP & UG		OP, UG & OPUG		Standalone for OP & UG	Standalone for OP & UG			
	Commodity		Gold	Confidential	Gold-silver-copper	Gold	Coal mining – kimberlite pipe	Gold	Iron	Gold - 4 different deposits	Hypothetical
	Notable Remarks	Mining sequence not considered	NPV compared to deterministic model	Optimality gap not improved						Deterministic not good	

2.6 Factors influencing mining options and transitions planning

The factors influencing mining options and transitions planning are also referred to as transition indicators. These transition indicators mostly depend on the shape and size of the mineral deposit and quantity of high grade ore present in the selective mining units. These indicators usually vary for different orebodies and commodities, and the philosophy of mine management. The indicators can be broadly grouped into geologic, operational, geotechnical, economic, and mine management strategy on global business economic outlook (Gabryk, et al., 2012; Opoku & Musingwini, 2013). These transition indicators are key to the identification and development of the set of constraints required for optimizing the OP-UG mining options and transitions planning problem (Roberts, et al., 2013; Ordin & Vasil'ev, 2014; Ben-Awuah, et al., 2016).

Some of the indicators to consider in OP-UG transition optimization are mining recovery, commodity price, mineral grade, cost of extracting ore and stripping waste, mining and processing capacities, and UG dilution (Musendu, 1995). Subsequent research highlighted some additional important parameters that must be considered during transition planning (Luxford, 1997). These include workforce requirement, shape and size of the orebody, and geotechnical properties of the rock formation. Economic parameters such as discounting rate of expected cash flows and OPUG mining costs, and primary factors including the competence of mine manage, characteristics of the geology of the orebody, stripping ratio, productivity rate, and capital cost requirements for the UG mining option will affect the decision for OP-UG transition as well (Hayes, 1997; Roberts, et al., 2013).

The OP-UG mining options and transitions planning problem becomes complex when critical indicators such as crown pillar positioning and other essential underground mining constraints including primary and secondary access development, ventilation development, and geotechnical requirements for the development openings and stopes are integrated into the optimization framework. These essential UG mining developments were previously not considered in the optimization process because of computational complexities and the difficulty of integrating them with open pit mining operations (Nhleko, et al., 2018). The importance of incorporating crown pillar positioning, geotechnical support activity sequencing, underground infrastructure development, and mine management strategy in OP-UG mining options optimization studies are essential to attaining realistic mine plans (Roberts, et al., 2013; Bakhtavar, et al., 2017).

When investigating OP-UG mining options, it is important to model all constraints and factors that have direct and remote impact on the economic and technical feasibility of the project. The strategic production plan is generally subject to several constraints that enforce the extraction sequence, blending requirements, and mining and processing capacity requirements for practical implementation.

2.7 Crown pillar and rock strength considerations in mining options and transitions planning

A crown pillar is the horizontal part of a rock formation between the first upper stope of an underground mine and an open pit excavation. A crown pillar is often provided to prevent the inflow of water from the OP floor to the UG workings, while reducing surface subsidence and caving. Finding the most suitable location of the crown pillar in a combined mining method of OPUG operations, is one of the most interesting problems for mining engineers today especially when the crown pillar must collapse (Bakhtavar, et al., 2012). It is however worthy to note that crown pillar usage in the OPUG mine is not universal as sometimes it is desirable for the crown pillar to collapse during the life of mine (Whittle, et al., 2018).

The positioning of the required crown pillar in the surface-underground mining options and transitions planning problem is key to the operations of such mines. Some researchers preselected the depth of the crown pillar (transition depth) before evaluating portions above the crown pillar for OP mining and portions below the crown pillar for underground mining (Ordin & Vasil'ev, 2014; Ben-Awuah, et al., 2015; De Carli & De Lemos, 2015; Ben-Awuah, et al., 2016; King, et al., 2017). This may lead to suboptimal solutions and will require evaluating multiple crown pillar locations in a scenario-based approach. A few authors have attempted to incorporate the positioning of the crown pillar in the optimization process (Whittle, et al., 2018; Afum & Ben-Awuah, 2019; Afum, et al., 2019a). Their models were good improvements over previous works but were missing some constraints like ventilation requirement and rock strength properties required for practical implementation. The transition from open pit to underground mining is a complicated geomechanical process which requires the consideration of rock mass properties (Fengshan, et al., 2012; Yardimci, et al., 2016).

In OP to UG transition, the challenge of deformation, displacement, and rock formation stability need to be meticulously investigated. These will ensure their effect on production, worker and equipment safety, and the working environment of the UG operation are highly secured (Ma, et al., 2012). When the crown pillar thickness is large, considerable quantity of mineral deposit is lost but when the pillar is undersized and thin, the probability of pillar failure and instability of the mine is eminent (Tavakoli, 1994). According to Ma, et al. (2012), ground movement and deformation study of OP mines such as geometrical, geomechanical, and analytical analyses are important in the transition study.

Optimizing the crown pillar dimensions and positioning is particularly significant in UG mining operations. Estimation of the optimum crown pillar thickness is a complicated study which involves the experience of the engineer and the use of numerical and empirical techniques (Tavakoli, 1994). Several parameters influence crown pillar stability. These parameters are broadly grouped into mining and geological (Brady & Brown, 2006). The mining parameters include the crown pillar geometry; stope surroundings; supporting methods employed (including backfilling); sequence of mining operations and stress redistribution resulting from material extraction. The geological parameters include the strength and deformation characteristics, and inclination of the hangingwall, footwall and orebody in general; geometry of the mineral deposits; virgin stress conditions and properties of the contact regions between the ore and country rock.

Crown pillar placement invariably defines the interface of the OP to UG transition. Appropriately defining a suitable location of the crown pillar is fundamental to the mining options optimization problem. Leaving an appropriate crown pillar thickness will minimize the destructive interference between the OP and UG mining operations, while maximizing ore recovery. The assumption of a uniform crown pillar of known height below the optimized pit bottom is common (Bakhtavar, et al., 2012). Subsequently, an ad-hoc branch-and-bound technique to exhaustively search the appropriate locations for crown and sill pillar placements before computing the relaxed linear programming (LP) model for the OPUG mining problem was developed by King, et al. (2017). A rounding heuristic was used to transform the relaxed LP solution into IP solution using satisfactory values of the objective function.

The IP solution was used to terminate the several potential crown and sill pillar placements and hence decrease the computational time required. The researchers further concluded that, only 40 out of the over 3500 crown and sill pillar placement options had a relaxed LP objective function value greater than the best-known IP objective function value.

In a recent research, MacNeil & Dimitrakopoulos (2017) pre-determined four possible crown pillar positions while evaluating a gold deposit for OPUG mining. Their approach resulted in four distinct transition points acting as potential candidates for a crown pillar. The size of the crown pillar however remained the same for each candidate, as the location is varied. This assumption does not support the geotechnical variability of the rock formation. According to King, et al. (2017), the crown pillar is usually positioned by industrial experts based on: (1) the optimal OP mining limit, or (2) the largest undiscounted profit resulting from the extraction technique for each 3-D discretization of the orebody and country rock. This is done after a detailed geotechnical assessment of the rock formation in the immediate vicinity of the crown pillar.

The stability and placement of the crown pillar or transition interface significantly affects the NPV of a mining project. Numerical simulation and machine learning are among the most effective techniques for studying the crown pillar stability (Wang & Zheng, 2010; Shi, et al., 2011; Bo-lin, et al., 2014). The Fast Lagrangian Analysis of Continua (FLAC) software based on numerical modeling and a hybrid support vector regression (v-SVR) analysis based on supervised machine learning algorithm are used to examine deformation characteristics of surrounding rocks in complex conditions of OP to UG mining transition. These analyses can provide approximation of rock strength properties to be incorporated in the transitions planning optimization problem.

2.8 Existing evaluation tools for mining options and transitions planning

Most of the existing optimization models and algorithms for evaluating OP-UG mining options and transitions planning have been incorporated into software packages for easy implementation. These are mostly based on the fundamental evaluation techniques discussed in Section 2.4. Some of the models and algorithms are Lerchs-Grossman algorithm, Seymour algorithm, floating cone technique, dynamic programming, neural network, theory of graphs and network flow algorithm. These models and algorithms are used in software packages including NPV Scheduler, Whittle Four-X, MineSched, Vulcan, MineScape and MineSight (Achireko, 1998; Ordin & Vasil'ev, 2014).

GEOVIA Whittle® software which is based on the Lerchs-Grossman algorithm is commonly used to optimize the OP limit prior to assessing the remaining mineral resource outside the pit boundary for underground extraction (Opoku & Musingwini, 2013). Similarly, Blasor pit optimization software which is developed based on a combinatorial mathematical programming model is also used to identify the independent OP mining outline before UG extraction evaluation for remaining mineral resources (Roberts, et al., 2013). Mathematical programming frameworks based on integer programming (IP), mixed integer linear programming (MILP), and dynamic programming models have been formulated and implemented with commercial optimization solvers for the transitions planning problem (Dagdelen & Traore, 2014; Ben-Awuah, et al., 2016; Bakhtavar, et al., 2017; MacNeil & Dimitrakopoulos, 2017).

After obtaining the OP outline and thereafter knowing the extent of the UG mining limits, production schedules are generated for each mining option. Software packages based on heuristics and mathematical programming models including GEOVIA Whittle, OptiMine®, and COMET® have been used to produce strategic production schedules for the OP portion of the mine (Opoku & Musingwini, 2013; Dagdelen & Traore, 2014; Ben-Awuah, et al., 2016). Similarly, software packages including Snowden's Evaluator, XPAC®, and OptiMine® have been used to produce the strategic production schedule for the UG portion of the mine (Opoku & Musingwini, 2013; Roberts, et al., 2013; Dagdelen & Traore, 2014; Ben-Awuah, et al., 2016; Bakhtavar, et al., 2017; King, et al., 2017). A software package that interrogates a mineral deposit amenable to OPUG mining and simultaneously determines the preferred optimal mining option, the transition depth, and the strategic production schedule for each mining option in a global optimal solution provides the basis for this PhD research.

2.9 Limitations of current models and algorithms for OP-UG mining options and transitions planning

Primary challenges to the mining options optimization problem include the optimization approach used, the exhaustive consideration of contributing variables to the models and algorithms, and geotechnical considerations in defining the transition interface and its contribution to the mining operation. Progressively integrating geotechnical models in strategic mine plans at the prefeasibility stage similar to how geologic models are incorporated will improve the reliability of the mine plan (Jakubec, 2001; McCracken, 2001). However, the information required to produce a detailed geotechnical model at the prefeasibility stage is limited and therefore difficult to model. Some of the limitations with current models and algorithms for OP-UG mining options optimization include one or more of the following:

1. Consideration of rock support and reinforcement in the optimization process;
2. Consideration of essential infrastructural development in the optimization process;
3. Consideration of stochastic variables such as grade and metal price;
4. Comprehensiveness and efficiency of models; and
5. Optimality assessment.

2.9.1 Consideration of rock support and reinforcement in the optimization process

Rock support and reinforcement are essential to the stability of openings during the development of an underground mine. The term support generally refers to the various types of geotechnical rock support used to protect underground workers and may include steel mesh, shotcrete, fibrecrete, and a variety of types of steel straps. Reinforcement on other hand refers to the various types of rock reinforcement to help prevent rock movement and may include a variety of types of rockbolts, cablebolts, rebar, and dowel. Cablebolting in stope development can particularly be very costly and may introduce considerable time delays. Most cablebolts need 30 days for the portland cement grout to properly set for the cablebolts to be fully functional.

Several authors have acknowledged the importance of incorporating geotechnical constraints to the OP-UG transition problem (King, 1999; Bakhtavar, et al., 2008; Bakhtavar, et al., 2012; Finch, 2012; Roberts, et al., 2013; Ordin & Vasil'ev, 2014; MacNeil & Dimitrakopoulos, 2017). To verify the impact of geotechnical constraints on the optimal solution, Roberts, et al. (2013) recommended that, such constraints need to be incorporated in subsequent studies. Moving beyond determining transition depth and incorporating geotechnical parameters in locating and sizing the transition zone will have direct impact on the feasibility and sustainability of the OP-UG mining project.

These existing models do not integrate the geomechanical classification of the rock formation in the surface-underground mining option and transition problem. Although the geotechnical characteristics of the rock formation are significant to underground mining operations, their added complexities make it difficult to be included in the surface-underground mining option and transition optimization models. According to Bullock (2011b), mine planning is an iterative process that requires looking at many options and determining which, in the long run, provide the optimum results. Using such iterative process could lead to some inferior solution(s) or sub optimal solution(s) that do not constitute the global optimal solution.

Rock support and reinforcement in the development openings and stopes will increase the operational costs and time (delay the mine life), and further affect the quantity and sequence of rock material extracted from the stopes to the processing plants. To incorporate the rock formation's strength into the formulation, rock mass classification systems (Kaiser & Cai, 2012; Abbas & Konietzky, 2015) such as the Geological Strength Index (GSI), Rock Structure Rating (RSR), Rock Mass Rating (RMR), and Q system are determined to characterize the rock formation, and then Kriging is applied for spatial interpolation in order to populate the block model. According to Abbas & Konietzky (2015), these classification systems could be grouped as qualitative or descriptive (e.g. GSI) and quantitative (e.g. Q system, RMR, and RSR) with RMR being more applicable to tunnels and mines. According to Kaiser & Cai (2012), data obtained from the geology and geomechanics of the rock formation are fundamental to mine planning and development designs. This is because the behavior of the rock formation varies in the mine and therefore rock mass domaining based on stress data, and geology and geometric data becomes essential. Knowledge of the strength of the rock mass and its behavior are important for the engineering design of all kinds of support for underground excavations (Edelbro, 2004; Ozturk & Nasuf, 2013).

2.9.2 Consideration of essential infrastructural development in the optimization process

The typical UG mine is interspersed with important infrastructure development that ensures the facilitations of the UG mining operations. These infrastructures include but not limited to primary access development, secondary access development, ventilation development, ore pass development, sumps, maintenance and refuge chambers, and fuel station bays. Primary access developments are usually the main development that links the entire UG mine to the surface and could be vertical or inclined shaft(s), decline(s) or adit(s) or tunnel(s). The shaft(s) are often equipped with facilities to transport humans (workers) and materials for the UG operations. Secondary access development includes the construction of lateral openings such as levels, ore and waste drives, crosscuts, etc. to link the UG operational activities to the primary access(es) while ventilation development often incorporates a series of bored raises and lateral drive development, and the construction of ventilation controls. Ventilation controls are a range of objects such as regulators, doors and walls which need considerable time and money to create. Ore pass development entails the construction of vertical or near vertical openings to link the various UG levels to the main ore hoisting station. Similarly, sumps, maintenance and refuge chambers, and fuel station bays are constructed to enable the removal of water being used for the UG operations and the provision of several essential services to operating equipment UG rather than they being transported to the surface for such services.

2.9.3 Consideration of stochastic variables

In recent studies, the industrial practice has been that, the production for OP mine and UG mine are independently scheduled and merged into one for the OPUG mine. According to King, et al. (2017), this approach creates a myopic solution. However, the discussions of their approach were limited to open stoping and extraction sequence. No stochastic variables like grade and price uncertainty were included in their model. The methodology in handling the transition problem require further work to handle the applicability, accuracy and reduction of the optimality gap was further acknowledged. Grade uncertainty has been identified to have a significant impact on the determination of the transition point (Chung, et al., 2015).

Although MacNeil & Dimitrakopoulos (2017) incorporated grade uncertainty in their work, they further identified some important notable geological uncertainties such as the rock formation, metal content and relevant rock properties and their impact on the strategic long term planning of a mining project. MacNeil & Dimitrakopoulos (2017) further recommended that, financial uncertainty should be incorporated into future studies to improve on their solution method. Ben-Awuah, et al. (2016) did not consider uncertainty in their model formulation and further recommended that pre-production capital expenditure (CAPEX) and geological uncertainties should be added to the mining options evaluation. By considering stochastic variables in a risk-based mining options optimization framework, mine plans that can stand the test of time can be generated.

2.9.4 Comprehensiveness and efficiency of models

Comprehensiveness and efficiency of the models relates to the ability of the optimization framework to exhaustively formulate various scenarios of the OP-UG mining options problem and deliver practical results in a reasonable time frame. Such models are exhaustive and are applicable in a wide range of mining systems. Stacey & Terbrugge (2000) indicated that a complete model for handling the transition problem remained a challenge. They further reckoned that the transition study for a mine should start at the onset of the mine life and not be deferred to latter years since the planning and implementation could sometimes take up to 20 years for completion. Shinobe (1997) developed a software based on a mathematical programming model for this challenge but assumed that the existence of underground reserves has been confined and that their extraction is technically feasible. He later recommended that the results of the program should be viewed only as a preliminary level indication of the economics of underground conversion. No final decision to proceed with the conversion should be taken, solely based on the program's output.

Majority of the current work on the transition problem lack some constraints and solution to a more general problem. This includes consideration of the design capacities and depth as primary indicators for transition in the joint evaluation problem (Ordin & Vasil'ev, 2014). The approach however was limited in scope in relation to a more generic framework. NPV curves and transition depth for OP to UG mining for the Botuobinskaya pipe deposit were generated to solve the transition problem. From their results, the total NPV of the OPUG mining operation was higher than the standalone NPVs of OP and UG mines for the same mining depth. In subsequent research, re-handling cost proved to be insignificant when incorporated into the transition model (King, et al., 2017). Additionally, it was observed that there exist undesirable fluctuations in the OPUG production schedules that must be smoothened to achieve a more practical solution. In their work, MacNeil & Dimitrakopoulos (2017) incorporated the constraints for mining, processing, metal content and precedence relationships in their model. According to the constraints affecting the transition problem identified in the works of Opoku & Musingwini (2013), those constraints considered are not exhaustive since UG mining capital expenditures were not considered in the model application.

2.9.5 Optimality assessment

Optimality assessment is a real challenge to current heuristic and meta-heuristic models and algorithms for OP-UG mining options optimization. The underlying optimization approach fundamentally affects the optimality of the resource evaluation. Unless additional steps are put in place to define an upper bound, the extent of optimality of the solution from these models is unknown. Some of the current models can solve the transition problem, usually producing near optimal solutions (Bakhtavar, et al., 2012; Finch, 2012). These existing models and algorithms assume open pit mining operations must surely be an option in the evaluation process and usually precedes the underground mining. Similarly, few models assess the resource with the assumption that underground mining could precede open pit mining. However, a resource assessment that is devoid of the traditional arrangement of the mining options but allows the optimization process to decide the choice and sequence of the mining option(s) is key to achieving global optimal solution. Bakhtavar, et al. (2009a) noted that, few methods (algorithms) have some disadvantages and deficiencies in finding the optimal transition depth. According to Askari-Nasab, et al. (2011), heuristic algorithms may not produce optimal solutions. This could lead to loss of major financial benefits resulting from implementing sub-optimal mine plans.

Finch (2012) also highlighted that, the effort of producing OPUG mining schedules for the possible candidates of transition interface for all the various mining and processing capacities could be time consuming and costly. The process commonly leads to the generation of sub-optimal solutions since the problem is not thoroughly investigated. According to Richard & Stefan (2011), designs optimized for deterministic cases are often sub-optimal when uncertainties are recognized and their effects understood. By applying mathematical programming models with current high computing resources (Huang, et al., 2020b), optimal or near optimal solutions with known optimality gap can be obtained for the OP-UG mining options optimization problem in a practical time frame and at an acceptable computational cost.

2.10 Modeling techniques for optimization problems

The literature review conducted for this study indicates that mining options and transitions optimization problems are modeled and solved using different evaluation techniques. Understanding the various classifications of optimization models and their advantages are essential to the development of an optimization framework capable of handling the notable limitations and gaps associated with the current models and algorithms for surface-underground mining options and transitions optimization (SUMOTO) problems. These optimization models may be grouped into four broad types according to how well they are able to define a given problem: (1) operational exercise models, (2) gaming models, (3) simulation models, and (4) analytical or mathematical programming models. Simulation and analytical models are widely used for mining optimization problems due to their practicality, rigorousness and efficiency (Ben-Awuah, 2013; Maremi, 2020). Table 2-2 shows classification of certainty and uncertainty models.

Table 2-2: Classification of certainty and uncertainty models (Orlin, 2017)

Class	Strategy evaluation	Strategy generation
Certainty	Deterministic simulation Econometric models Systems of simultaneous equations Input-output models	Linear programming Network models Integer and mixed integer programming Nonlinear programming Control theory
Uncertainty	Monte Carlo simulation Econometric models Stochastic processes Queueing theory Reliability theory	Decision theory Dynamic programming Inventory theory Stochastic programming Stochastic control theory

2.10.1 Operational exercise models

Operational exercise is a modeling method that relates variables to the actual environment where the study decision will be applied (Orlin, 2017). This method has the highest degree of realism when compared to other techniques of modeling. It is usually prohibitive, expensive to implement, and associated with extreme challenge when alternatives must be assessed. This then leads to sub-optimization of the final solution. A human decision-maker forms part of the modeling processes of operational exercises.

2.10.2 Gaming models

Gaming is a modeling approach constructed to represent more simple or abstract entities in the real world (Orlin, 2017). This method gives the mine planner the opportunity to try several variations of the decisions to make. A human decision-maker is part of the modeling processes for gaming models.

2.10.3 Simulation models

Simulation models provide several ways to assess the performances of alternatives outlined by the planner. They do not allow much interferences from human interactions during the computational analyses stage of implementation (Orlin, 2017). Simulation models could sometimes be compared to gaming models. However, they involve the application of logical arithmetic performed in a particular sequence usually by computer programs. If the problem is exclusively defined in an analytical form, much flexibility and realistic results are attained. This is essential if uncertainties are critical components of the decisions being made. A human decision-maker is external or not part of the modeling processes of simulation and analytical models.

2.10.4 Analytical or mathematical programming models

The fourth model category is analytical models. These models represent the problem completely in mathematical forms, usually by means of a criterion or objective subject to series of constraints that impact on the decision being made (Orlin, 2017). The mathematical formulation helps to determine the optimal solution in the presence of the set of constraints. Although mathematical models highly simplify the problem, they are less expensive and easy to develop.

Mathematical programming is a significant method when decision variables must be quantified for planning. The problem is defined with mathematical expressions in a well-defined structure to find an optimal solution based on performance evaluation criteria including cost, profit and time. The optimal solution is obtained from a feasible region of alternative results. The expressions are parameters (input data) and variables which represent the optimization results or outcome. When multiple criteria decisions are required for any complex problem, a multi-objective mathematical programming model involving more than one objective function is deployed simultaneously. The main advantages of MPMs are: a) comparatively simple with high approximations of complicated problems, and b) ability to search the feasible solution spaces among competing variables and alternatives (Chandra & Grabis, 2016). Common MP techniques are stochastic programming, deterministic programming, dynamic programming, LP, nonlinear programming, IP, and MILP.

2.10.4.1 Stochastic programming

Stochastic programming is a special case of programming in which some of the constraints or parameters depend on random variables. This type of programming is used to solve problems that involve uncertainty and allow stochastic variables to be accounted for (Chandra & Grabis, 2016).

2.10.4.2 Deterministic programming

Deterministic programming is a form of MP that is rigorous and solves problems in finite time. The optimization model is deterministic when the parameters considered in the model are known constants (Orlin, 2017). The method is useful when global solution is required and is extremely difficult to find a feasible solution.

2.10.4.3 Dynamic programming

Dynamic programming is a form of mathematical programming used to solve multistage complicated problems (Chandra & Grabis, 2016). A large-scale problem is simplified by breaking it down into simpler sub-problems in a nested recursive manner as opposed to deterministic programming.

2.10.4.4 Linear programming

This is a distinct case of convex programming where the model objective and the equality and inequality constraints are expressed as linear functions. The feasible solution set is usually a polytope or polyhedron with connected sets of polygonal faces and convex. The optimal solution could be a single vertex, edge or face, or even sometimes the entire feasible region when dealing with high dimension problems. Typical LP solutions could arrive as infeasible or unbounded (Nocedal & Wright, 2006).

2.10.4.5 Nonlinear programming

Nonlinear programming is a special case of programming which could be convex or nonconvex with nonlinear objective functions or nonlinear constraints or (Nocedal & Wright, 2006). Some assumptions are often made on the shape and function behavior when solving nonlinear programming problems (Orlin, 2017). Nonparametric and simulation-based optimization techniques have been used to explicitly solve models with nonlinear constraints.

However, computational feasibility have been a major disadvantage in the development and implementation of MPMs (Chandra & Grabis, 2016).

2.10.4.6 Integer programming

Integer programming (IP) is a special type of LP where some or of all the decision variables are constrained to take on integers and therefore not continuous. If all the variables are discrete or integers or binaries, the model is referred to as pure integer programming. Essentially, IP problems are hard problems because they are difficult to solve and therefore referred to as combinatorial analysis than LP (Orlin, 2017).

2.10.4.7 Mixed integer linear programming

This is a special form of IP in which some decision variables are constrained to take on integers and others continuous. Continuous variables indicate that the variables could take on fractions as part of the solution regime. Due to the strength of MILP formulations, it is proposed as the model for solving the SUMOTO problem in this research. The MILP formulation structure is well defined with objective function, and a set of constraints to achieve the mining decisions that are usually made up of continuous variables and integers. The formulated MILP model in this research and the general form for implementation with MATLAB (Mathworks, 2018) and IBM ILOG CPLEX (ILOG, 2015) is presented in CHAPTER 3.

2.11 Summary and conclusions

Relevant literature review on the surface-underground mining options and transitions optimization (SUMOTO) problem has been conducted and documented. A summary table of notable solution techniques employed for the SUMOTO problem in the past decade is presented in Table 2-1. Research works in this area have primarily focused on different variations of determining the transition depth between open pit (OP) and underground (UG) mines, and the subsequent optimization of the strategic schedule for each mining option. Heuristics and exact solution methods have both been used in the past to attempt the SUMOTO problem.

Notable limitations of current models and algorithms for the SUMOTO problem include: (1) consideration of rock support in the optimization process; (2) consideration of essential infrastructural development in the optimization process; (3) consideration of stochastic variables in the model; (4) comprehensiveness and efficiency of model; and (5) optimality assessment of model solution. These identified limitations may often lead to sub-optimal global solution to the SUMOTO problem thereby affecting the viability of the mining project. It is therefore essential to develop a rigorous optimization framework that attempts to address some of these deficiencies. This research employs the strength of mixed integer linear programming (MILP) to formulate and evaluate near-surface mineral deposits that extends to greater depths and are amenable to both OP and UG mining options. MILP models are typical mathematical programing model (MPM) structured in the form of an objective function to be attained, subject to a set of constraints that defines the practical mining environment. A typical large-scale complex problem could be formulated as a multi-objective MILP optimization problem. The proposed MILP model to be developed will provide a framework for evaluating a typical mineral deposit amenable to both OP and UG mining options during prefeasibility studies for a mining project. The management and engineering of such mining projects will be improved by the implementation of the proposed MILP model for mineral resource extraction evaluation.

CHAPTER 3

MILP THEORETICAL FORMULATION

3.1 Background

This chapter introduces the competitive economic evaluation (CEE) approach and the multi-objective mixed integer linear programming (MILP) optimization framework for solving the open pit-underground mining options and transitions optimization problem. The mathematical formulation including a representation of the problem, conceptual mine, multi-objective function and constraints are documented in this chapter. The model maximizes the net present value (NPV) of the mining project that is amenable to open pit mining methods or underground mining methods or both. A schematic network of the workflow and problem definition is shown in Figure 1-2.

A multi-objective MILP framework based on the proposed Competitive Economic Evaluation (CEE) technique used in solving the mining options and transitions optimization problem is formulated (Afum & Ben-Awuah, 2019; Afum, et al., 2019a). According to Afum, et al. (2019a), the CEE optimization strategy is an unbiased approach that provides fair opportunity to each mining block in the block model for selection through either an independent open pit (OP) mining, an independent underground (UG) mining, a simultaneous open pit and underground (OPUG) mining, a sequential OPUG mining, or combinations of simultaneous and sequential OPUG mining. Underground development is a necessary part of underground mining as it provides the infrastructure with which production of ore can be undertaken (Tuck, 2011). Each mining option has a respective independent surface stockpile (OP stockpile and UG stockpile) that accommodate excess mineralized material mined but can be processed in the future.

In UG mining, two development types exist: primary access or capital development and secondary access or operational development. The life expectancy of a development differentiates between the two types. Primary access development is the development of more permanent openings of a mine while operational development is temporary in nature and tends to be associated with the needs of a production stope(s). Capital or primary access development includes shafts, declines, raises, ore handling development, and other mine accesses such as main level accesses, while operational development includes ore and waste drives, and cross-cuts. Primary access development such as mine ventilation development and geotechnical rock mass support in the operational development and stopes that could change the dynamics of the surface-underground mining options and transitions optimization problem have been included in the MILP formulation as constraints.

The estimated rock strength characterization values for the various rock formation in the block model are used as determinants to estimate the cost and delay in providing rock support and reinforcement in the operational development openings and stopes of the underground mine. It will usually cost the mine more to provide support and reinforcement in the weak rock zones compared to hard rock zones. Similarly, more delay is expected during support and reinforcement works in the weak rock zones compared to high strength rock zones. The estimated delay and sequence of providing rock support in the operational development openings and stopes are integrated as operational constraints in the optimization process.

The time spent in installing geotechnical rock support and reinforcement are modelled as delay factors. For any operational development opening or stope, if significant time is spent to secure the area (delays are encountered), the available time required to excavate the opening or extract ore from the stope will reduce. This will not only increase the cost of operation but will also reduce the total length excavated for the operational development opening or the total ore tonnage extracted from the stope per unit time. These geotechnical constraints are aimed at improving the practical resource recovery factor for deposits that are amenable to OPUG extraction. The multi-objective function of the proposed MILP model determines the following:

1. Position of the required crown pillar;
2. Most suitable mining option(s) for the deposit;
3. Main primary access development schedule;
4. Main ventilation shaft/raises schedule;
5. Operational development schedule;
6. Rock support and reinforcement schedule for operational development;
7. Rock support and reinforcement schedule for stopes;

8. Extraction schedule for the optimal mining option;
9. Schedule of mineralized material delivered from OP mine to processing plant or OP stockpile;
10. Schedule of mineralized material delivered from UG mine to processing plant or UG stockpile;
11. Schedule of mineralized material delivered from OP stockpile to processing plant;
12. Schedule of mineralized material delivered from UG stockpile to processing plant;
13. Life of mine; and
14. Net present value (NPV) of the mining project.

3.2 Competitive economic evaluation (CEE) optimization approach

For a block model made up of several unit blocks, the CEE optimization process evaluates each unit block of the mineral deposit and economically decides (a) blocks suitable for OP mining, (b) blocks suitable for UG mining, (c) blocks left as unmined in the ground and (d) blocks left as unmined in the crown pillar (Figure 3-1). Each block becomes a candidate for selection by the various categories, hence, the competitiveness of the optimization strategy. The CEE optimization strategy is an unbiased approach that provides fair opportunity to each mining block for selection by a mining option subject to the extraction environment constraints. Illustration of the block extraction options in the CEE optimization approach is shown in Figure 3-1. For a unit block $x_{i,j}^t$ shown in Figure 3-1, the block is competitively available for selection as OP mine block, UG mine block, unmined block left in the ground, or unmined block left in the crown pillar. The CEE optimization approach forms the basis for the proposed MILP formulation framework in this research.

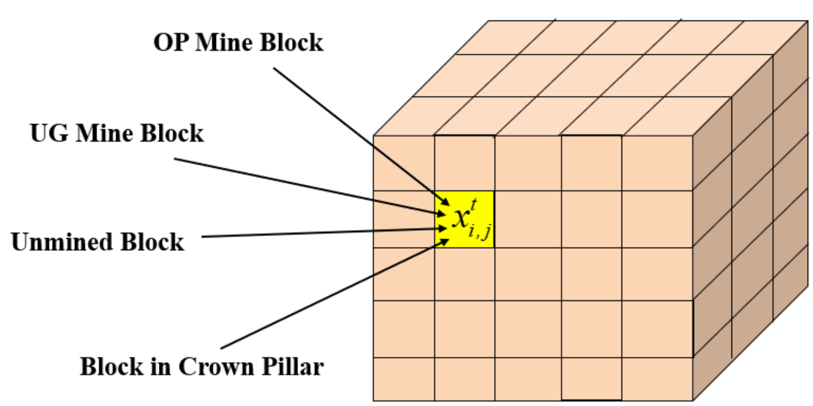


Figure 3-1: Block extraction options in the CEE optimization approach

3.3 MILP formulation

The mixed integer linear programming (MILP) framework is introduced in this section. The set and indices used in the formulation have been explained and the parameters and decision variables required to implement the MILP model are defined. The modelling procedure of the economic and rock support parameters have also been discussed, as well as the objective function and the sets of constraints in the MILP framework.

3.3.1 Sets

$\boldsymbol{J} = \{1,\ldots,J\}$ set of all open pit mining-cuts in the model.

$\boldsymbol{J_s} = \{1,\ldots,J_s\}$ set of all open pit mining-cuts on a level in the model.

$\boldsymbol{P} = \{1,\ldots,P\}$ set of all underground stopes in the model.

$\boldsymbol{P_s} = \{1,\ldots,P_s\}$ set of all underground stopes on a level in the model.

$\boldsymbol{C} = \{1,\ldots,C\}$ set of all levels (crown pillars) in the model.

$\boldsymbol{OD_s} = \{1,\ldots,OD_s\}$ set of all underground operational development on a level in the model.

$\boldsymbol{RD} = \{1,\ldots,RD\}$ set of all underground operational development geotechnical rock support and reinforcement in the model.

$\boldsymbol{RP} = \{1,\ldots,RP\}$ set of all underground stopes geotechnical rock support and reinforcement in the model.

$\boldsymbol{SF} = \{1,\ldots,SF\}$ set of sectional underground primary development in the model.

$\boldsymbol{VD} = \{1,\ldots,VD\}$ set of sectional underground main ventilation development in the model.

$\boldsymbol{S} = \{1,\ldots,S\}$ set of immediate predecessor variables (S) in the model.

$\mathrm{M_j}(\mathrm{S})$ for each open pit cut j, there is a set $M_j(S) \subset \boldsymbol{J}$, defining the immediate predecessor cuts that must be extracted prior to extracting cut j, where S is the total number of cuts in the set $\mathrm{M_j}(\mathrm{S})$.

$\mathrm{U_p}(\mathrm{S})$	for each underground stope *p*, there is a set $U_p(S) \subset \boldsymbol{P}$, defining the immediate predecessor stopes that must be extracted prior to extracting stope *p*; where *S* is the total number of stopes in the set $\mathrm{U_p}(\mathrm{S})$.
$\mathrm{C_j}(\mathrm{S})$	for each level, there is a set $C_j(S) \subset \boldsymbol{J}$, defining the number of cuts on that level that is available for open pit extraction, or left as unmined level, or crown pillar (*c*); where *S* is the total number of cuts in the set $\mathrm{C_j}(\mathrm{S})$.
$\mathrm{C_p}(\mathrm{S})$	for each level, there is a set $C_p(S) \subset \boldsymbol{P}$, defining the number of stopes on that level that is available for underground extraction, or left as unmined level, or crown pillar (*c*); where *S* is the total number of stopes in the set $\mathrm{C_p}(\mathrm{S})$.
$\mathrm{OD_{od}}(\mathrm{S})$	for each level, there is a set $OD_{od}(S) \subset \boldsymbol{OD_s}$, defining the number of underground operational development on that level that must be advanced before a stope (*p*) is extracted; where *S* is the total number of operational developments on the level in the set $\mathrm{OD_{od}}(\mathrm{S})$.
$\mathrm{D}_{sf}(\mathrm{S})$	for each level, there is a set $D_{sf}(S) \subset \boldsymbol{SF}$, defining the number of underground primary development (shaft) that must be advanced before operational development on that level can be started; where *S* is the total number of capital developments in set $\mathrm{D}_{sf}(\mathrm{S})$.

3.3.2 Indices

A general parameter *f* can take a maximum of four indices in the format of $f_{j,k}^{op,t}$. Where:

$\mathrm{t} \in \{1,\ldots,\mathrm{T}\}$	index for scheduling periods.
$\mathrm{j} \in \{1,\ldots,\mathrm{J}\}$	index for open pit mining-cuts in the model.
$\mathrm{p} \in \{1,\ldots,\mathrm{P}\}$	index for underground stopes in the model.
$\mathrm{sf} \in \{1,\ldots,\mathrm{SF}\}$	index for underground primary access development (shaft) in the model.
$\mathrm{od} \in \{1,\ldots,\mathrm{OD}\}$	index for underground operational development in the model.

$\text{rd} \in \{1,\ldots,\text{RD}\}$ index for underground operational development geotechnical rock support and reinforcement in the model.

$\text{rp} \in \{1,\ldots,\text{RP}\}$ index for underground stopes geotechnical rock support and reinforcement in the model.

$\text{vd} \in \{1,\ldots,\text{VD}\}$ index for underground main ventilation development in the model.

$\text{c} \in \{1,\ldots,\text{C}\}$ index for crown pillars in the model.

$\text{s} \in \{1,\ldots,\text{S}\}$ index for the set of immediate predecessor variables in the model.

op index for open pit mining option.

ug index for underground mining option.

opug index for combined open pit and underground mining option.

sk index for stockpile.

3.3.3 Parameters

r processing recovery, the proportion of mineral content recovered.

sp^t selling price of mineral commodity in present value terms.

sc^t selling cost of mineral commodity in present value terms.

pc^t extra cost in present value terms per tonne of ore for mining and processing in period t.

i discount rate, the factor used to discount future cashflows back to the present day.

$v_j^{op,t}$ the open pit (*op*) discounted revenue generated by selling the final product within mining-cut j in period t minus the discounted extra cost of extracting mining-cut j as ore and processing it.

$v_{j,sk}^{op,t}$ discounted revenue from OP stockpile $x_{sk,pr}^{op,t}$ minus the extra cost of rehandling material from OP stockpile to processing plant.

$v_{p}^{ug,t}$ the underground (*ug*) discounted revenue generated by selling the final product within stope p in period t minus the discounted extra cost of extracting stope p as ore and processing it.

$v_{p,sk}^{ug,t}$ discounted revenue from UG stockpile $x_{sk,pr}^{ug,t}$ minus the extra cost of rehandling material from UG stockpile to processing plant.

$q_{j}^{op,t}$ the open pit (*op*) discounted cost of mining all the material in mining-cut j in period t as waste.

$q_{j,sk}^{op,t}$ discounted extra cost of mining and transporting mineralized material from OP mine to OP stockpile in period t.

$q_{p}^{ug,t}$ the underground (*ug*) discounted cost of mining all the material in stope p in period t as waste.

$q_{p,sk}^{ug,t}$ discounted extra cost of mining and transporting mineralized material from UG mine to UG stockpile in period t.

$q_{sf}^{ug,t}$ discounted cost of constructing the main underground capital development length sf in period t.

$q_{od}^{ug,t}$ discounted cost of constructing operational development length od in period t.

$q_{vd}^{ug,t}$ discounted cost of constructing the main underground ventilation development length vd in period t.

$q_{rd}^{ug,t}$ discounted cost for underground operational development geotechnical rock support and reinforcement in period t.

$q_{rp}^{ug,t}$ discounted cost for underground stopes geotechnical rock support and reinforcement in period t.

cm_{j}^{t} cost in present value terms of extracting a tonne of rock material by open pit mining in period t.

cm_p^t cost in present value terms of extracting a tonne of rock material by underground mining in period t.

cd_{sf}^t cost in present value terms per primary access development (shaft) length sf in period t.

cd_{od}^t cost in present value terms per operational development length od in period t.

cd_{vd}^t cost in present value terms per main ventilation shaft development length vd in period t.

cg_{rd}^t cost in present value terms of providing geotechnical rock support and reinforcement per underground operational development length od in period t.

cg_{rp}^t cost in present value terms of providing geotechnical rock support and reinforcement per underground stope tonnage p in period t.

ch_j^t cost in present value terms of transporting mineralized material from OP stockpile to the ore processing plant.

ch_p^t cost in present value terms of transporting mineralized material from UG stockpile to the ore processing plant.

g_j estimated grade of element in ore portion of mining-cut j.

g_p estimated grade of element in ore portion of stope p.

m_j mineralized material in cut j.

m_p mineralized material in stope p.

n_j non mineralized material in cut j.

n_p non mineralized material in stope p.

pd_{sf} capital (primary access) development length (shaft) sf.

od_{od} operational development length od.

cg_{rd} underground operational development geotechnical rock support and reinforcement delay factor per rock mass classification.

cg_{rp} underground stopes geotechnical rock support and reinforcement delay factor per rock mass classification.

pd_{vd} main ventilation shaft development length *vd* per vertical length of a unit mining block.

$g_{lb}^{op,t}$ lower bound on acceptable average grade of element for open pit mining (*op*) in period *t*.

$g_{lb}^{ug,t}$ lower bound on acceptable average grade of element for underground mining (*ug*) in period *t*.

$g_{ub}^{op,t}$ upper bound on acceptable average grade of element for open pit mining (*op*) in period *t*.

$g_{ub}^{ug,t}$ upper bound on acceptable average grade of element for underground mining (*ug*) in period *t*.

$T_{pr,lb}^{op,t}$ lower bound on ore processing capacity requirement from open pit mining in period *t*.

$T_{pr,lb}^{ug,t}$ lower bound on ore processing capacity requirement from underground mining in period *t*.

$T_{pr,ub}^{op,t}$ upper bound on ore processing capacity requirement from open pit mining in period *t*.

$T_{pr,ub}^{ug,t}$ upper bound on ore processing capacity requirement from underground mining in period *t*.

$T_{m,lb}^{op,t}$ lower bound on available open pit mining capacity in period *t*.

$T_{m,lb}^{ug,t}$ lower bound on available underground mining capacity in period *t*.

$T_{m,ub}^{op,t}$ upper bound on available open pit mining capacity in period *t*.

$T_{m,ub}^{ug,t}$ upper bound on available underground mining capacity in period *t*.

$T_{pr,lb}^{opug,t}$ lower bound on ore processing capacity requirement from both open pit and underground mining in period *t*.

$T_{pr,ub}^{opug,t}$ upper bound on ore processing capacity requirement from both open pit and underground mining in period *t*.

$T_{sf,lb}^{ug,t}$ lower bound on capital or primary access development length for underground mining in period *t*.

$T_{sf,ub}^{ug,t}$ upper bound on capital development length for underground mining in period *t*.

$T_{od,lb}^{ug,t}$ lower bound on operational development length for underground mining in period *t*.

$T_{od,ub}^{ug,t}$ upper bound on operational development length for underground mining in period *t*.

$T_{vd,lb}^{ug,t}$ lower bound on main ventilation shaft development length for underground mining in period *t*.

$T_{vd,ub}^{ug,t}$ upper bound on main ventilation development length for underground mining in period *t*.

$T_{sk,ub}^{op,t}$ maximum capacity of the open pit stockpile in period *t*.

$T_{sk,ub}^{ug,t}$ maximum capacity of the underground stockpile in period *t*.

3.3.4 Decision variables

$x_j^{op,t} \in \{0,1\}$ continuous variable, representing the portion of cut *j* to be extracted as mineralized material in period *t* from OP mining.

$x_p^{ug,t} \in \{0,1\}$ continuous variable, representing the portion of stope *p* to be extracted as mineralized material in period *t* from UG mining.

$y_j^{op,t} \in \{0,1\}$ continuous variable, representing the portion of cut *j* to be mined in period *t* through OP mining; fraction of *y* characterizes both mineralized and non-mineralized material included in the cut.

$y_{p}^{ug,t} \in \{0,1\}$ continuous variable, representing the portion of stope p to be mined in period t through UG mining; fraction of y characterizes both mineralized and non-mineralized material included in the stope.

$x_{j,pr}^{op,t} \in \{0,1\}$ continuous variable, representing the portion of cut j to be extracted as mineralized material and processed in period t from OP mining.

$x_{j,sk}^{op,t} \in \{0,1\}$ continuous variable, representing the portion of cut j to be extracted as mineralized material and delivered to the OP stockpile in period t from OP mining.

$x_{sk,pr}^{op,t} \in \{0,1\}$ continuous variable, representing the portion of mineralized material to be delivered from the OP stockpile to the processing plant in period t.

$x_{p,pr}^{ug,t} \in \{0,1\}$ continuous variable, representing the portion of stope p to be extracted as mineralized material and processed in period t from UG mining.

$x_{p,sk}^{ug,t} \in \{0,1\}$ continuous variable, representing the portion of stope p to be extracted as mineralized material and delivered to the UG stockpile in period t from UG mining.

$x_{sk,pr}^{ug,t} \in \{0,1\}$ continuous variable, representing the portion of mineralized material to be delivered from the UG stockpile to the processing plant in period t.

$d_{sf}^{ug,t} \in \{0,1\}$ continuous variable, representing the portion of capital development (shaft) *sf* to be advanced in period t for UG mining.

$d_{vd}^{ug,t} \in \{0,1\}$ continuous variable, representing the portion of main ventilation development *vd* to be advanced in period t for UG mining.

$d_{od}^{ug,t} \in \{0,1\}$ continuous variable, representing the portion of operational development *od* to be advanced in period t for UG mining.

$d_{rd}^{ug,t} \in \{0,1\}$ continuous variable, representing the rock support and reinforcement of the operational development *rd* to be provided in period t for UG mining.

$y_{rp}^{ug,t} \in \{0,1\}$ binary integer variable, representing the rock support and reinforcement of the operational stope *rp* to be provided in period t for UG mining.

$y_c^t \in \{0,1\}$ binary integer variable, equal to one if a level c is left as crown pillar in period t, otherwise zero.

$y_j^t \in \{0,1\}$ binary integer variable, equal to one if a cut j or all cuts J_s on a level are extracted through OP mining in period t, otherwise zero.

$y_p^t \in \{0,1\}$ binary integer variable, equal to one if a stope p or all stopes P_s on a level are extracted through UG mining in period t, otherwise zero.

$y_{od}^t \in \{0,1\}$ binary integer variable, equal to one if operational development od on a level is advanced in period t, otherwise zero.

$b_j^t \in \{0,1\}$ binary integer variable controlling the precedence of extraction of cut for OP mining. b_j^t is equal to one if extraction of cut j has started by or in period t, otherwise it is zero.

$b_p^t \in \{0,1\}$ binary integer variable controlling the precedence of extraction of stope for UG mining. b_p^t is equal to one if extraction of stope p has started by or in period t, otherwise it is zero.

$b_{sf}^t \in \{0,1\}$ binary integer variable controlling the precedence of primary access development for UG mining. b_{sf}^t is equal to one if primary access development sf has started by or in period t, otherwise it is zero.

$b_{od}^t \in \{0,1\}$ binary integer variable controlling the precedence of operational development for UG mining. b_{od}^t is equal to one if operational development od has started by or in period t, otherwise it is zero.

3.3.5 Modeling economic and rock support parameters

A block model of the mineral deposit with estimated grade, density, and rock type characteristics (rock strength) serves as input to the MILP optimization framework. The discounted revenues obtained by selling the final commodity within each block being exploited in period t by OP mining $v_j^{op,t}$ and $v_{j,sk}^{op,t}$, and UG mining $v_p^{ug,t}$ and $v_{p,sk}^{ug,t}$, are evaluated by Equations (1), (2), (3), and (4) respectively. The revenue from a block is a function of the available mineralized material, grade, price and selling cost of the commodity, processing recovery, and any extra cost of mining and processing the mineralized material in the block from the mine or stockpile.

Computations of the discounted costs of mining all rock material within each block as waste in period t by OP mining $q_{j}^{op,t}$ and UG mining $q_{p}^{ug,t}$ are respectively given in Equations (5) and (6). The cost of mining a block as waste is computed by multiplying the extraction cost per tonne of rock material (mineralized or non-mineralized material or both) to the total rock material tonnage in the block. Similarly, the discounted costs of constructing the main UG capital (primary access) development (shaft) $q_{sf}^{ug,t}$, operational development layouts $q_{od}^{ug,t}$ and the main UG ventilation development (shaft) $q_{vd}^{ug,t}$ are respectively defined by Equations (7), (8) and (9). The cost of constructing the main UG access shaft is obtained by multiplying the cost per length of access shaft development to the length of capital development. The cost of constructing operational development layouts in the UG mine is obtained by multiplying the cost per length of developing a vertical level length, ore or waste drives, and crosscuts to the length of operational development excavation. The cost of constructing the main UG ventilation shaft is obtained by multiplying the cost per length of ventilation shaft development to the length of ventilation development.

The discounted cost of providing rock supports and reinforcements for the operational development openings $q_{rd}^{ug,t}$ and stopes $q_{rp}^{ug,t}$ are given in Equations (10) and (11). During the excavation of operational development layouts, the openings are geo-supported and reinforced according to the rock formation strength and the proposed rock support design for such section of the opening. Providing supports in the stopes are necessary to ensure the safe extraction of ore material from the stopes. The costs of providing rock supports and reinforcements in the operational development openings and stopes were modelled as a function of the rock type characteristics (rock strength) and the proposed rock support design.

The discounted extra cost of extracting material from the OP mine to the OP stockpile $q_{j,sk}^{op,t}$ is given in Equation (12) while the discounted extra cost of extracting material from the UG mine to the UG stockpile $q_{p,sk}^{ug,t}$ is given in Equation (13). These costs are mainly made up of the cost of rehandling mineralized rock material from the respective mining option (OP or UG) to their respective stockpiles during the operational life of the mine.

$$v_j^{op,t} = \frac{1}{(1+i)^t}\left[\sum_{j=1}^{J} m_j \times g_j \times r \times (sp - sc) - \sum_{j=1}^{J} m_j \times pc\right] \tag{1}$$

$$v_{j,sk}^{op,t} = \frac{1}{(1+i)^t}\left[\sum_{j=1}^{J} m_j \times g_{sk} \times r \times (sp - sc) - \sum_{j=1}^{J} m_j \times pc\right] \tag{2}$$

$$v_p^{ug,t} = \frac{1}{(1+i)^t}\left[\sum_{p=1}^{P} m_p \times g_p \times r \times (sp - sc) - \sum_{p=1}^{P} m_p \times pc\right] \tag{3}$$

$$v_{p,sk}^{ug,t} = \frac{1}{(1+i)^t}\left[\sum_{p=1}^{P} m_p \times g_{sk} \times r \times (sp - sc) - \sum_{p=1}^{P} m_p \times pc\right] \tag{4}$$

$$q_j^{op,t} = \frac{(m_j + n_j) \times cm_j^t}{(1+i)^t} \tag{5}$$

$$q_p^{ug,t} = \frac{(m_p + n_p) \times cm_p^t}{(1+i)^t} \tag{6}$$

$$q_{sf}^{ug,t} = \frac{pd_{sf} \times cd_{sf}^t}{(1+i)^t} \tag{7}$$

$$q_{od}^{ug,t} = \frac{od_{od} \times cd_{od}^t}{(1+i)^t} \tag{8}$$

$$q_{vd}^{ug,t} = \frac{pd_{vd} \times cd_{vd}^t}{(1+i)^t} \tag{9}$$

$$q_{rd}^{ug,t} = \frac{cg_{rd} \times od_{od} \times cg_{rd}^t}{(1+i)^t} \tag{10}$$

$$q_{rp}^{ug,t} = \frac{cg_{rp} \times (m_p + n_p) \times cg_{rp}^t}{(1+i)^t} \tag{11}$$

$$q_{j,sk}^{op,t} = \frac{m_j \times ch_j^t}{(1+i)^t} \tag{12}$$

$$q_{p,sk}^{ug,t} = \frac{m_p \times ch_p^t}{\left(1+i\right)^t} \tag{13}$$

3.3.6 Objective function of the MILP model

The objective function of the MILP model seeks to:

1. Maximize the discounted revenue from direct OP mining using Equation (14);
2. Maximize the discounted revenue from direct UG mining using Equation (15);
3. Maximize the discounted revenue from OP stockpile using Equation (16);
4. Maximize the discounted revenue from UG stockpile using Equation (17);
5. Minimize the discounted cost of OP mining using Equation (18);
6. Minimize the discounted cost of UG mining using Equation (19);
7. Minimize the discounted cost of constructing UG primary access development using Equation (20);
8. Minimize the discounted cost of constructing UG ventilation development using Equation (21);
9. Minimize the discounted cost of UG operational development using Equation (22)
10. Minimize the discounted cost of providing rock support and reinforcement in UG operational development using Equation (23);
11. Minimize the discounted cost of providing rock support and reinforcement in UG stopes using Equation (24);
12. Minimize the extra discounted cost of sending material from the OP mine to OP stockpile using Equation (25);
13. Minimize the extra discounted cost of sending material from the UG mine to UG stockpile using Equation (26);

$$Max \sum_{t=1}^{T} \sum_{j=1}^{J} \left(v_j^{op,t} \times x_{j,pr}^{op,t} \right) \tag{14}$$

$$Max \sum_{t=1}^{T} \sum_{p=1}^{P} \left(v_p^{ug,t} \times x_{p,pr}^{ug,t} \right) \tag{15}$$

$$Max\sum_{t=1}^{T}\sum_{j=1}^{J}\left(v_{sk,pr}^{op,t}\times x_{sk,pr}^{op,t}\right) \tag{16}$$

$$Max\sum_{t=1}^{T}\sum_{p=1}^{P}\left(v_{sk,pr}^{ug,t}\times x_{sk,pr}^{ug,t}\right) \tag{17}$$

$$Min\sum_{t=1}^{T}\sum_{j=1}^{J}\left(q_{j}^{op,t}\times y_{j}^{op,t}\right) \tag{18}$$

$$Min\sum_{t=1}^{T}\sum_{p=1}^{P}\left(q_{p}^{ug,t}\times y_{p}^{ug,t}\right) \tag{19}$$

$$Min\sum_{t=1}^{T}\sum_{sf=1}^{SF}\left(q_{sf}^{ug,t}\times d_{sf}^{ug,t}\right) \tag{20}$$

$$Min\sum_{t=1}^{T}\sum_{vd=1}^{VD}\left(q_{vd}^{ug,t}\times d_{vd}^{ug,t}\right) \tag{21}$$

$$Min\sum_{t=1}^{T}\sum_{od=1}^{OD}\left(q_{od}^{ug,t}\times d_{od}^{ug,t}\right) \tag{22}$$

$$Min\sum_{t=1}^{T}\sum_{rd=1}^{RD}\left(q_{rd}^{ug,t}\times d_{rd}^{ug,t}\right) \tag{23}$$

$$Min\sum_{t=1}^{T}\sum_{rp=1}^{RP}\left(q_{rp}^{ug,t}\times y_{rp}^{ug,t}\right) \tag{24}$$

$$Min\sum_{t=1}^{T}\sum_{j=1}^{J}\left(q_{j,sk}^{op,t}\times x_{j,sk}^{op,t}\right) \tag{25}$$

$$Min\sum_{t=1}^{T}\sum_{p=1}^{P}\left(q_{p,sk}^{ug,t}\times x_{p,sk}^{ug,t}\right) \tag{26}$$

Equations (14) to (26) are combined into a multi-objective function of the MILP formulation as shown in Equation (27). The objective function maximizes the NPV of the mining project and determines the schedules for OP mineralized material delivered directly to the processing plant $x_{j,pr}^{op,t}$, UG mineralized material delivered from the UG mine directly to the processing plant $x_{p,pr}^{ug,t}$, OP mineralized material delivered from OP stockpile to the processing plant $x_{sk,pr}^{op,t}$, and UG mineralized material delivered from UG stockpile to the processing plant $x_{sk,pr}^{ug,t}$. The objective function minimizes the cost of the mining project and further determines the respective block extraction strategy of OP mining $y_j^{op,t}$ and UG mining $y_p^{ug,t}$, and further determines the schedules for primary access development $d_{sf}^{ug,t}$, main ventilation development $d_{vd}^{ug,t}$, operational development (level, drives, crosscut) $d_{od}^{ug,t}$, and the rock support provisions for operational development layouts $d_{rd}^{ug,t}$ and stopes $y_{rp}^{ug,t}$. The cost of OP mining mineralized material delivered to OP stockpile and UG mining mineralized material delivered to UG stockpile are also minimized by the objective function through Equation (25) and Equation (26) respectively. During optimization, if OP mining only or UG mining only is selected, the objective function of the unselected mining option and the associated decision variables become zeros.

$$Max\sum_{t=1}^{T}\begin{bmatrix} \sum_{j=1}^{J}\left(v_j^{op,t}\times x_{j,pr}^{op,t}-q_j^{op,t}\times y_j^{op,t}\right)+\sum_{p=1}^{P}\left(v_p^{ug,t}\times x_{p,pr}^{ug,t}-q_p^{ug,t}\times y_p^{ug,t}\right) \\ +\sum_{j=1}^{J}\sum_{sk=1}^{SK}\left(v_{sk,pr}^{op,t}\times x_{sk,pr}^{op,t}-q_{j,sk}^{op,t}\times x_{j,sk}^{op,t}\right)+\sum_{p=1}^{P}\sum_{sk=1}^{SK}\left(v_{sk,pr}^{ug,t}\times x_{sk,pr}^{ug,t}-q_{p,sk}^{ug,t}\times x_{p,sk}^{ug,t}\right) \\ -\sum_{sf=1}^{SF}\left(q_{sf}^{ug,t}\times d_{sf}^{ug,t}\right)-\sum_{vd=1}^{VD}\left(q_{vd}^{ug,t}\times d_{vd}^{ug,t}\right)-\sum_{od=1}^{OD}\left(q_{od}^{ug,t}\times d_{od}^{ug,t}\right) \\ -\sum_{rd=1}^{RD}\left(q_{rd}^{ug,t}\times d_{rd}^{ug,t}\right)-\sum_{rp=1}^{RP}\left(q_{rp}^{ug,t}\times y_{rp}^{ug,t}\right) \end{bmatrix} \tag{27}$$

3.3.7 MILP model constraints

The material extraction and geotechnical constraints of the MILP model are given in Equations (28) to (85). The main constraints are grouped as follows: OP mining constraints; OP stockpile management; UG mining constraints; UG stockpile management; OP and UG mining interaction constraints; crown pillar positioning constraints; capital development (primary access) constraints; operational development constraints; main ventilation shaft (requirement) constraints; and non-negativity constraints.

3.3.7.1 Open pit mining constraints

$$T_{m,lb}^{op,t} \le \sum_{j=1}^{J}\left[\left(m_j + n_j\right)\times y_j^{op,t}\right] \le T_{m,ub}^{op,t} \tag{28}$$

$$T_{pr,lb}^{op,t} \le \sum_{j=1}^{J}\left[m_j \times \left(x_{j,pr}^{op,t} + x_{sk,pr}^{op,t}\right)\right] \le T_{pr,ub}^{op,t} \tag{29}$$

$$g_{lb}^{op,t} \le \left[\frac{\sum_{j=1}^{J}\left(g_j \times m_j \times x_j^{op,t}\right)}{\sum_{j=1}^{J}\left(m_j \times x_j^{op,t}\right)}\right] \le g_{ub}^{op,t} \tag{30}$$

$$x_j^{op,t} - y_j^{op,t} \le 0 \tag{31}$$

$$b_j^t - \sum_{t=1}^{T} y_s^{op,t} \le 0 \tag{32}$$

$$\sum_{t=1}^{T} y_j^{op,t} - b_j^t \le 0 \tag{33}$$

$$b_j^t - b_j^{t+1} \le 0 \tag{34}$$

$$\sum_{t=1}^{T} y_j^{op,t} \le 1 \tag{35}$$

$$\left(\sum_{j=1}^{J}\frac{1}{Js}\times b_j^t\right) - y_j^t \le 0 \tag{36}$$

$$y_j^t - y_{j-1}^t \le 0 \tag{37}$$

Equations (28) to (37) control the block extraction dynamics for the open pit (OP) operation. The open pit block extraction dynamics as shown in Equation (28) defines the mining capacity constraint for OP extraction. Open pit extraction is controlled by the continuous decision variable $y_j^{op,t}$. The total tonnage of rock material mined in each period is constrained within the acceptable lower and upper limits of the total available equipment capacity for the open pit mining operation.

Equation (29) is the processing capacity constraint that controls the quantity of mineralized material being delivered from the open pit mining operation and OP stockpile to the processing plant. The processing capacity constraint is controlled by the continuous decision variables $x_{j,pr}^{op,t}$ and $x_{sk,pr}^{op,t}$. This inequality ensures that uniform mineralized material from the OP mining operation is fed to the processing plant throughout the mine life within the acceptable lower and upper targets of the ore processing capacity of the plant in each period.

Equation (30) specifies the quality of mineralized material in terms of grade content extracted from the open pit $x_{j}^{op,t}$ and delivered to either the processing plant $x_{j,pr}^{op,t}$ or OP stockpile $x_{j,sk}^{op,t}$ in each period. Higher grades of mineralized material are prioritized for extraction over lower grades of mineralized material for plant processing. This constraint does not specify the head grade of the processing plant but controls the extraction grade from the OP mine. The minimum and maximum available mineralized material grades in the block model are used to respectively define the lower and upper grade targets for OP mining allowing the optimizer to decide the economic cut-off grade for extraction. Equation (30) ensures that the contribution from OP mining operation towards a blended processing plant head grade can be controlled.

Equation (31) outlines the relation between the mineralized material portion of the mining-cut (block) and the total block tonnage (both mineralized and non-mineralized) extracted by OP mining. The continuous variable $x_{j}^{op,t}$ should always be smaller than or equal to the continuous variable $y_{j}^{op,t}$. Equations (32) to (34) control the vertical precedence relation of mining-cut extraction following the appropriate geotechnical mining slope for the open pit mining option. For open pit mining, nine overlying mining-cuts should be extracted before the underlying mining-cut is removed.

Equation (35) ensures that a block is extracted once in the life of the OP mine. Equation (36) ensures that a level belongs to OP mining option when one or more mining-cuts on that level is extracted by OP mining, while Equation (37) ensures that when a level (*j*) is considered for OP mining, the immediate level above it (*j-1*) has already been evaluated for OP mining option.

3.3.7.2 Open pit mining stockpile management constraints

$$\sum_{j=1}^{J}\left[m_j \times \left(x_{j,sk}^{op,t} - x_{sk,pr}^{op,t}\right)\right] \le T_{sk,ub}^{op,t} \tag{38}$$

$$x_{j,sk}^{op,t} + x_{j,pr}^{op,t} - x_{j}^{op,t} \le 0 \tag{39}$$

$$\sum_{t=t}^{T} x_{sk,pr}^{op,t} - x_{j,sk}^{op,t} \le 0 \tag{40}$$

Equation (38) ensures that open pit mineralized material at the OP stockpile during any time do not exceed the specified capacity of the delineated mineralized material pad. This ensures that there is an accountable balance between the quantity of mineralized material delivered from the OP mining to the OP stockpile $x_{j,sk}^{op,t}$ and the quantity of mineralized material moved from OP stockpile to the processing plant $x_{sk,pr}^{op,t}$. Equation (39) ensures that the quantity of mineralized material extracted from the open pit $x_{j}^{op,t}$ is either stockpiled for later processing $x_{j,sk}^{op,t}$ or immediately delivered to the plant for processing $x_{j,pr}^{op,t}$, while Equation (40) ensures that the quantity of mineralized material delivered from OP mining to OP stockpile is available to be hauled to the processing plant.

3.3.7.3 Underground mining constraints

$$T_{m,lb}^{ug,t} \le \left\{\sum_{p=1}^{P}\left[\left(m_p + n_p\right)\times y_p^{ug,t}\right] + \sum_{rp=1}^{RP}\left[cg_{rp}\times\left(m_p + n_p\right)\times y_{rp}^{ug,t}\right]\right\} \le T_{m,ub}^{ug,t} \tag{41}$$

$$T_{pr,lb}^{ug,t} \le \sum_{p=1}^{P}\left[m_p \times \left(x_{p,pr}^{ug,t} + x_{sk,pr}^{ug,t}\right)\right] \le T_{pr,ub}^{ug,t} \tag{42}$$

$$g_{lb}^{ug,t} \le \left[\frac{\sum_{p=1}^{P}\left(g_p \times m_p \times x_p^{ug,t}\right)}{\sum_{p=1}^{P}\left(m_p \times x_p^{ug,t}\right)}\right] \le g_{ub}^{ug,t} \tag{43}$$

$$x_p^{ug,t} - y_p^{ug,t} \le 0 \tag{44}$$

$$b_p^t - \sum_{t=1}^{T} y_s^{ug,t} \le 0 \tag{45}$$

$$\sum_{t=1}^{T} y_p^{ug,t} - b_p^t \le 0 \tag{46}$$

$$b_p^t - b_p^{t+1} \le 0 \tag{47}$$

$$\sum_{t=1}^{T} x_p^{ug,t} \le 1 \tag{48}$$

$$y_p^{ug,t} - \sum_{t=1}^{T} y_{rp}^{ug,t} \le 0 \tag{49}$$

$$\sum_{t=1}^{T} y_{rp}^{ug,t} \le 1 \tag{50}$$

$$\left(\sum_{p=1}^{P} \frac{1}{Ps} \times b_p^t \right) - y_p^t \le 0 \tag{51}$$

Equations (41) to (51) control the underground (UG) mining operations. Equation (41) is the capacity constraint for the UG mining operation controlled by the continuous decision variable $y_p^{ug,t}$, and the delay associated with providing support and reinforcement in the stopes $y_{rp}^{ug,t}$. This inequality ensures the total tonnage of rock material mined in each period is within acceptable lower and upper limits of the total available equipment capacity for the UG mining operation. The inequality further ensures that, delays associated with providing reinforcement in the stopes are factored into the stope production schedule. Thus, the quantity of rock material extracted in each period is controlled by the mining rate and associated geotechnical delay rate or time spent in providing rock support and reinforcement in the stopes.

Equation (42) is the processing capacity constraint that controls the quantity of mineralized material delivered from the UG mining operation to the processing plant. The processing capacity constraint is controlled by the continuous decision variable $x_{p,pr}^{ug,t}$ and $x_{sk,pr}^{ug,t}$. This inequality ensures the mineralized material sent from the UG mine directly to the processing plant $x_{p,pr}^{ug,t}$, and sent from the UG stockpile to the processing plant $x_{sk,pr}^{ug,t}$, are controlled.

Equation (42) ensures the contribution of mineralized material production from the UG mine to the overall OPUG processing capacity does not exceed a pre-defined limit.

Equation (43) controls the quality of ore grade being extracted from UG mining operation. Equation (43) ensures that rock material with high grades is prioritized over rock material with low grades during stope mining sequencing. This constraint does not control the limiting grade requirement or head grade of the processing plant but only controls the mineralized material quality extracted from each stope. The minimum and maximum available grade of mineralized material in the block model are used to respectively define this lower and upper grade targets for the UG mining operation allowing the optimizer to decide the economic cut-off grade for extraction.

Equation (44) defines the relation between mineralized material tonnage in the stope and the stope tonnage (both mineralized and non-mineralized) controlling the UG mining and processing decisions. Thus, the continuous variable $x_p^{ug,t}$ should always be smaller than or equal to the continuous variable $y_p^{ug,t}$. Equations (45) to (47) control the lateral and vertical (or inter-level) precedence relation of stope extraction on each level for the UG mining option. For UG mining, stope extraction sequence is implemented in either a retreating or advancing manner towards the main mine entrance for each underground level depending on the rock formation and mine management strategy. Stope extraction is also implemented in either an overhand or underhand stoping sequence. The choice of inter-level interactions dictates the underground mining method for the orebody and the neighboring rock formation.

Equation (48) defines the reserve constraint for UG mining. This inequality ensures that each stope $x_p^{ug,t}$ on a level y_p^t is extracted once in the life of the UG mine. Eq. (49) defines the relations between the stoping activity and the delay associated with providing support and reinforcement in the stopes. The constraint ensures that the delays and costs of providing support in the stope is considered if that stope must be exploited. That is, the stope support variable $y_{rp}^{ug,t}$ becomes non-zero is a stope $y_p^{ug,t}$ is being exploited (thus, non-zero).

Eq. (50) defines the [0,1] constraint of the stope supporting activity for the UG mine. This inequality ensures that a stope y_p^t is supported once in the life of the UG mine. Equation

(51) ensures that a level belongs to UG mining option when one or more stopes on that level is extracted by UG mining.

3.3.7.4 Underground mining stockpile management constraints

$$\sum_{p=1}^{P}\left[m_p \times \left(x_{p,sk}^{ug,t} - x_{sk,pr}^{ug,t}\right)\right] \le T_{sk,ub}^{ug,t} \tag{52}$$

$$x_{p,sk}^{ug,t} + x_{p,pr}^{ug,t} - x_{p}^{ug,t} \le 0 \tag{53}$$

$$\sum_{t=t}^{T} x_{sk,pr}^{ug,t} - x_{p,sk}^{ug,t} \le 0 \tag{54}$$

Equation (52) ensures that the mineralized material at the UG stockpile during any time do not exceed the specified capacity of the delineated mineralized material pad or area. This equation ensures that there is an accountable balance between the quantity of mineralized material delivered from the underground mine to UG stockpile $x_{p,sk}^{up,t}$ and the quantity of mineralized material moved from UG stockpile to the processing plant $x_{sk,pr}^{ug,t}$. Equation (53) ensures that the quantity of mineralized material extracted from UG mining $x_{p}^{ug,t}$ is either stockpiled for later processing $x_{p,sk}^{ug,t}$ or immediately sent to the plant for processing $x_{p,pr}^{ug,t}$, while Equation (54) ensures that the quantity of mineralized material delivered from UG mining to UG stockpile is available to be hauled to the processing plant.

3.3.7.5 Open pit and underground mining interactions constraints

$$b_j^t - b_p^t \le 0 \tag{55}$$

$$T_{pr,lb}^{opug,t} \le \left\{\sum_{j=1}^{J}\left[m_j \times \left(x_{j,pr}^{op,t} + x_{sk,pr}^{op,t}\right)\right] + \sum_{p=1}^{P}\left[m_p \times \left(x_{p,pr}^{ug,t} + x_{sk,pr}^{ug,t}\right)\right]\right\} \le T_{pr,ub}^{opug,t} \tag{56}$$

$$y_j^t + y_c^t + y_p^t \le 1 \tag{57}$$

Equations (55) to (57) manage the interactions between the OP and UG mining operations. Equation (55) represents the interaction of OP mining-cuts with UG stopes. This inequality ensures that each block (mining-cut/stope) is extracted by only one mining option or left as unmined block in the crown pillar or within the extraction limit. Equation (56) is the combined processing capacity constraint that controls the overall mill feed. This inequality represents the contribution of ore production from both OP and UG mining options to the processing plant. Equation (57) ensures that, a level or bench is either considered for OP mining, UG mining, crown pillar, or unmined level within the extraction limit. Equation (57) further ensures that, one level or bench cannot simultaneously be present in the open pit mine, underground mine and crown pillar.

3.3.7.6 Crown pillar constraints

$$y_c^t - y_{j-1}^t \le 0 \tag{58}$$

$$y_{c-1}^t - y_p^t \le 0 \tag{59}$$

$$y_c^t - y_c^{t+1} \le 0 \tag{60}$$

$$\sum_{c=1}^{C} y_c^t = 1 \tag{61}$$

Equations (58) to (61) control the positioning of the required crown pillar or transition depth and its relation to the location of the OP and UG mining operations. Equation (58) ensures that the crown pillar is positioned immediately at the bottom of the OP mine when OP operations ends while Equation (59) ensures the crown pillar is always positioned above the level being considered for UG mining operations. Equation (60) ensures that a level acting as crown pillar is unmined but stays at the same location throughout the life of the mining operation while Equation (61) ensures that one level always acts as unmined crown pillar.

3.3.7.7 Capital development (primary access) constraints

$$T_{sf,lb}^{ug,t} \le \sum_{sf=1}^{SF} \left(cd_{sf} \times d_{sf}^{ug,t} \right) \le T_{sf,ub}^{ug,t} \tag{62}$$

$$b_{sf}^t - \sum_{t=1}^{T} d_{sf,s}^{ug,t} \le 0 \tag{63}$$

$$\sum_{t=1}^{T} d_{sf}^{ug,t} - b_{sf}^{t} \leq 0 \tag{64}$$

$$b_{sf}^{t} - b_{sf}^{t+1} \leq 0 \tag{65}$$

$$d_{od,s}^{ug,t} - \sum_{t=1}^{T} d_{sf}^{ug,t} \leq 0 \tag{66}$$

$$\sum_{t=1}^{T} d_{sf}^{ug,t} \leq 1 \tag{67}$$

The primary access development constraints defined in Equations (62) to (67) ensure that the required primary access development is considered when UG mining operation is the preferred or part of the preferred extraction option. Equation (62) defines the primary access development capacity constraints for UG mining. This inequality ensures that the total length of primary access development required in each period is within the acceptable lower and upper limits of the available equipment capacity for developing the UG mine.

Equations (63) to (66) control the precedence relations between sections of primary access development leading to each level and the operational development on each level. Equations (63) to (65) ensure that sets of primary access development representing sections above a level must be completed before the primary access development b_{sf}^{t} of that level commences. Equation (66) ensures that development of the operational level $d_{od,s}^{ug,t}$ linking the primary access development could only commence after completion of a set of required primary access development $\sum_{t=1}^{T} d_{sf}^{ug,t}$ above and on that level. Equation (67) ensures that each section of primary access development (shaft or decline) is extracted ones in the life of the UG mine.

3.3.7.8 Operational development constraints

$$T_{od,lb}^{ug,t} \leq \left[\sum_{od=1}^{OD} \left(od_{od} \times d_{od}^{ug,t} \right) + \sum_{rd=1}^{RD} \left(cg_{rd} \times od_{od} \times d_{rd}^{ug,t} \right) \right] \leq T_{od,ub}^{ug,t} \tag{68}$$

$$b_{od}^{t} - \sum_{t=1}^{T} d_{od,s}^{ug,t} \leq 0 \tag{69}$$

$$\sum_{t=1}^{T} d_{od}^{ug,t} - b_{od}^{t} \leq 0 \tag{70}$$

$$b_{od}^{t} - b_{od}^{t+1} \leq 0 \tag{71}$$

$$x_{p}^{ug,t} - \sum_{t=1}^{T} d_{od,s}^{ug,t} \leq 0 \tag{72}$$

$$\left(\sum_{od=1}^{ODs} \frac{1}{OD} \times b_{od,s}^{t} \right) - y_{od}^{t} \leq 0 \tag{73}$$

$$y_{c}^{t} + y_{od}^{t} \leq 1 \tag{74}$$

$$d_{od}^{ug,t} - \sum_{t=1}^{T} d_{rd}^{ug,t} \leq 0 \tag{75}$$

$$\sum_{t=1}^{T} d_{od}^{ug,t} \leq 1 \tag{76}$$

$$\sum_{t=1}^{T} d_{rd}^{ug,t} \leq 1 \tag{77}$$

The set of constraints from Equations (68) to (77) define the operational development requirements including the type and length of each operational development (level, ore and waste drives, crosscuts) and lateral precedence relations with the stope extraction sequence. Equation (68) defines the operational development capacity constraint for UG mining, ensuring that the total length of operational development (level, ore and waste drives, crosscuts) required in each period is within the acceptable lower and upper limits of the available equipment capacity for developing the UG mine. The inequality further ensures that, delays associated with providing geotechnical rock support and reinforcement in the operational development are factored into the operational development schedule. Thus, the length of operational development advanced in each period is controlled by the operational development rate and geotechnical delay rate or time spent in providing rock support and reinforcement for the development openings.

Equations (69) to (71) control the lateral precedence relations of the UG operational development required for exploiting the orebody. Equation (72) ensures that a set of operational development $\sum_{t=1}^{T} d_{od,s}^{ug,t}$ is completed before exploiting a stope $x_p^{ug,t}$ in any period. In the case of Equation (73), an underground level is activated when operational development and rock support activities have commenced or completed for that level while Equation (74) ensures that a level selected by the optimization process as the crown pillar would not be available for operational development and vice versa. Equation (75) constraint ensures that the geotechnical rock support and reinforcement provided in the operational development is completed immediately after completion of the operational development. Equations (76) and (77) respectively ensure that the operational development (level, ore and waste drives, crosscuts) and rock support at any section of the mine is advanced ones in the life of the UG operation.

3.3.7.9 Main ventilation development constraints

$$T_{vd,lb}^{ug,t} \le \sum_{vd=1}^{VD} \left(cd_{vd} \times d_{vd}^{ug,t} \right) \le T_{vd,ub}^{ug,t} \tag{78}$$

$$b_{vd}^{t} - \sum_{t=1}^{T} d_{vd,s}^{ug,t} \le 0 \tag{79}$$

$$\sum_{t=1}^{T} d_{vd}^{ug,t} - b_{vd}^{t} \le 0 \tag{80}$$

$$b_{vd}^{t} - b_{vd}^{t+1} \le 0 \tag{81}$$

$$d_{od,s}^{ug,t} - \sum_{t=1}^{T} d_{vd,s}^{ug,t} \le 0 \tag{82}$$

$$\sum_{t=1}^{T} d_{vd}^{ug,t} \le 1 \tag{83}$$

The main ventilation development constraints defined in Equations (78) to (83) ensure that the required main ventilation development is considered when UG mining operation is the preferred or part of the preferred extraction option. Equation (78) defines the main ventilation development capacity constraints for UG mining operation. This inequality ensures that the total length of the main ventilation development required in each period is within the acceptable lower and upper limits of the available equipment capacity for UG mine ventilation development.

Equations (79) to (81) control the precedence relations between sections of main ventilation development leading to each level and the development on each level (drives, crosscuts). These constraints ensure that sets of main ventilation development representing sections above a level must be completed before the ventilation development b_{vd}^{t} of that level commences. Equation (82) ensures that development of the operational level $d_{od,s}^{ug,t}$ linking the main ventilation development could only commence after completion of a set of required main ventilation development $\sum_{t=1}^{T} d_{vd,s}^{ug,t}$ above and on that level. Equation (83) ensures that each section of the main ventilation development is excavated ones in the life of the UG mine.

3.3.7.10 Non-negativity constraints

$$x_{j}^{op,t}, x_{j,sk}^{op,t}, x_{sk,pr}^{op,t}, x_{j,pr}^{op,t}, y_{j}^{op,t}, x_{p}^{ug,t}, x_{p,sk}^{ug,t}, x_{sk,pr}^{ug,t}, x_{p,pr}^{ug,t}, y_{p}^{ug,t}, d_{sf}^{ug,t}, d_{od}^{ug,t}, d_{vd}^{ug,t}, d_{rd}^{ug,t}, d_{rp}^{ug,t} \geq 0 \qquad (84)$$

$$b_{j}^{t}, b_{p}^{t}, b_{sf}^{t}, b_{od}^{t}, b_{vd}^{t}, y_{j}^{t}, y_{p}^{t}, y_{c}^{t}, y_{od}^{t} \geq 0 \text{ and integers} \qquad (85)$$

Equations (84) and (85) ensure that the decision variables for OP and UG mining, OP and UG processing, crown pillar, OP mining benches, UG mining levels, UG operational development, UG primary access development, UG ventilation development, UG operational development rock support and reinforcement, and UG stope rock support and reinforcement are non-negative and continuous. These inequality constraints further define that the binary variables controlling the sequence of geotechnical rock support, operational development, capital development, ventilation development, and extraction in the OP and UG mining operations are non-negative and integers.

3.4 MILP formulation implementation

The MILP is formulated by first defining the objective function and establishing the conceptual mine based on the technical and economic constructs of the mining operation. The objective function maximizes the NPV and minimizes the cost of mining, stockpiling, development, and geotechnical support. Mining-cuts are used as elements to control the OP mining operation while stopes are used as elements to control the UG mining operation. For a given conceptual mining model, the objective function and constraints interact with the economic block model during the optimization process to attain the set objectives. The MILP framework implementation commences with the identification of the appropriate numerical modelling platform before the problem structure is set-up and solved in a realistic time period. MATLAB 2018a (Mathworks, 2018) is employed as the mathematical programming environment and IBM ILOG CPLEX (ILOG, 2015) is used as the optimization solver.

3.4.1 Numerical modeling

Numerical modeling of the MILP formulation is implemented in MATLAB 2018a (Mathworks, 2018). The following serve as the main inputs to the MILP model: (a) geologic block model; (b) technical mining parameters such as mining, stockpile and processing capacities, primary access, ventilation and operational development capacities, geotechnical rock support and reinforcement requirements; (c) economic parameters such as commodity price, cost of development, mining, stockpiling and processing; and (d) mining strategy (sequence of OP mining operation and UG mining operation). The numerical models of the objective function and constraints are generated in MATLAB in the form of matrices to be passed on to IBM ILOG CPLEX. The IBM ILOG CPLEX solver (ILOG, 2015) uses the branch and cut algorithm to solve large-scale mine planning optimization problems based on mathematical programming models. The technique is a combination of branch and bound, and cutting plane methods (Horst & Hoang, 1996). Wolsey (1998) provides detailed explanation to the branch and cut algorithm, and its application. The user (mine planner) establishes an optimization criterion to terminate CPLEX. This criterion is called gap tolerance (EPGAP). The EPGAP is the optimality measure which defines the absolute tolerance on the difference between the best integer solution and the solution derived from the best node remaining in the branch and cut algorithm. CPLEX terminates based on the established EPGAP when a feasible integer solution is found within the defined gap.

3.4.2 General definition of MILP formulation for CPLEX

The basic general form for defining MILP optimization problems in MATLAB for IBM ILOG CPLEX is specified by Equations (86) to (88).

$$\min f(x) = c^T \cdot x \tag{86}$$

subject to:

$$b_L \leq A \cdot x \leq b_U \tag{87}$$

$$x_L \leq x \leq x_U \tag{88}$$

where

- **c** is the coefficients of objective function of the MILP model; a vector $j \times 1$.
- **x** is the decision variables used in the MILP model; a vector $j \times 1$.
- **T** is the index for scheduling periods.
- **A** signifies the coefficients of the MILP constraints; a matrix $i \times j$.
- $\mathbf{b}_L$ and $\mathbf{b}_U$ describe the lower and upper bounds of the constraints, respectively; vectors $i \times 1$.
- $\mathbf{x}_L$ and $\mathbf{x}_U$ define the lower and upper bounds of the decision variables, respectively; vectors $j \times 1$.
- Equality constraints are defined by setting the lower bounds equal to the upper bounds for the respective elements of vectors $\mathbf{b}_L$ and $\mathbf{b}_U$.

Detailed formulation and programming techniques in deploying MILP models for mine optimization problems can be found in Ben-Awuah, et al. (2018) and Askari-Nasab, et al. (2010).

3.5 Summary and conclusions

This chapter has presented the proposed optimization approach (the CEE approach) and the MILP model for surface-underground mining options and transitions optimization. The CEE optimization technique ensures an unbiased global optimization solution is obtained from the mathematical program. The MILP framework has the capacity to select the optimal mining options for exploiting a mineral deposit at the prefeasibility stage of a mining project. The model is flexible in defining surface mining methods such as open pit mining and quarrying, and underground mining methods such as sublevel stoping, open stoping, vertical crater retreat (VCR), vein mining, longwall caving, sublevel caving, and top slicing. The

MILP model expands the frontiers of integrated strategic mine planning and optimization for resource extraction evaluation of mineral deposits. The interactions of the operations associated with both surface and underground mining, and the interrelations between the various mining processes and procedures of a typical deposit amenable to both OP and UG mining options were discussed. The model provides a global optimization solution with an optimum strategic production schedule that maximizes the NPV of the mining project in the presence of essential underground infrastructure and crown pillar positioning requirements. To achieve this objective, the theoretical model considers assumptions and limitations in relation to OP-UG mine planning and optimization methods. The model is deployed with suitable concepts and strategies for implementation of case studies.

CHAPTER 4

APPLICATIONS OF THE MILP MODEL AND DISCUSSION OF RESULTS

4.1 Background

The mixed integer linear programming (MILP) model was implemented on three case studies. As an initial step, the mathematical programming framework was implemented with an experimental or laboratory prepared dataset before implementing on actual deposits. The first case study was a laboratory prepared copper dataset. The second and third case studies were actual gold deposits. The third case study was further evaluated with an industry standard optimization software, Whittle. The results from Whittle were compared to that from the MILP model to evaluate the performance of the proposed framework. This chapter discusses the applications and results from the synthetic copper dataset and two gold deposit case studies.

To implement the case studies, the mathematical programming framework was defined in MATLAB 2018a (Mathworks, 2018) environment and the large scale IBM ILOG CPLEX Optimization solver (ILOG, 2015) was integrated into MATLAB to solve the MILP problem at a gap tolerance of 5%. The initial results from the synthetic copper datasets support literature findings that block sequencing technique in formulating and solving MILP models are not solution-efficient because they are typically non-deterministic polynomial-time hard problems (Tran & Killat, 2008). To mitigate the hardness of the MILP model solution computations, the formulation was normalized during the implementation of the second and third case studies.

4.2 Conceptual mining system

The MILP model was implemented for a mineral resource amenable to both open pit and underground mining. In open pit mining, the ore is exploited from the top to the bottom with a 45^{o} slope to ensure the geotechnics of the mine is controlled. An isometric view of the block model showing the block extraction precedence for an open pit mine is shown in Figure 4-1. In Figure 4-1, to mine Block B5 which is directly below Block A5, the overlying

Blocks A1, A2, A3, A4, A5, A6, A7, A8 and A9, need to be extracted first. Thus, for a block to be extracted, 9 blocks overlying that block must be primarily mined to give way for the underlying block to be extracted. The total number of overlying blocks is dictated by the overall slope of the pit.

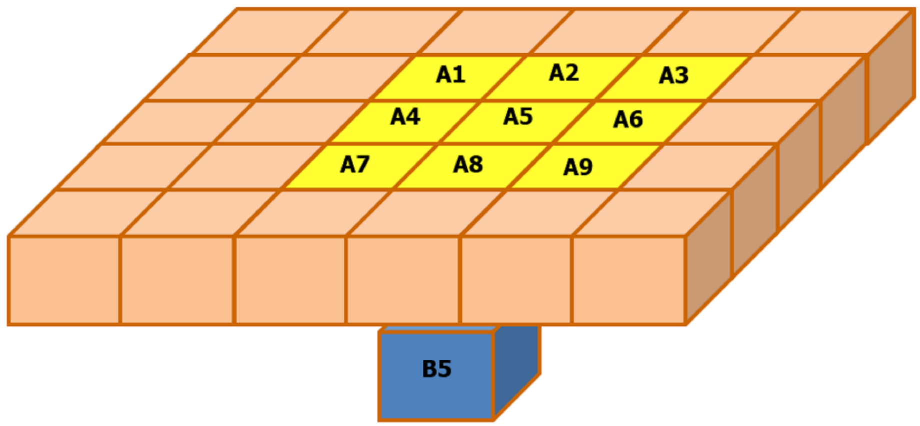


Figure 4-1: An isometric view of the block extraction precedence for OP mining in the MILP model.

The MILP model was implemented for an underground (UG) mine primarily accessed by a shaft. The main ventilation is achieved by a shaft system hosting the main ventilation fans with auxiliary fans serving as booster fans in the mine working areas. Lateral development including levels, waste and ore drives, and crosscuts are developed to link the mining areas for extraction of stopes. The level development extends from the main shaft to the ore and waste drives, and the crosscuts into the various stopes. It is assumed that the proposed model will be applicable for open stoping underground mining methods and caving methods but not configured for underground supported mining methods.

There are different directions of mining that dictate the extraction sequence in an underground mine. A combination of these mining sequences may also be implemented in a typical UG mine. The sequence of extraction is classified based on the vertical and lateral planes of the mine. If the extraction face extends from the lower levels towards the upper levels of the mine, the mining sequence is termed overhand (Bullock, 2011a; Stephen, 2011). However, if the extraction face extends from the upper levels towards the lower levels of the mine, the mining sequence is called underhand. In the lateral plane, if the extraction face moves away from the primary access towards the end of the mining limit, the mining sequence is termed advancing. However, if the extraction face moves from the end of the limit of mining towards the primary access, the mining sequence is referred to as retreating

(Bessinger, 2011; Stace, 2011). Figure 4-2 illustrates a typical overhand and retreating mining sequence while Figure 4-3 depicts a typical underhand and retreating mining sequence in underground mining. Figure 4-4 illustrates a retreating mining sequence while Figure 4-5 depicts an advancing mining sequence in underground mining.

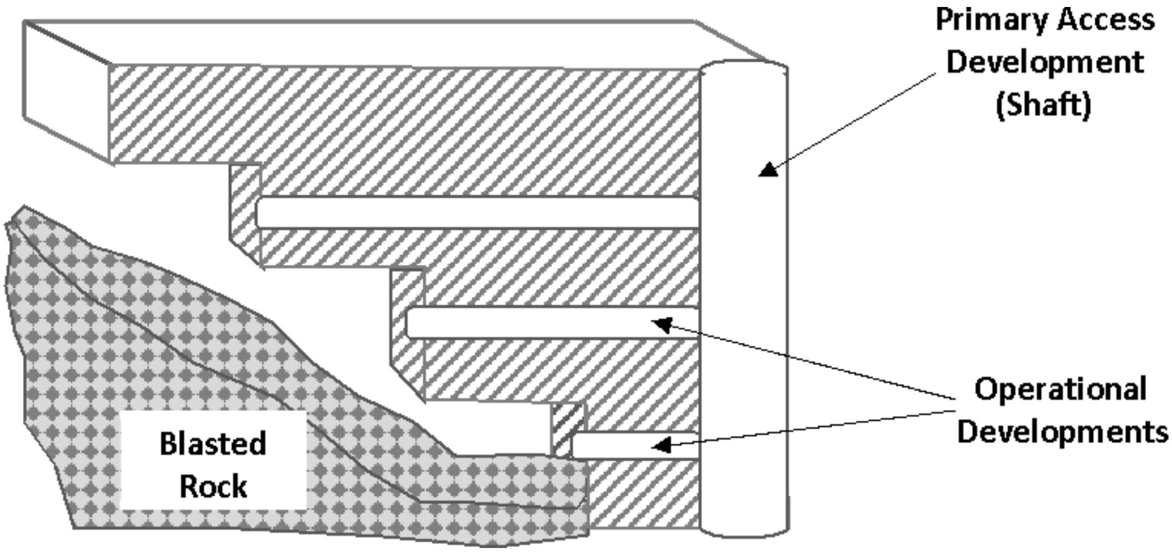


Figure 4-2: Schematic illustration of an overhand and retreating mining sequence in underground mining.

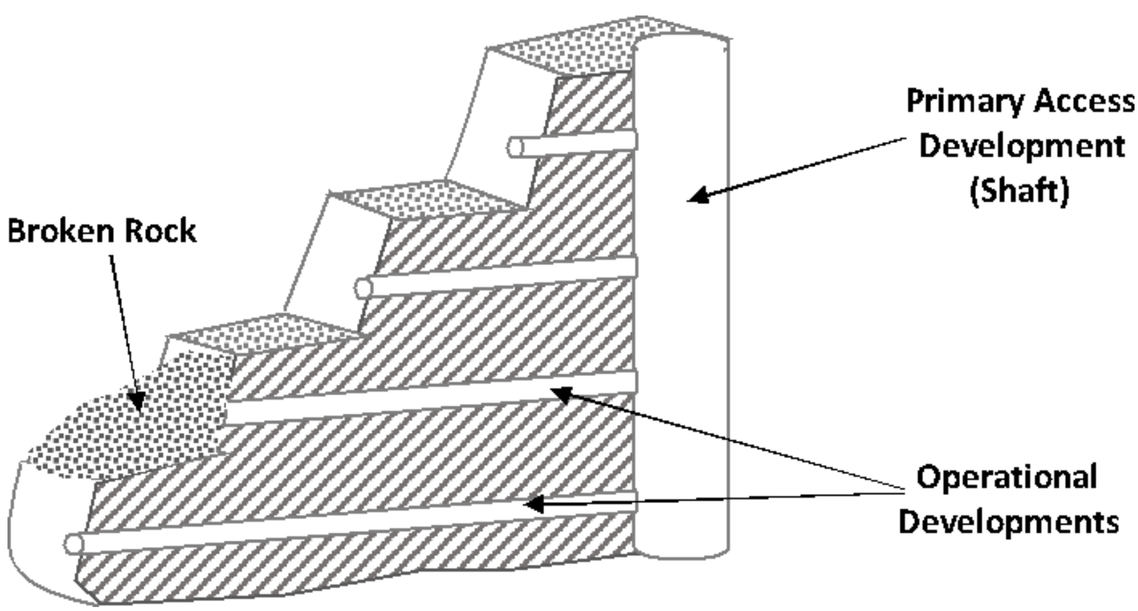


Figure 4-3: Schematic illustration of an underhand and retreating mining sequence in underground mining.

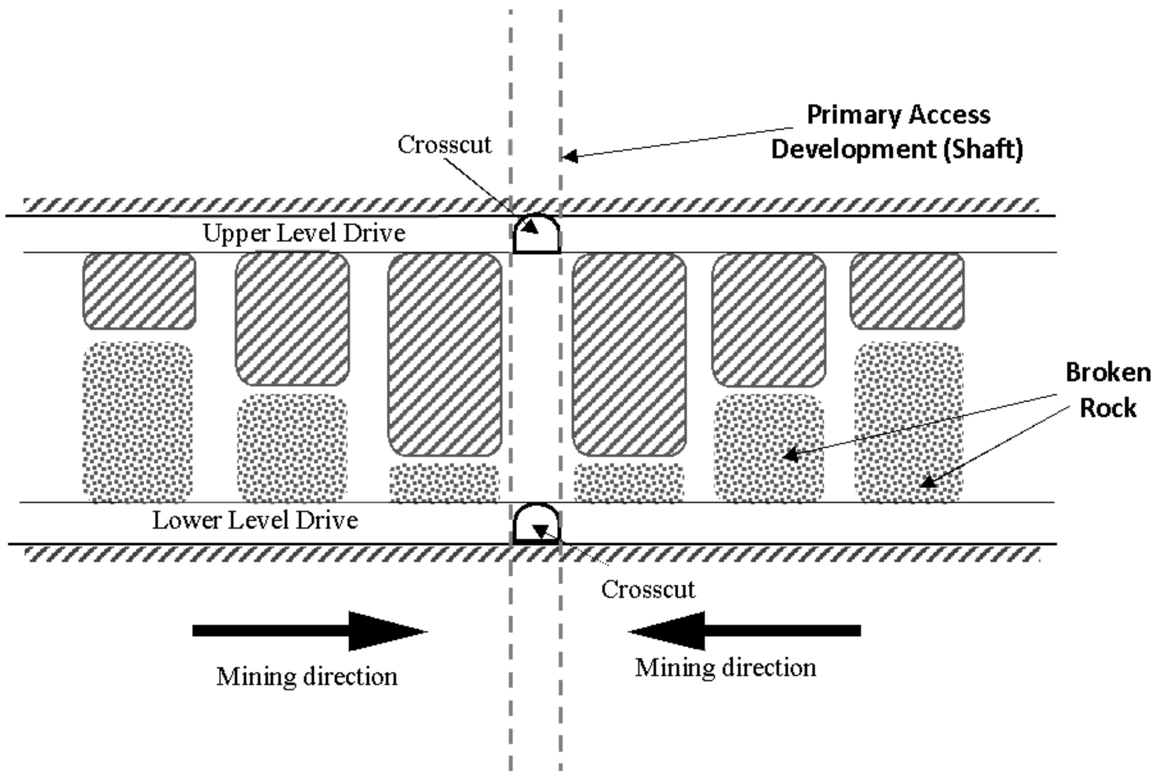


Figure 4-4: Sectional view of a retreating mining sequence in underground mining.

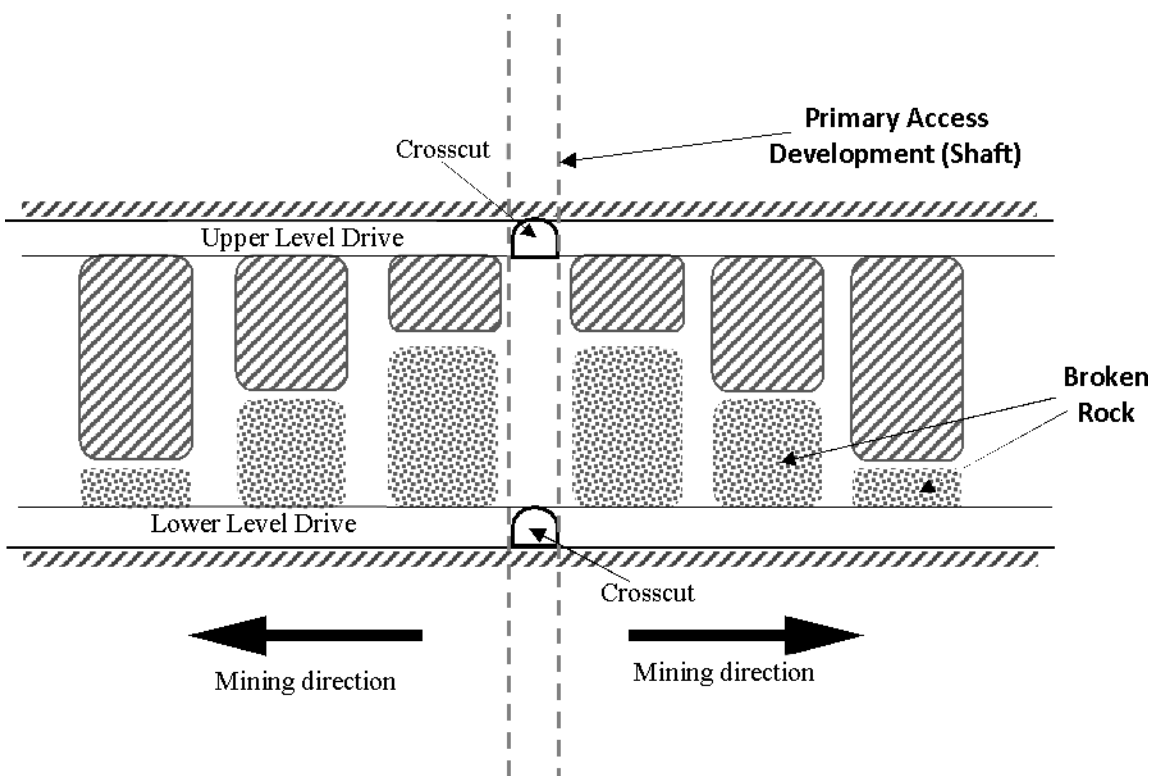


Figure 4-5: Sectional view of an advancing mining sequence in underground mining.

The underhand and retreating sequence of stope extraction is implemented in this conceptual mining system. Thus, for each level, the operational developments are constructed to the end of the mining limit and stope extraction begins at the end of the mining limit and moves toward the main entrance or primary access (shaft) of the mine. During the operational development, the openings are reinforced or supported to secure the rock mass from failing before advancing towards the end of the mining limit. The rock mass reinforcement activity delays the rate of developing the operational openings in the mine. Similarly, the stopes are reinforced during the extraction process to ensure the stability of the stopes. Delays associated with the provision of stope support affect the operational time of the stoping activity.

In many situations, backfilling is integrated into the system of mining to improve ground conditions in the adjoining mining areas. The backfill material holds back the walls of the stopes, and further provides working platforms for mine workers and equipment (Huang, et al., 2020a). Backfill material movement, associated delays in filling void stopes, and the cost of backfilling activities will then impact the operational activities schedule and the net present value of the UG mining project. Backfilling, however, was not considered in the current MILP formulation and will be part of future model extension. Figure 4-6 is an isometric representation of a block model showing the unit blocks (SMUs) and the plan view of the UG primary access (shaft), operational development, end of mining limit, ventilation shaft, and ore extraction sequence.

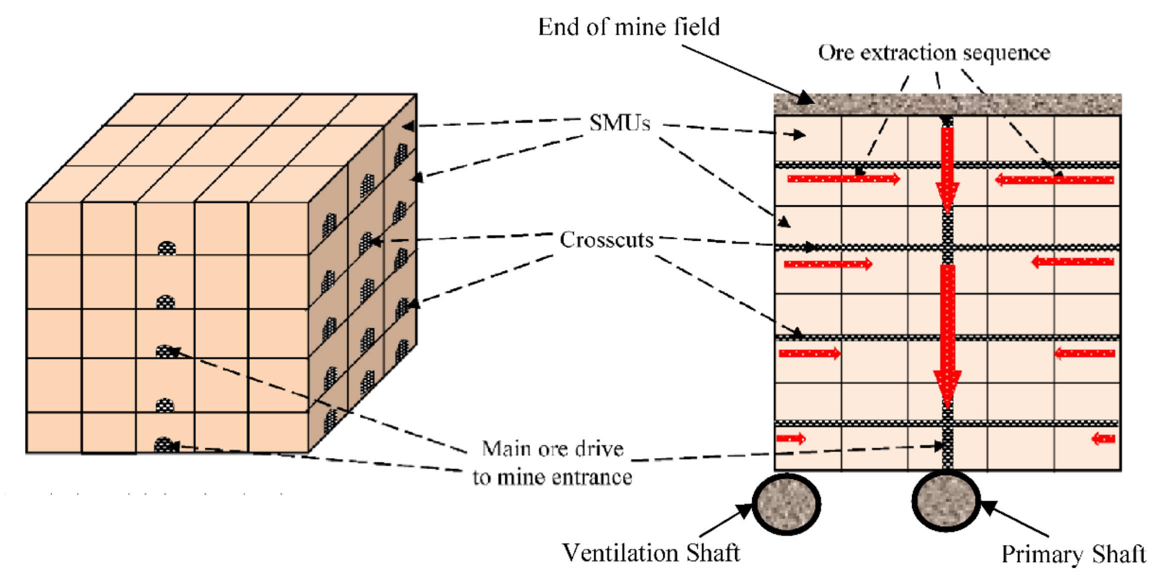


Figure 4-6: Isometric representation of a block model (left), and plan view of UG development layouts, mine workings and arrows showing ore extraction sequence on a level (right) modified after Afum, et al. (2019a).

4.3 Rock support and reinforcement for underground mining

A typical underground mining operation is interspersed with rock supporting and reinforcement systems for the various development openings and stopes. The rock support and reinforcement required for development openings and operating stopes are incorporated in the implementation of the MILP model using the appropriate rock mass classification for each unit block in the block model. For the gold deposit case study, the waste material is associated with a more competent rock mass compared to the ore material. Thus, the assumed corresponding rock mass rating (RMR) value for the ore blocks is 60 while the RMR value for the waste blocks is 72 (Singh, et al., 2015). It is important to note that RMR values are variable within the rock mass, and the values employed in the model implementation is a simplified assumption. For any block that contains both ore and waste, the dominant material type is used in assigning the RMR value. With knowledge of the geotechnical considerations and mining constraints, support systems are designed for the operational development openings in the ore rock formation and waste rock formation, respectively. Similarly, supports are designed for the stopes when it comes into operation.

The cost and time (or delay) expended in providing support and reinforcement in the operational development openings and stopes were estimated based on existing mining practices and incorporated as constraints in the MILP model. The minimum time spent in installing rock support and reinforcement per length of advancing operational development opening and per rock tonnage extracted from a stope in a day were used to determine delay factors associated with providing geotechnical support per period for each case study. Rock support delay factors per operational development length and per stope tonnage were assumed as 0.25 and 0.1, respectively. Thus, for a typical 10-hour shift, about 2.5 hours is used to provide support and reinforcement during excavation of the operational development. Similarly, for a 10-hour shift, about 1 hour is used to support the stopes and prepare it for safe operation. The time spent in providing support in the operational development openings and the stopes are mandatory and therefore constitute delays to those activities. Increasing these delays in a shift will affect the length of operational development openings constructed in the shift and consequently in the year.

In general, decreasing a delay factor decreases the geotechnical support delays and hence increases the project NPV until the time spent in providing the support cannot be reduced any further. The details on the estimation of the RMR values for each unit block in the block model and the delay factors associated with providing geotechnical rock support and reinforcement are outside the scope of this research. An assessment and application of geostatistical methodology in modeling the rock mass quality using RMR is detailed by Egana & Ortiz (2013).

4.4 Case study 1 – synthetic copper deposit

The initial framework of the MILP model was implemented for Case study 1 without the following constraints: i) main UG ventilation development; ii) rock support and reinforcement of the operational development openings and stopes; iii) UG vertical mining direction control; and iv) stockpile management. The implementation did not also factor in mining dilution and recovery for each mining option. The synthetic copper dataset is represented by a geologic block model which is a 3D array of cubical blocks containing 605-unit blocks. These unit blocks represent the selective mining units (SMUs) for OP and UG mining. The orebody in the block model is irregularly shaped with a total mineral resource of 116.10 Mt at an average Cu grade of 1.05%, minimum grade of 0.72% and maximum grade of 3.0%. Figure 4-7 represents a layout of the synthetic copper deposit showing mineralized blocks.

Table 4-1 and Table 4-2 respectively show the statistical description of the deposit, and distribution of metal content in the deposit. The quality of the copper deposit is shown by a few blocks occurring at the top portions of the ore body with higher metal contents. The bottom portions of the deposit show a similar mineralization while low grade blocks occur in the middle sections. The copper grade distribution in the deposit suggests it could be exploited with open pit mining for earlier financial benefit. However, the incremental cost of open pit mining with depth may cause underground mining to compete as the better option at a certain depth during the mineral exploitation process. Thus, making this deposit a good candidate for OP-UG mining options and transitions evaluation. In implementing the basic MILP model on the synthetic copper dataset, constraints for ventilation development, and geotechnical support and stockpile management were excluded from this initial deployment.

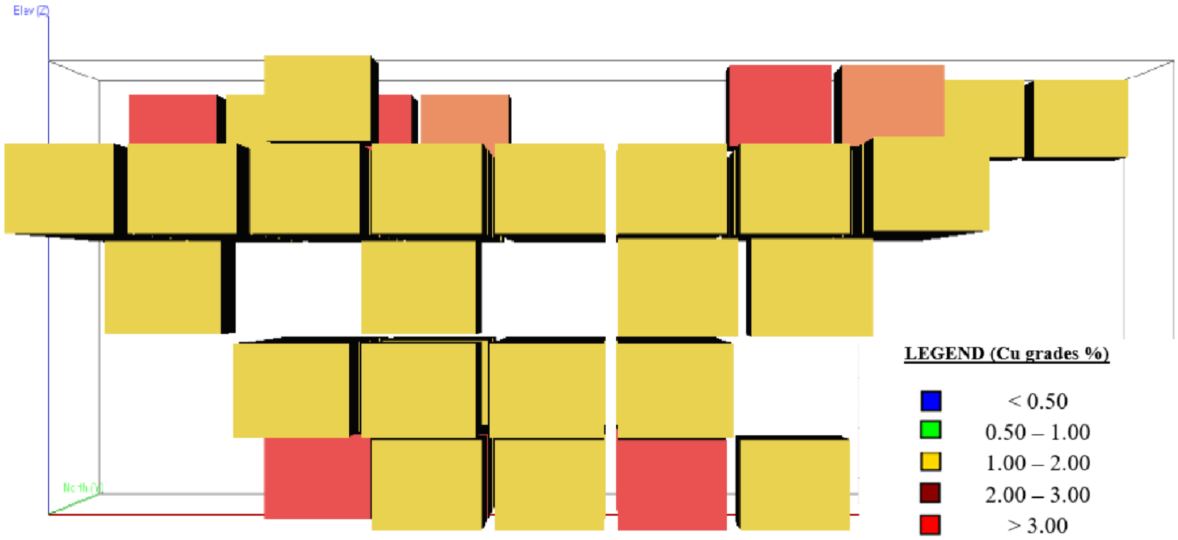


Figure 4-7: Layout of the synthetic copper deposit showing mineralized blocks (Afum & Ben-Awuah, 2019).

Table 4-1: Statistical description of the synthetic copper deposit.

Description	Value
Total mineralized material (Mt)	116.10
Minimum value of Cu (%)	0.719
Maximum value of Cu (%)	3.000
Average value of Cu (%)	1.051
Variance ($\%^2$)	0.394
Standard deviation (%)	0.628
Number of levels/benches	5

Table 4-2: Distribution of metal content in the deposit.

Level no.	Total Cu metal content (Mt)	Total ore tonnage (Mt)	No. of mineralized blocks
1	0.1910	12.15	9
2	0.4022	52.65	39
3	0.405	5.40	4
4	0.1574	20.25	15
5	0.4284	25.65	19

4.4.1 Economic, mining and processing data for Case study 1

The yearly processing capacities are determined based on the proposed plant capacity for the mine while the yearly mining capacities are deduced from the ore and waste proportions of the deposit. An incremental bench cost of $ 4.0 per 15 m bench was used as the open pit mining variable cost as the pit extends deeper. The economic, mining and processing data used for evaluating the copper deposit as summarized in Table 4-3 were estimated using data from CostMine (2016) and pre-feasibility study reports of two mining companies in Canada

(Puritch, et al., 2016; Sirois & Gignac, 2016). All economic parameters used in the computations were in Canadian dollars.

Table 4-3: Economic, mining and processing data for evaluating the copper deposit.

Parameter	Value
Open pit mining cost ($/t)	8.0
Underground mining cost ($/t)	300.0
Processing cost ($/t)	15.0
Selling cost ($/lb)	1.50
Selling price of copper ($/lb)	4.65
Discount rate (%)	10.0
Processing recovery (%)	95.0
Max open pit (OP) ore extraction capacity (Mt/year)	8.0
Min open pit (OP) ore extraction capacity (Mt/year)	0.0
Max open pit (OP) rock mining capacity (Mt/year)	20.0
Min open pit (OP) rock mining capacity (Mt/year)	0.0
Max underground (UG) ore extraction capacity (Mt/year)	3.0
Min underground (UG) ore extraction capacity (Mt/year)	0.0
Max underground (UG) rock mining capacity (Mt/year)	3.0
Min underground (UG) rock mining capacity (Mt/year)	0.0
Max open pit & underground (OPUG) processing plant capacity (Mt/year)	8.0
Min open pit & underground (OPUG) processing plant capacity (Mt/year)	0.0
Incremental bench cost ($/15 m)	4.0
Operating development cost ($/m)	7,000
Capital development cost ($/m)	15,000
Max operating development (m/year)	5,000
Max capital development (m/year)	30

4.4.2 Results and discussion for Case study 1

Evaluation of the synthetic copper deposit with the integrated multi-objective Mixed Integer Linear Programming (MILP) model indicates that a combined sequential and simultaneous open pit and underground (OPUG) mining operation is the preferred mining option with the highest Net Present Value (NPV) of $ 2.66 billion. The integrated MILP model was modified to evaluate the ore body for an independent open pit (OP) mining option and an independent underground (UG) mining option with the required crown pillar, primary access and secondary (operational) developments. The NPV of the OPUG mining option is about 14.4% better than an independent OP mining option and about 24.1% better than an independent UG mining option. The evaluated results are shown in Table 4-4. The copper deposit is therefore best exploited with a combined open pit and underground (OPUG) mining option for a mine life of 14 years with Level 3 acting as the unmined crown pillar.

Table 4-4: Results from the integrated MILP model for Case study 1.

Mining option	Net present value ($ B)	NPV of OPUG compared (%)	Processed ore tonnage (Mt)	Resource depletion (%)	Mine life (years)
Open pit mining	2.28	-14.4	116.00	100.0	25
Underground mining	2.02	-24.1	103.95	89.60	15
Combined open pit and underground mining	2.66	-	100.05	86.30	14

With a total ore production of 100.05 Mt from the combined OPUG mining option with crown pillar, the OP mining operation contributes 64.80 Mt of ore while the UG mining operation contributes 35.25 Mt of ore. The remaining 15.95 Mt of the available mineral resource is either left in the crown pillar (5.40 Mt) or left as unmined low-grade stopes (10.55 Mt). In a more practical underground mine, where the crown pillar is recovered during the operational life of mine, the 5.40 Mt of ore will be mined to further improve the total net present value (NPV) of the project if the ore quality is economical. The mine life of the independent OP mining option was 25 years, about 40% and 44% higher than the mine life of the independent UG mining option and the combined OPUG mining option, respectively.

A sectional view through the ore body showing the open pit limit, location of the required crown pillar, an unmined low-grade stope and the mineralized zone extending beyond the bottom of the open pit limit is shown in Figure 4-8. The crown pillar is located on Level 3; thus, the first 2 levels (or benches) are extracted by open pit mining while Levels 4 and 5 are extracted by underground mining.

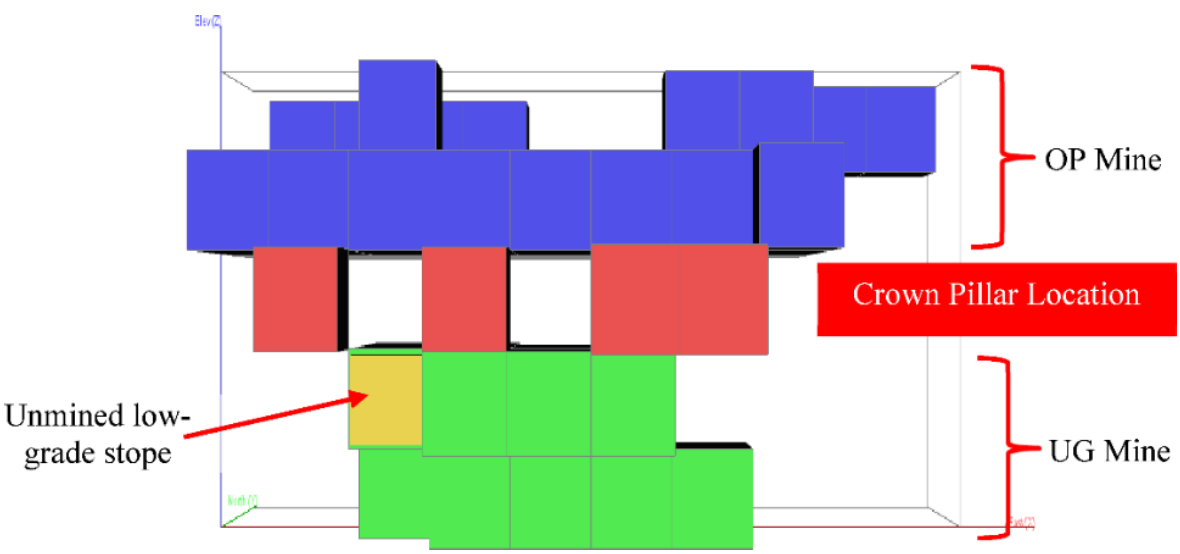


Figure 4-8: A sectional view through the block model showing the open pit limit, crown pillar and underground mining regions for Case study 1 (Afum & Ben-Awuah, 2019).

The ore and rock extraction schedules for the combined open pit and underground mining (OPUG) option with a crown pillar are shown in Figure 4-9 and Figure 4-10 respectively. The ore is extracted by an independent open pit mining option in the first 2 years of the mine life before underground ore production begins simultaneously from the 3rd year through to the 12th year. The ore production schedule switches completely to an independent underground mining option in the remaining 2 years of the mine life. In summary, the ore extraction profile for the combined OPUG mining option is a blend of sequential and simultaneous open pit and underground mining options for a mine life of 14 years. Due to the outcrop of the ore body, the ore extraction schedule in the first year of the mine life satisfies the full capacity of the plant requirement of 8.0 Mt.

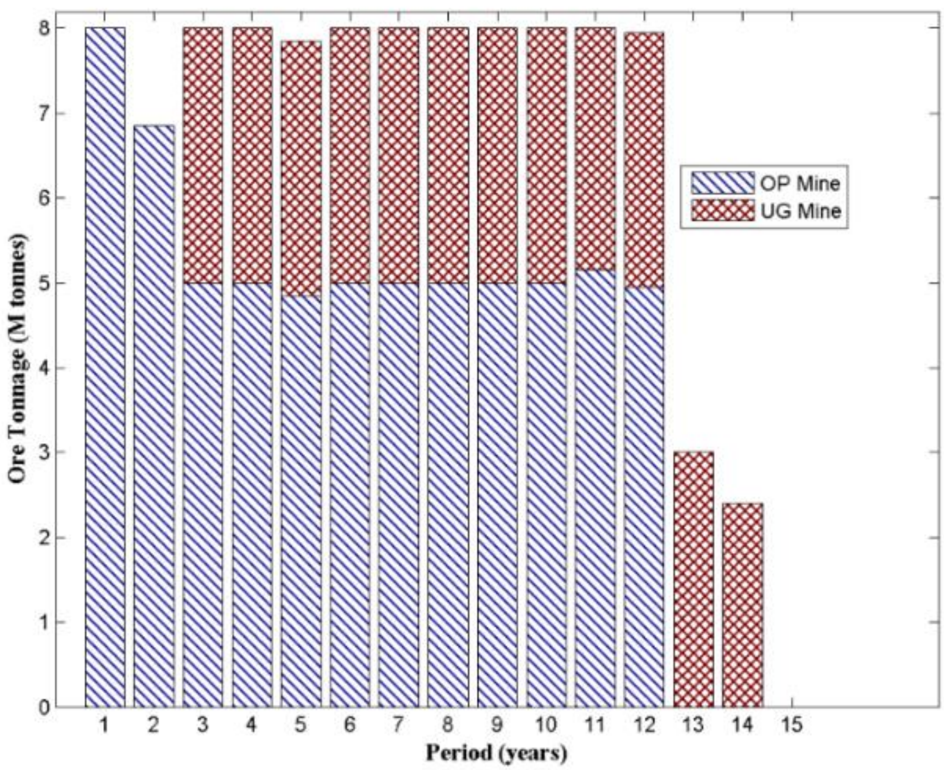


Figure 4-9: Yearly ore production schedule for the OPUG mining option (Afum & Ben-Awuah, 2019).

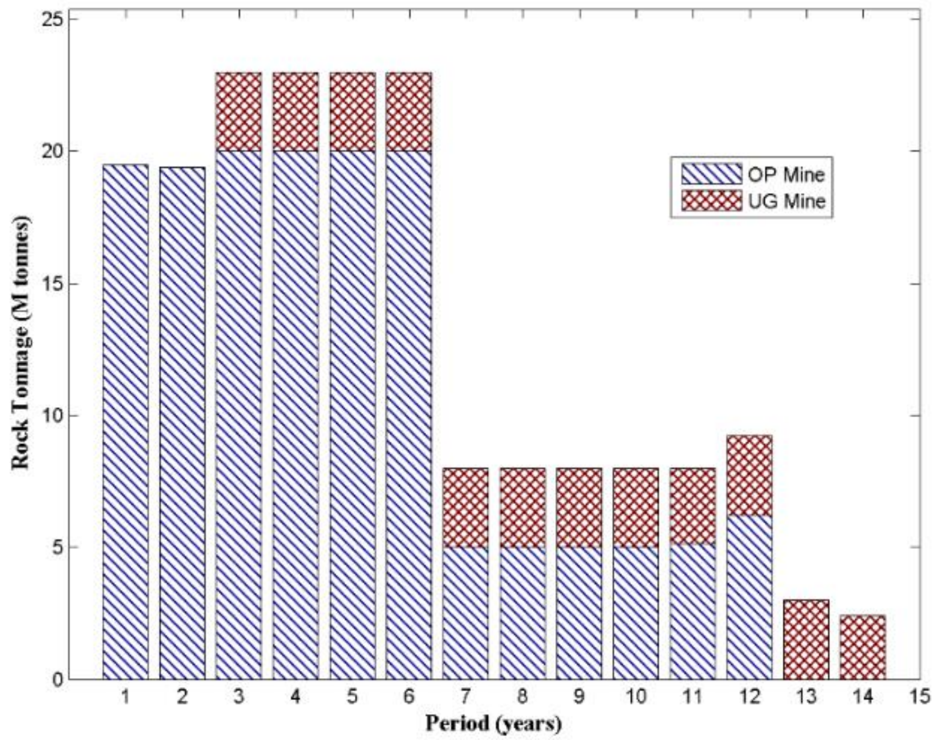


Figure 4-10: Yearly rock production schedule for the OPUG mining option (Afum & Ben-Awuah, 2019).

The average grade of Cu ore processed in each period for the combined open pit and underground mining option with a crown pillar is shown in Figure 4-11. In Figure 4-11, the yearly average grade of processed ore for the underground mining operation is generally higher compared to the open pit mining operation. This shows that underground mining prioritizes the extraction of high-grade ores to cover the high cost of mining by generating higher revenues. It is convincing that the grade blending constraints prioritizes higher grades over lower grades in exploiting the mineral deposit by any of the mining options. Thus, the open pit mining operation targeted the outcropped high-grade ore while the underground mining operation targeted the deep-seated high-grade ore in the early years of mine life.

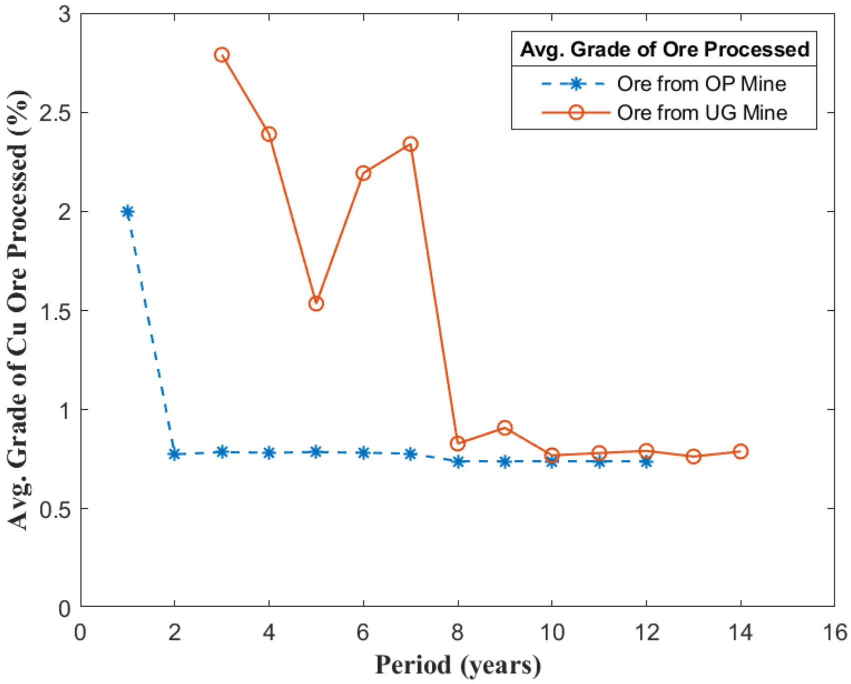


Figure 4-11: Yearly average grade of processed ore for OPUG mining option (Afum & Ben-Awuah, 2019).

To highlight the nature of the underground mining operation and obtain further information on the strategic exploitation method, the levels contributing to the mineral extraction schedule are shown in Figure 4-12. With the crown pillar located on Level 3, ore extraction by the underground mining operation started on the last level, Level 5, from the 3rd year through to the 7th year of the mine life before continuing with ore production from Level 4 to the end of mine life. With this knowledge on the mining sequence from each level (Figure 4-12) and the geotechnics of the rock formation, any suitable underhand method of ore extraction could be further evaluated and selected as the appropriate underground exploitation method to develop this deposit.

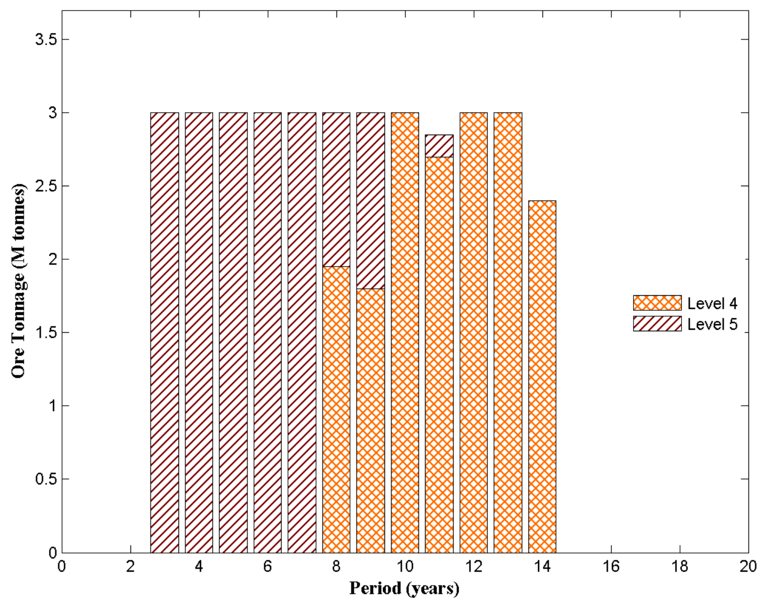


Figure 4-12: Yearly ore production tonnage extracted per level for the UG mining operation in the OPUG mining option (Afum & Ben-Awuah, 2019).

The operational development schedule for the underground mine is shown in Figure 4-13. With a total operational development capacity of 10,000 m per year, advancement starts immediately after completion of the capital development (shaft) in the 3rd year but ends in the 6th year before commencing again from Years 8 to 12. From Figure 4-9, additional operational development capacity may be required in Years 3 and 8 through contract mining to support the operational development schedule.

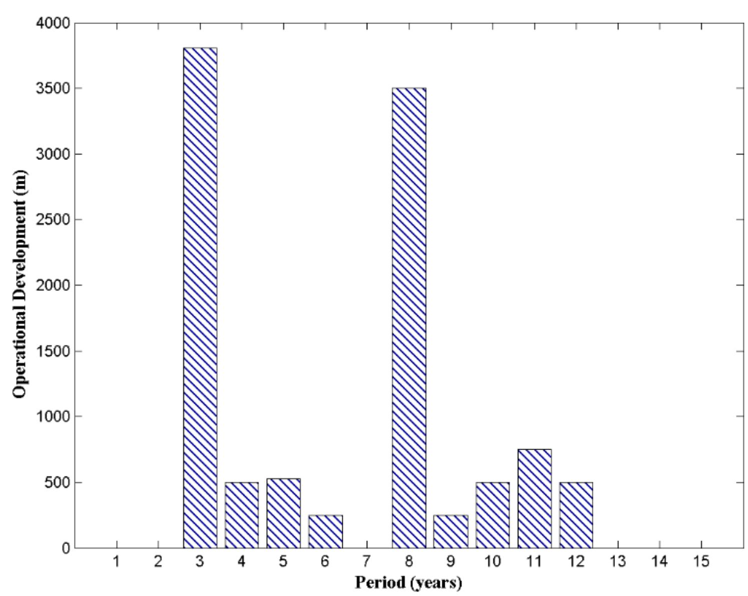


Figure 4-13: Operational development schedule for the UG mining operation in the OPUG mining option (Afum & Ben-Awuah, 2019).

4.4.3 Sensitivity analysis for Case study 1

The sensitivity of the results to copper price, underground processing capacity and underground capital development (shaft) is evaluated with the MILP model. This sensitivity analysis was conducted to examine the selected technical or economic parameters that influence the choice of mining option(s) for the deposit. The sensitivity plot is shown in Figure 4-14. In Figure 4-14, the MILP model is most sensitive to the market price of copper, followed by the quantity of ore processed from the UG mining operation and the completion rate of capital development.

For this case study, about 9% decrease in the copper price (from $ 3.18 to $ 2.89 per lb) will change the optimal mining option from combined OPUG mining to an independent OP mining option for a reduced NPV of $ 2.28. Similarly, when the underground processing capacity decreases by about 73% (0.81 Mt), the combined OPUG mining will change to an independent OP mining option. The synthetic copper deposit is however less sensitive to the completion rate of underground capital development (shaft) as it did not cause a change in mining option within the changes tested.

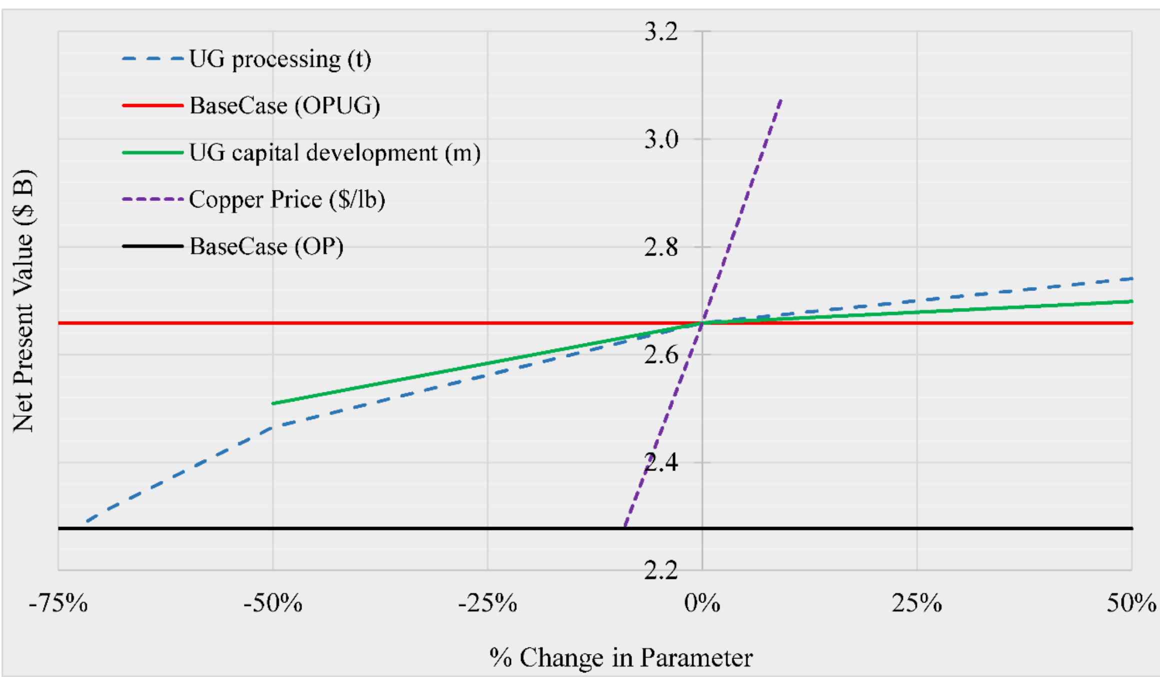


Figure 4-14: Sensitivity assessment of selected technical and economic parameters used in the MILP implementation (Afum & Ben-Awuah, 2019).

4.4.4 Summary findings from Case study 1

The proposed MILP model interrogated the synthetic copper deposit and determined the optimal extraction strategy for a combination or one of these mining options: (a) independent OP mining, (b) independent UG mining with crown pillar, (c) simultaneous OPUG mining with crown pillar, (d) sequential OPUG mining with crown pillar, and (e) combinations of simultaneous and sequential OPUG mining with crown pillar. The location of the 3D crown pillar together with the required capital and operational development schedules are decided by the optimization process. The results from the case study showed a combined sequential and simultaneous OPUG mining option with crown pillar was selected as the optimal option to exploit the deposit. The ore and rock extraction schedules for both mining operations together with the operational and capital development schedules were determined for the synthetic copper project. The output of the model in mapping-out the ore extraction per level in each period further provided more insight into the mining sequence for the appropriate UG mining method. The sensitivity analysis conducted on the selected technical and operational parameters indicated that the determination of the optimal mining option using the MILP model is most sensitive to the selling price of copper, followed by the quantity of ore processed by the UG mining operation and the completion rate of the UG capital development (shaft).

4.5 Case study 2 – gold deposit

For Case study 2, the MILP framework was implemented without the following constraints: rock support and reinforcement of the operational development openings and stopes, UG vertical mining direction control, and stockpile management. Mining dilution and recovery were also not included in this implementation. The geologic block model of the gold deposit has unit block sizes of 30 m x 30 m x 20 m. To implement the MILP model, it is assumed that the unit block sizes represent the mining-cut sizes of the OP mining operation and the stope sizes of the UG mining operation. The total mineral resource is 19.20 Mt at an average gold grade of 4.39 g/t. Figure 4-15 is a layout of the gold deposit showing mineralized blocks (Afum, et al., 2019a). The gold deposit shows high-grade mineralization at the top and bottom sections of the block model, while the middle sections show lower grades. The Rock Mass Rating (RMR) assumed for ~~of~~ the ore and waste zones in the block model indicates that the waste rocks are stronger than the ore rocks. Table 4-5 is a parametric description of the gold deposit. This gold deposit is similar in size to the Pinarbasi chromite mine in Turkey (Elevli, et al., 2002). Due to the shallow depth of the chromite deposit, Elevli, et al. (2002) reported that traditional method of shaft sinking for small scale mines by means of drilling, blasting, mucking, hoisting, pumping, ventilation and lining was employed. The total shaft depth of approximately 120 m from the surface was completed over two years with sinking buckets ranging in capacity from 0.5 to 1.5 tonnes.

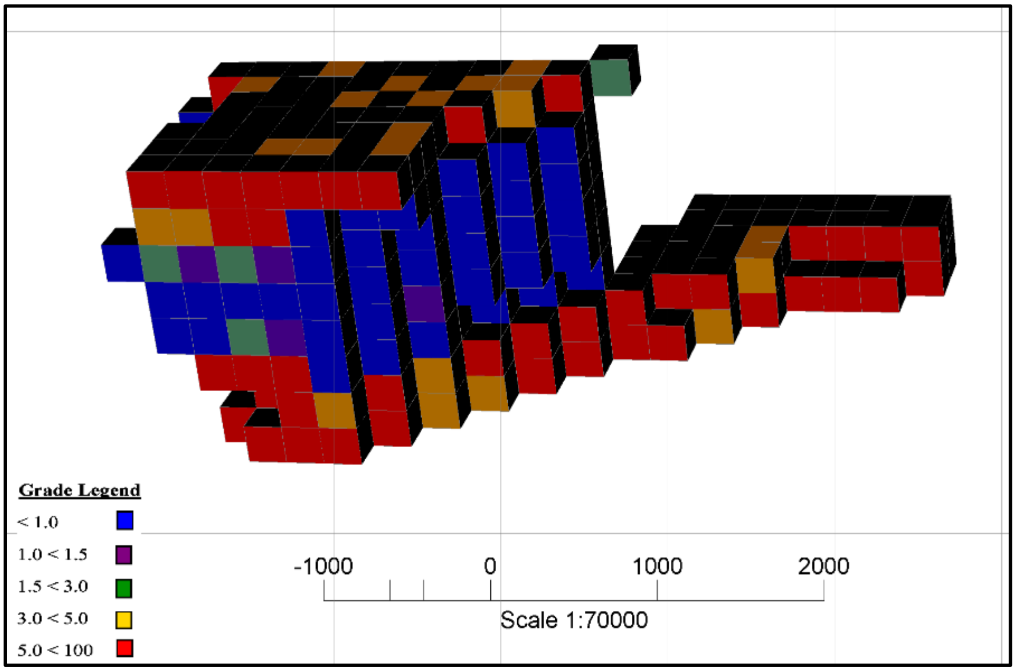


Figure 4-15: Layout of the gold deposit showing mineralized blocks modified after Afum, et al. (2019a).

Table 4-5: Parametric description of the gold deposit

Description (units)	Value
Total mineralized material (Mt)	19.15
Minimum value of Au (g/t)	0.01
Maximum value of Au (g/t)	14.78
Average value of Au (g/t)	4.39
Variance $(g/t)^2$	16.54
Standard deviation (g/t)	4.07
Number of levels/benches	8
Rock mass rating of ore blocks	60
Rock mass rating of waste blocks	72

4.5.1 Economic and technical data for Case study 2

The technical and economic data used for implementing the MILP model to evaluate the gold deposit was estimated from CostMine (CostMine, 2016) and prefeasibility reports of similar gold mining operating companies in Canada (Puritch, et al., 2016; Sirois & Gignac, 2016). The annual processing capacities are based on the proposed processing plant capacity for the mine while the yearly mining capacities are deduced from the ore and waste proportions of the gold deposit. Incremental bench cost of $ 2.00 per 10 m, following the NI 43-101 report of Centerra Gold Inc. and Premier Gold Mines Ltd. (Sirois & Gignac, 2016), was used as the open pit variable cost as the pit transcends downwards. The incremental bench cost is a necessary variable cost in the reporting of mineral reserves in the mining industry and it is an important parameter in the implementation of the MILP model. It was assumed that there was no external stope dilution and mining recovery losses. Table 4-6 details the technical and economic data used for the implementation of the MILP model for the gold deposit case study. All economic parameters used in the computations were in Canadian dollars

Table 4-6: Economic, mining and processing data for evaluating the gold deposit

Parameter (units)	Values
Open pit mining cost ($/t)	8.00
Underground mining cost ($/t)	200.00
Processing cost ($/t)	15.00
Selling cost ($/oz)	50.00
Selling price of gold ($/oz)	1,860.00
Discount rate (%)	5
Processing recovery (%)	90
Max open pit (OP) ore extraction capacity (Mt/year)	2.000
Min open pit (OP) ore extraction capacity (Mt/year)	0.000
Max open pit (OP) rock mining capacity (Mt/year)	5.000
Min open pit (OP) rock mining capacity (Mt/year)	0.000
Max underground (UG) ore extraction capacity (Mt/year)	1.125
Min underground (UG) ore extraction capacity (Mt/year)	0.000
Max underground (UG) rock mining capacity (Mt/year)	2.500
Min underground (UG) rock mining capacity (Mt/year)	0.000
Max open pit & underground (OPUG) processing plant capacity (Mt/year)	2.500
Min open pit & underground (OPUG) processing plant capacity (Mt/year)	0.000
Incremental bench cost ($/10 m)	2.00
Operational development cost ($/m)	7,000.00
Capital or primary access development cost ($/m)	16,000.00
Max operating development (m/year)	10,000.00
Min operating development (m/year)	0.0
Max capital or primary access (shaft) development (m/year)	40.0

Min capital or primary access (shaft) development (m/year)	0.0
Max main ventilation shaft development (m/year)	40.0
Min main ventilation shaft development (m/year)	0.0
Main ventilation development cost ($/m)	1,600.00
Cost of supporting the operational development openings ($/m)	1,000.00
Cost of supporting the stopes ($/tonne)	80.00
Rock support delay factor for operational development length per year	0.25
Rock support delay factor for stopes tonnage per year	0.10

4.5.2 Results and discussion for Case study 2

Figure 4-16 is a sectional view of the optimal OPUG mining option for Case study 2 showing the location of the shaft, operational development openings, crown pillar position, OP mining limit, UG mining limit, and periods of extraction shown by the numbering of the blocks. The blocks within the OP mining limit are exploited from the surface towards the bottom from Levels 1 to 3. The unmined crown pillar is located on Level 4. The stope extraction sequence demonstrated in Figure 4-16 shows a typical overhand mining system.

Figure 4-16: Sectional view of the selected combined OPUG mining option for Case study 2 (Afum, et al., 2019a).

The optimized extraction option suitable for the gold deposit case study using the integrated multi-objective mixed integer linear programming (MILP) model is combined sequential and simultaneous open pit and underground (OPUG) mining with a net present value (NPV) of $ 2.52 billion. The gold mining operation commences with an independent open pit (OP) mine in the first 3 years. Underground (UG) mining follows simultaneously with the OP mine in the 4th and 5th years. In the 6th year, the gold mining project transitions sequentially from simultaneous OPUG mining to an independent UG mine until the end of mine life.

Figure 4-17 shows the ore extraction (processing) strategy and the average ore grade processed by each mining option for the gold deposit case study. In Figure 4-17, the yearly trend of the average grade of ore extracted and processed by both mining options indicate that the optimization technique, as in all cases of mining, prioritized the relatively high-grade blocks over the lower grade blocks in the early years of mine life. The life of mine of the gold deposit case study is determined as 12 years. Ore mining recovery of 16.56 Mtonnes, constituting about 87% of the total available mineral deposit of 19.15 Mtonnes is achieved. About 7.61 Mtonnes of ore is extracted by OP mining operation while 8.95 Mtonnes of ore is extracted by UG mining operation. The remaining mineralized rock material of 2.59 Mtonnes (about 14%) is either left *in-situ*, lost in the unmined crown pillar or delivered to the waste dump as low-grade ore material.

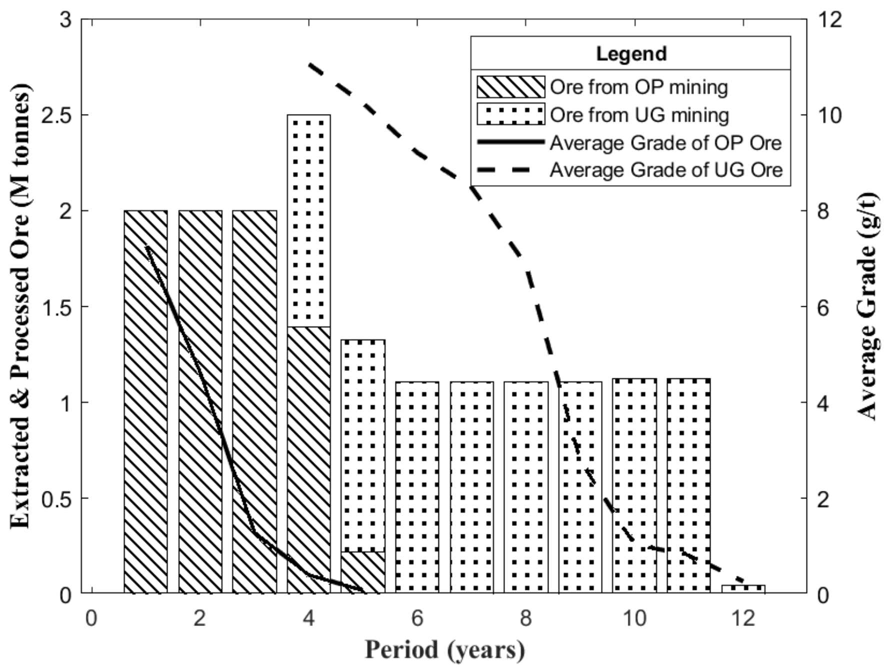


Figure 4-17: Ore extraction (processing) strategy and average ore grade processed by each mining option

The schedule for the total rock material extracted consisting of both ore and waste rocks is shown in Figure 4-18. The total rock material extracted by the OP operations indicate a gradual increase in waste tonnage to maintain a uniform plant feed. In the 3rd year of mine life, there is significant waste stripping compared to ore extraction (Figure 4-17 and Figure 4-18). Thus, there is some considerable pushback to be undertaken in the 3rd and 4th years of mine life to uncover ore material for OP extraction. It can be deduced that in the 4th year, when OP mining operations become less profitable due to significant waste stripping, UG mining operation takes over as it becomes more profitable.

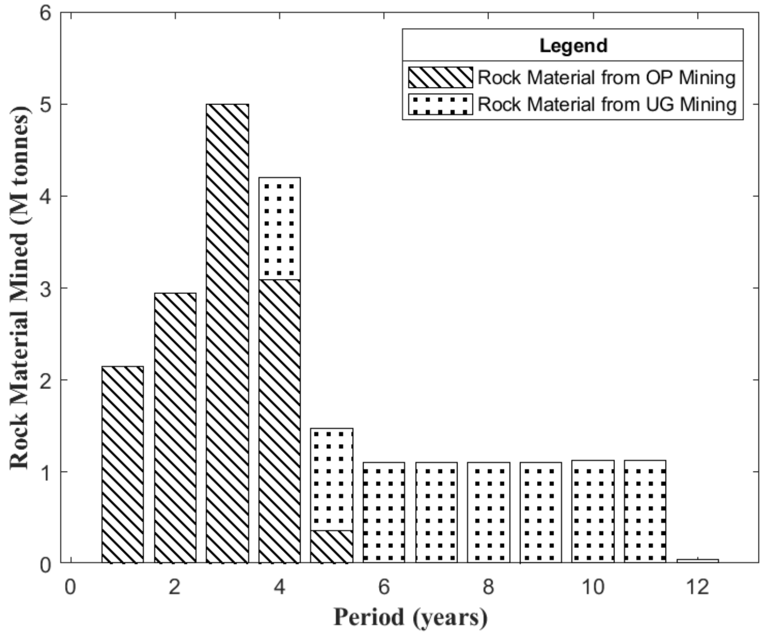


Figure 4-18: Mining schedule (ore and waste) for each mining option

The schedules for the development of the primary access and the lateral secondary or operational openings (levels, waste and ore drives, crosscuts) are shown in Figure 4-19. The primary access development starts in Year 1, during the OP mining operation and ends in Year 4, before secondary development starts. The secondary development which constitutes the operational development, occur from the 4th year to a year before the end of mine life.

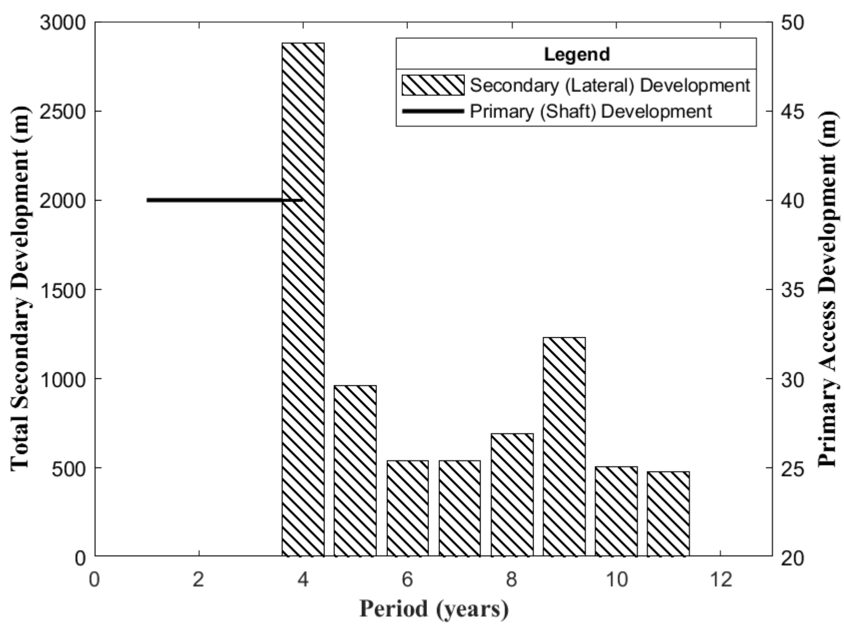


Figure 4-19: Primary access development and lateral secondary or operational development schedules for the UG mining option

Figure 4-20 shows the ore extraction schedule on each level of the UG mining option. The UG mining operation is located from Levels 5 to 8. The unmined crown pillar is positioned on Level 4 and the UG ore extraction starts from the high-grade mineralized zones on Level 8 towards the low-grade mineralized zones on Level 5. The ore extraction proceeds upward from Level 8 to Level 5 through Levels 7 and 6 in that order. On each level, the stope mining is retreating from the end of the mining limit towards the primary access (shaft) using open stoping mining methods.

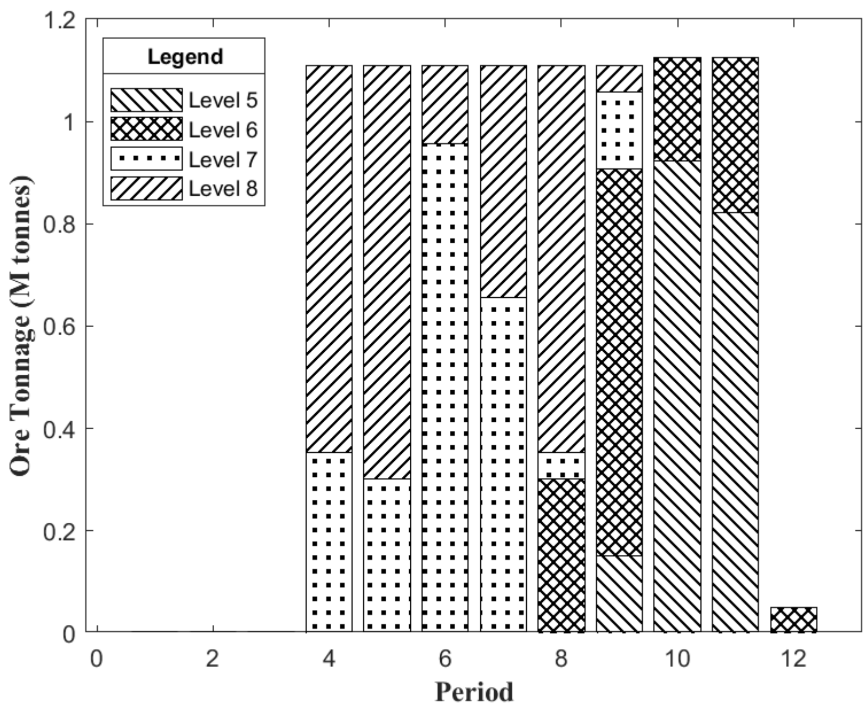


Figure 4-20: Yearly ore extraction schedule on each level of the UG mining option

The main ventilation development schedule and the delay schedules associated with providing rock support and reinforcement in the secondary or operational development openings and stopes are shown in Figure 4-21. As expected, development of the main ventilation shaft, a primary development, is completed before the development of the secondary openings and the associated support and reinforcement required for the operational development. The delays associated with supporting the operational development openings and stopes per period, converted to days show that much time is required in supporting the development and open stopes to ensure smooth ore delivery.

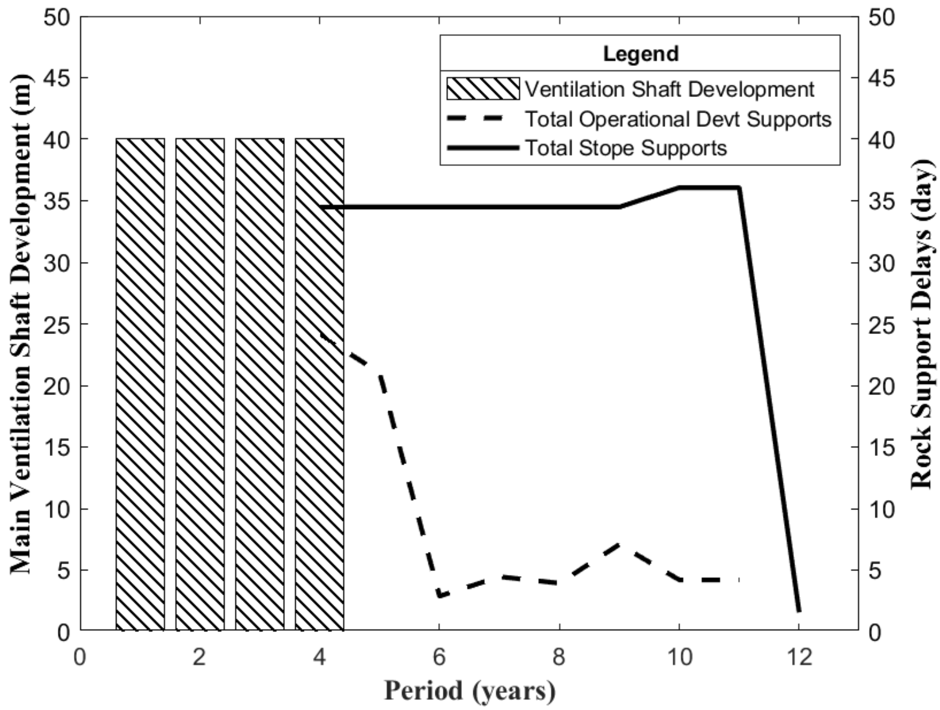


Figure 4-21: Schedules for main ventilation development and geotechnical rock support and reinforcement provided in the secondary development openings and stopes

4.5.3 Sensitivity analysis for Case study 2

Due to the practical variations in technical and economic parameters used in the MILP model implementation, sensitivity analysis is conducted on some selected parameters. These include: i) the delay factors for geotechnical rock support and reinforcement for operational development openings and stopes; ii) the quantity of ore being delivered from the UG mining operation to the processing plant; and iii) the price of the commodity. The model is deployed for OP mining option only when the quantity of ore being extracted and processed from the UG mine is constrained to zero. About 11% reduction in the gold price, from \$ 1,860.00 to \$ 1,660.00, changed the preferred optimal OPUG mining option with NPV of \$2.52 billion and mineral resource recovery of 16.60 Mt (about 87%) to an independent OP mining with NPV of \$2.23 billion and mineral resource recovery of 19.00 Mt (about 99%). Figure 4-22 shows the sensitivity analysis of selected parameters used in the evaluation of the gold deposit case study. The analysis shows the extent of influence of these selected parameters on the NPV of the optimal OPUG mining option. The optimal OPUG mining option was used as the baseline.

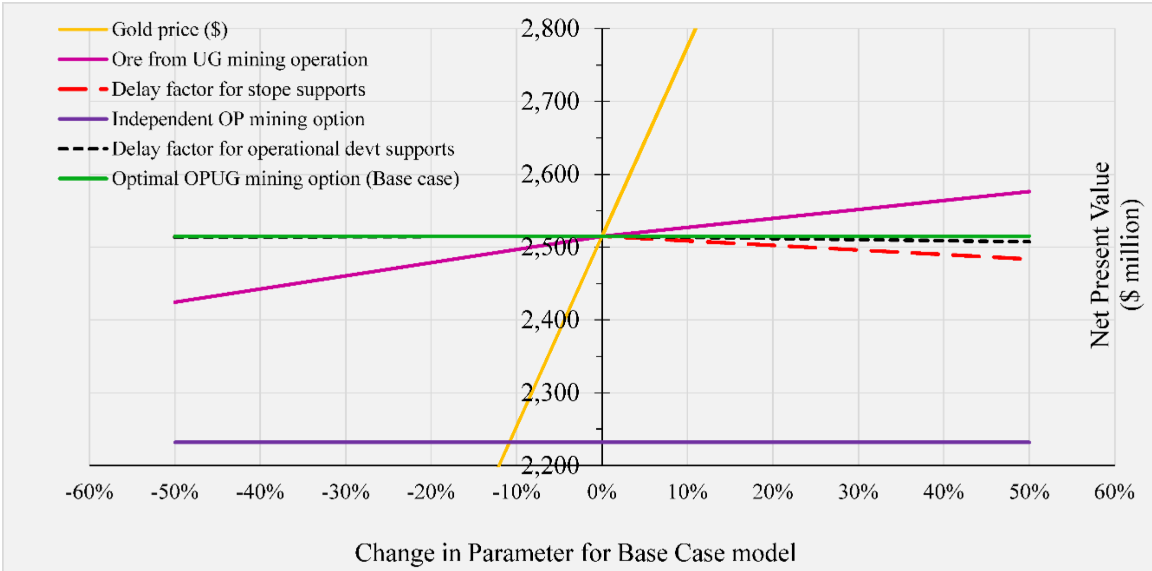


Figure 4-22: Sensitivity analyses for delay factors and ore processed from UG mining operation. The computed NPVs are compared to the optimal OPUG mining option as baseline

From Figure 4-22, the NPV of the optimal mining option is significantly sensitive to the quantity of ore being delivered from the UG mining operation to the processing plant. The higher the quantity of high-grade ore being processed from UG mining operation, the more likelihood UG and/or OPUG mining option become(s) favorable for selection as the optimal mining option. The sensitivity of the quantity of ore from the UG mining operation in an OPUG mining project was also noted by Afum & Ben-Awuah (2019) in their experimental work. It is further noted that, positive changes in the delay factors (which implies additional delays) associated with operational development support and mining stope support have more impact on the NPV than negative changes. As expected, increasing the delay associated with the installation of support and reinforcement in the operational development openings and stopes (increasing delay factor) reduces the NPV of the optimal mining option. Figure 4-22 further shows that, there is always unavoidable minimum constant delay associated with providing some form of support to the operational development openings and stopes for the defined mining and processing targets. This does not give room for any further reduction in rock support and reinforcement delays to increase the NPV of the optimal UG/OPUG mining option.

4.5.4 Summary findings from Case study 2

The objective function of the MILP model determines: (a) position of the required crown pillar; (b) primary access development schedule; (c) ventilation development schedule; (d) operational development schedule; (e) rock support and reinforcement schedule for operational development; (f) rock support and reinforcement schedule for operational stopes; (g) extraction strategy and schedule for the optimal mining option(s); (h) life of mine; and (i) net present value (NPV) of the mining operation. Results from the implementation indicated that the gold deposit case study (Case study 2) is optimally exploited by a combined sequential and simultaneous OPUG mining option with crown pillar. The model determined the NPV, life of mine, location of the crown pillar, and schedules for primary access development, operational development, main ventilation shaft, and rock support and reinforcement for the gold project. Ore extraction starts through independent OP mining from Year 1 to Year 3, and after completion of the development of the main ventilation shaft, primary access (shaft) and operational openings, the mine transitions into a simultaneous OPUG mining in Years 4 and 5. Ore extraction switches completely from simultaneous OPUG mining to independent UG mining from Year 6 to Year 12. The Net Present Value (NPV) is estimated as $ 2.52 billion at a gap tolerance of 5% and a mine life of 12 years. Sensitivity analysis on gold price indicated that, the optimal OPUG mining option with resource recovery of about 87% and NPV of $ 2.52 billion will switch to an independent OP mine with NPV of $ 2.23 billion and a resource recovery of about 99% when the gold price falls by 10.8%. The NPV is proportionally sensitive to the quantity of ore being delivered from the UG mining operation. It is however only sensitive to an increase in the delays associated with providing rock support and reinforcement in the secondary or operational development openings and stopes.

Additional experiments will be undertaken in future by adjusting the shaft sinking rate for Case study 2 from 40 m/year to 1000 m/year which is typical in many shaft sinking operations in North America. The experiments will introduce this aggressive shaft sinking rate to assess its impact on the mining options planning strategy (Goodell, 2014).

4.6 Case study 3 – gold deposit

The complete MILP framework as presented from Equations (27) to (85) was implemented on a gold deposit, and the results compared to that from an industry optimization software (GEOVIA Whittle). The geologic block model of this gold deposit has unit block sizes of 30 m x 30 m x 20 m. The total mineral resource is 20.87 Mt with an average gold grade of 4.24 g/t. Figure 4-23 is a layout of Case study 3 gold deposit showing mineralized blocks.

The gold deposit shows high-grade mineralization at the top and bottom sections of the block model, while the middle sections show lower grades. The Rock Mass Rating (RMR) assumed for the ore and waste zones in the block model indicates that the waste rocks are stronger than the ore rocks. Table 4-7 is a parametric description of Case study 3 gold deposit. This gold deposit is similar to the Pinarbasi chromite mine in Turkey, hence, implementation followed Elevli, et al. (2002).

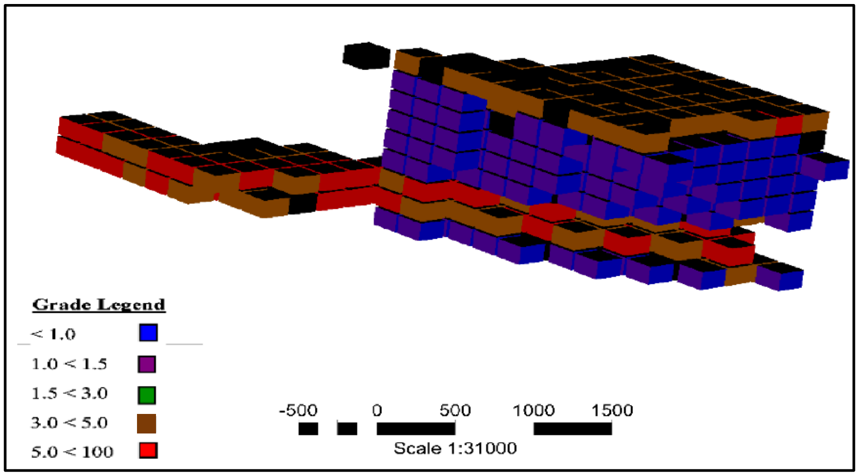


Figure 4-23: Layout of Case study 3 deposit showing mineralized blocks modified after Afum, et al. (2019a)

Table 4-7: Parametric description of Case study 3 gold deposit

Description (units)	Value
Total mineralized material (Mt)	20.87
Minimum value of Au (g/t)	0.01
Maximum value of Au (g/t)	14.8
Average value of Au (g/t)	4.24
Variance $(g/t)^2$	4.66
Standard deviation (g/t)	2.16
Number of levels/benches	9
Rock mass rating of ore blocks	60
Rock mass rating of waste blocks	72

4.6.1 Economic and technical data for Case study 3

The technical and economic data used for implementing the MILP model to evaluate this gold deposit was estimated from CostMine (CostMine, 2016) and prefeasibility reports of similar gold mining operating companies in Canada (Puritch, et al., 2016; Sirois & Gignac, 2016). The annual processing capacities are based on the proposed processing plant capacity for the mine while the yearly mining capacities are deduced from the ore and waste proportions of the gold deposit and proposed equipment schedule. Incremental bench cost of $ 2.00

per 10 m, following the NI 43-101 report of Centerra Gold Inc. and Premier Gold Mines Ltd. (Puritch, et al., 2016; Sirois & Gignac, 2016), was used as the open pit variable cost as the pit transcends downwards. For this case study, external stope dilution factor and mining recovery factor were assumed to be 20% and 90% respectively for the UG operations. Similarly, the extraction dilution factor and mining recovery factor for the open pit operations were set at 10% and 90% respectively. Table 4-8 details the technical and economic data used for the implementation of the MILP model for the gold deposit case study. All economic parameters used for the computations were in Canadian dollars.

Table 4-8: Economic, mining and processing data for evaluating Case study 3 gold deposit

Parameter (units)	Values
Open pit mining cost ($/t)	8.00
Underground mining cost ($/t)	150.00
Processing cost ($/t)	15.00
Selling cost ($/oz)	50.00
Selling price of gold ($/oz)	2,300.00
Discount rate (%)	5
Processing recovery (%)	90
Max open pit (OP) ore extraction capacity (Mt/year)	2.5
Max open pit (OP) rock mining capacity (Mt/year)	3.5
Max underground (UG) ore extraction capacity (Mt/year)	1.5
Max underground (UG) rock mining capacity (Mt/year)	2.5
Max open pit & underground (OPUG) processing plant capacity (Mt/year)	2.5
Incremental bench cost ($/10 m)	2.00
Operational development cost ($/m)	7,000.00
Primary access development cost ($/m)	16,000.00
Max operating development (m/year)	10,000
Max primary access (shaft) development (m/year)	80
Ventilation development cost ($/m)	1,600.00
Max main ventilation shaft development (m/year)	80
Cost of supporting the operational development openings ($/m)	1,000.00
Cost of supporting the stopes ($/tonne)	80.00
Rock support delay factor for operational development length per year	0.25
Rock support delay factor for stopes tonnage per year	0.10
Open pit (OP) mining dilution (%)	10
Open pit (OP) mining recovery (%)	90
Underground (UG) mining dilution (%)	20
Underground (UG) mining recovery (%)	90

4.6.2 Results and discussion for Case study 3

The optimized extraction option suitable for Case study 3 gold deposit using the integrated multi-objective mixed integer linear programming (MILP) model is combined sequential and simultaneous open pit and underground (OPUG) mining with a net present value (NPV) of $ 4.01 billion. The gold mining operation commences with an independent open pit (OP) mine in the first 2 years. Underground (UG) mining follows simultaneously with the OP mine from the 3rd year to 6th year. In the 7th year, the gold mining project transitions sequentially from simultaneous OPUG mining to an independent UG mine until the end of mine life. Figure 4-24 shows the ore extraction (processing) strategy and the average ore grade processed by each mining option in Case study 3 gold deposit.

In Figure 4-24, the yearly trend of the average grade of ore extracted and processed by both mining options indicate that the optimization technique, as in all cases of mining, prioritized the relatively high-grade material over the lower grade blocks in the early years of mine life. The life of mine is determined as 8 years. Ore mining recovery of 16.22 Mtonnes, constituting about 78% of the total available mineral deposit of 20.87 Mtonnes is achieved. About 7.61 Mtonnes of ore is extracted by OP mining operation while 8.61 Mtonnes of ore is extracted by UG mining operation. The remaining mineralized rock material of 4.65 Mtonnes (about 22%) was either left *in-situ*, lost in the unmined crown pillar or delivered to the waste dump as low-grade ore material.

The schedule for the total rock material extracted consisting of both ore and waste rocks is shown in Figure 4-25. Much waste is extracted in the first year before exposing the ore material for OP extraction. Similarly, there is significant waste striping in the 4th and 5th years to expose the ore material for OP extraction before switching to the more profitable UG mining option (Figure 4-25). Thus, there is some considerable pushback to be undertaken in the 4th and 5th years of mine life to uncover ore material for extraction which subsequently makes the OP mining operation less profitable. Even though the metal price for Case study 3 is comparatively higher to Case study 2, the effect of dilution and mining recovery loses resulted in less total ore tonnage being processed in a shorter mine life, while generating a higher NPV.

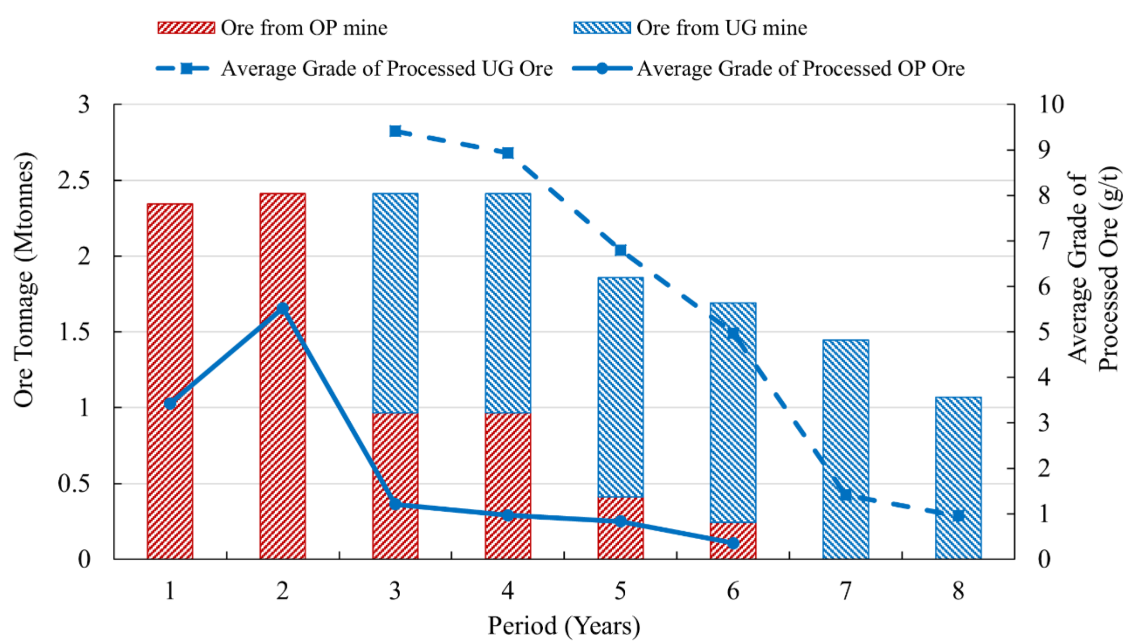


Figure 4-24: Ore extraction (processing) strategy and average ore grade processed by each mining option

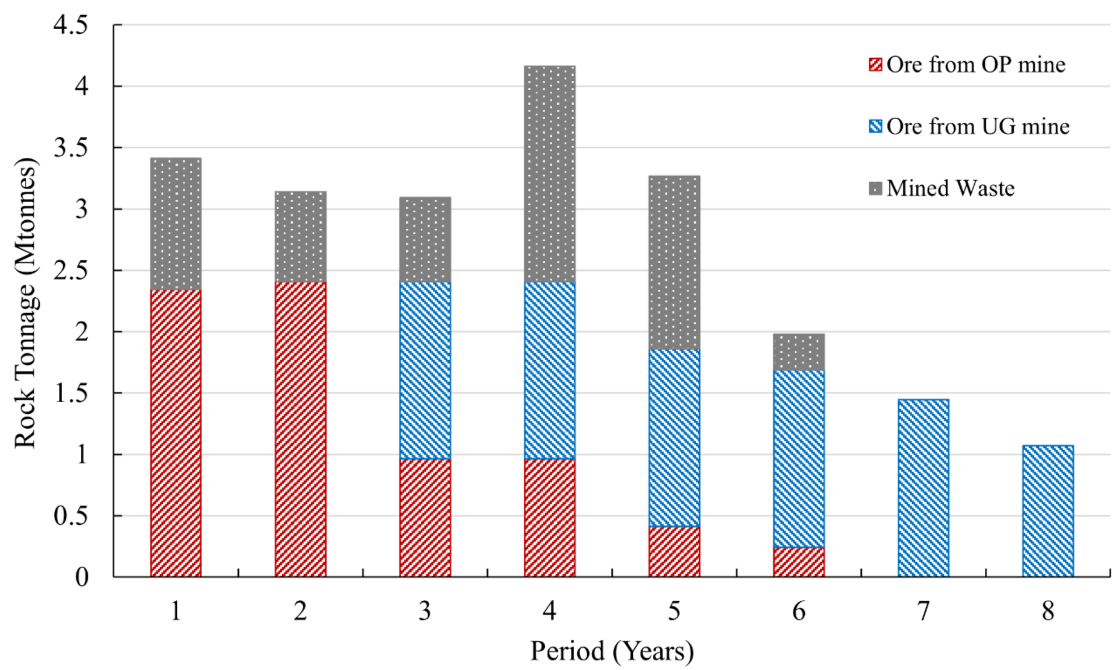


Figure 4-25: Mining schedule (ore and waste) for each mining option

The schedules for the primary access development and the lateral secondary or operational openings (levels, waste and ore drives, crosscuts) are shown in Figure 4-26. The primary access development starts in Year 1 during the OP mining operation and ends in Year 3 to make way for the commencement of the secondary development. The lower primary development rate in the first year before climaxing in the second and third years is to ensure primary and operational development cost is delayed until it is more competitive to start UG extraction in Year 3. This reduces the impact of development costs on the overall NPV of the mining project. The lateral secondary development which constitutes operational development, starts from the 3rd year to the end of mine life.

Additional experiments will be undertaken in future by adjusting the shaft sinking rate for Case study 3 from 80 m/year to 1000 m/year which is typical in many shaft sinking operations in North America (Goodell, 2014). The experiments will introduce this aggressive shaft sinking rate to assess its impact on the mining options planning strategy.

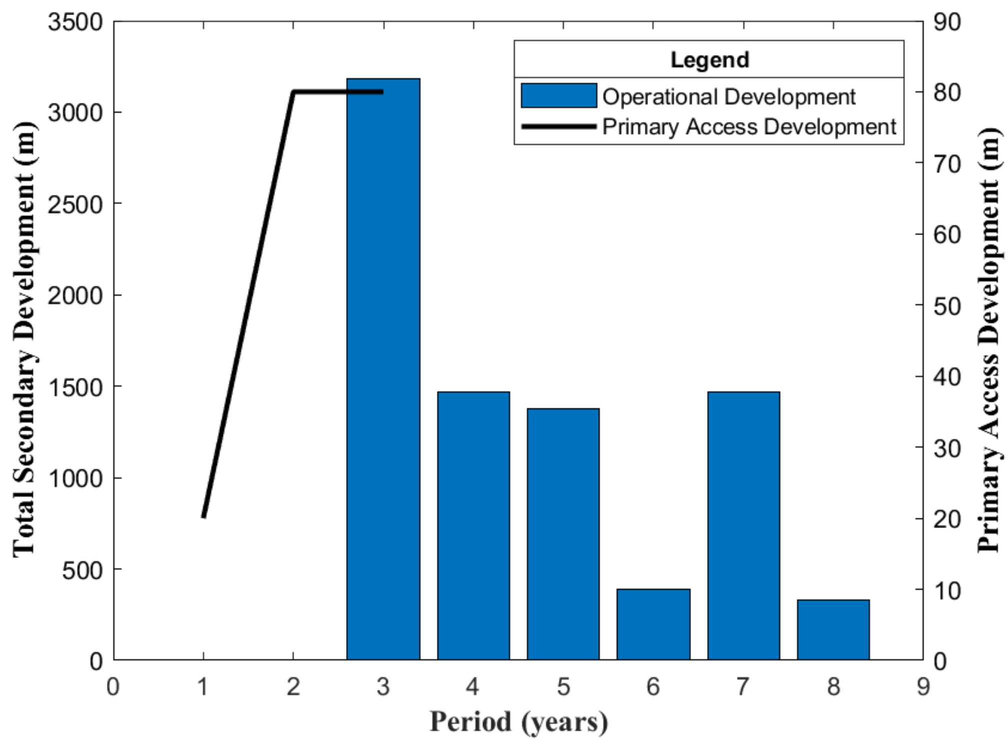


Figure 4-26: Primary access and lateral operational development schedules for the UG mining option

Figure 4-27 shows the ore extraction schedule on each level of the UG mining operation. The UG mining operation is located from Levels 5 to 9. The unmined crown pillar is positioned on Level 4 and the UG ore extraction starts from the high-grade mineralized zones on Levels 8 and 7 towards the low-grade mineralized zones on Level 5. The ore extraction sequence features an overhand mining method, and generally proceeds upward from Level 8 to Level 5 through Levels 7, 9, and 6 in that order. Due to the overhand sequence of mineral extraction in the implementation and the nature of the orebody, UG ore extraction shuffles between the various levels to ensure stopes in the lower levels are extracted before the upper levels. On each level, stope mining retreats from the end of the mining limit towards the primary access (shaft).

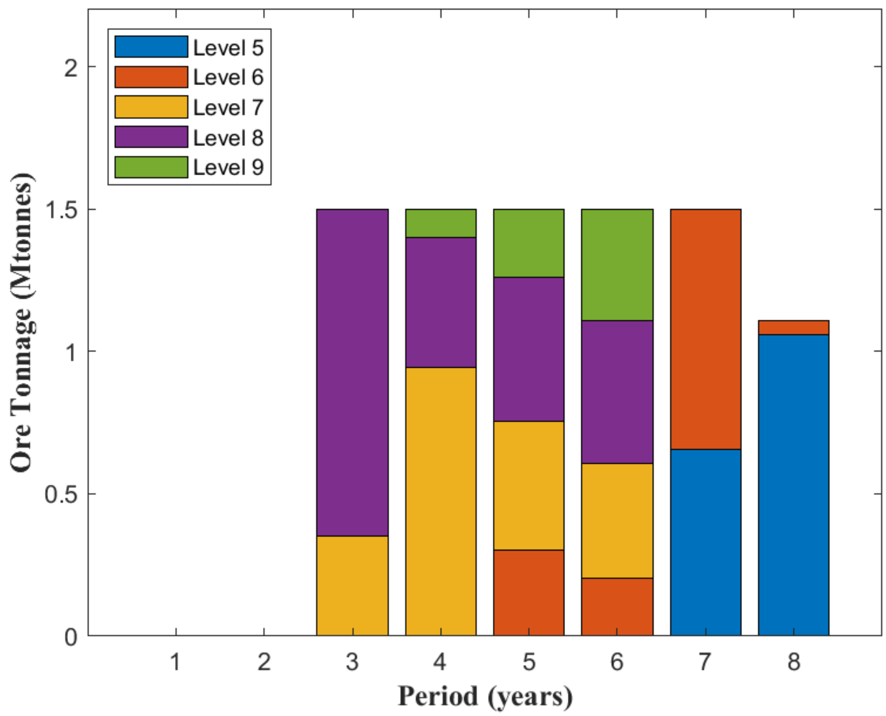


Figure 4-27: Ore extraction schedule on each level of the UG mining option

The main ventilation development schedule and the delay schedules associated with providing rock support and reinforcement in the secondary or operational development openings and stopes are shown in Figure 4-28. As expected, development of the main ventilation openings and the primary access development are completed before commencing the development of the secondary openings and the associated rock support and reinforcement required for the operational development. The lower ventilation shaft development rate in the first year before climaxing in the second and third years is to ensure ventilation shaft development cost is delayed until it is more competitive to start UG extraction in Year 3. This reduces the impact of main ventilation development cost on the overall NPV of the mining project. The delays associated with supporting the operational development openings and stopes in each period, converted to days show that much time is required in supporting the development and stopes to ensure continuous ore delivery.

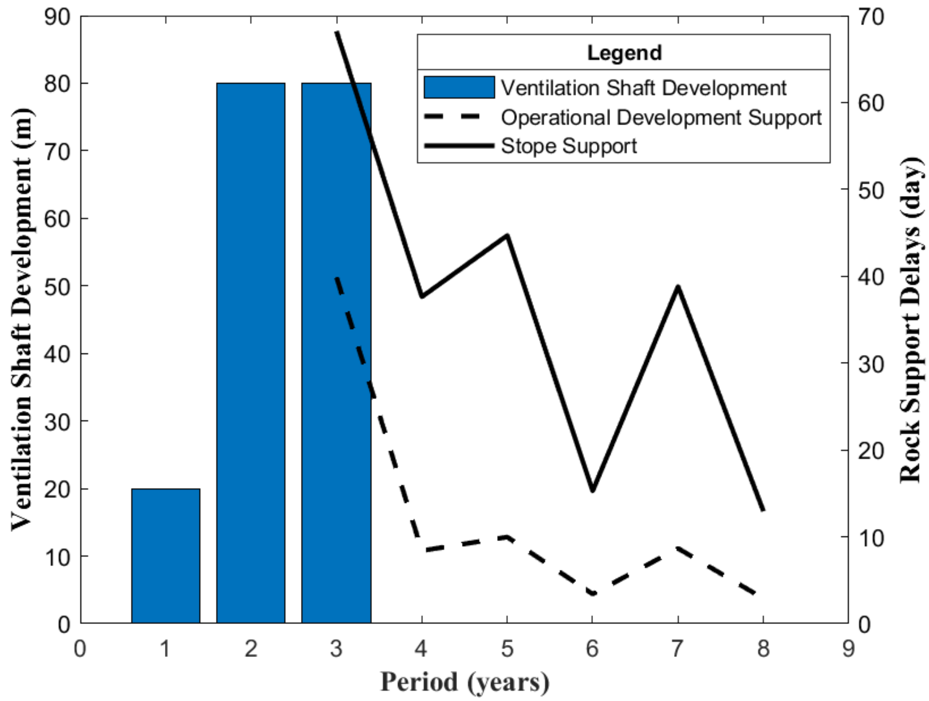


Figure 4-28: Schedules for main ventilation development and geotechnical rock support and reinforcement provided in the secondary development openings and stopes

4.6.3 Comparison of MILP model with industry standard optimization tool

Whittle is currently one of the most used standard industry software for open pit mine planning (Whittle, et al., 2018). It is the world's most trusted strategic mine planning software used to determine and optimize the economics of open pit mining projects (Dassault, 2020a). The MILP model is validated by conducting Whittle optimization runs on Case study 3 gold deposit. The main objective is to evaluate Case study 3 with the Whittle optimizer and the results compared to that of the MILP model in Section 4.6.2 to assess its performance. With the parametric analysis used in Whittle, optimality is not guaranteed though it presents a strong heuristic tool for locating high grade ore blocks in the deposit and for maximizing NPV. Even though Whittle is primarily designed for evaluating deposits amenable to open pit mining, this comparative analysis will provide insights into potential pitfalls that can occur during mining options analysis in prefeasibility studies. In most cases, decisions on when open pit mining starts and ends and when underground mining starts and ends are either based on the experience of the mining engineer or underground mining is considered after the optimal open pit outline is defined and there is still mineralized material below the ultimate pit shell. This MILP model implementation highlights the importance of the global optimization of the resource governance process towards simultaneous improved NPV and resource depletion ratio. The geologic block model used for Case study 3 (Figure 4-23 and Table 4-7), and the economic, mining and processing data used for Case study 3 (Table 4-9) were used for the Whittle run. The pit limit optimization summary from the Whittle run is shown in Table 4-9. Based on the results (Table 4-9), Pit 17 was selected as the optimal pit for production scheduling because its revenue factor is found between 0.94 and 1.02. The rock schedules and grade profiles for the Milawa NPV, Milawa Balanced and Fixed Lead scheduling algorithms of Whittle are shown in Figure 4-29, Figure 4-30 and Figure 4-31 respectively.

Table 4-9: Pit limit summary results from Whittle optimization for Case study 3 gold deposit

Pit	Min Rev Factor	Max Rev Factor	Rock Tonnes	Ore Tonnes	Strip Ratio	Max Bench	Min Bench	AU Grams	AU Grade
1	0.30	0.30	4,284,000	4,241,160	0.01	11	8	22,234,020	5.24
2	0.32	0.32	4,384,800	4,340,952	0.01	11	8	22,416,049	5.16
3	0.34	0.38	4,483,800	4,390,848	0.02	11	8	22,534,393	5.13
4	0.40	0.42	4,534,200	4,440,744	0.02	11	8	22,608,013	5.09
5	0.44	0.44	21,983,400	10,128,888	1.17	11	2	40,569,438	4.01
6	0.46	0.46	48,924,000	13,621,608	2.59	11	2	64,581,932	4.74
7	0.48	0.48	55,263,600	14,419,944	2.83	11	2	70,035,656	4.86
8	0.50	0.52	56,484,000	14,769,216	2.82	11	2	71,159,041	4.82
9	0.54	0.54	59,578,200	15,268,176	2.90	11	2	73,640,778	4.82
10	0.56	0.60	60,460,200	15,517,656	2.90	11	2	74,475,946	4.80
11	0.62	0.62	60,850,800	15,966,720	2.81	11	2	74,914,577	4.69
12	0.64	0.70	61,583,400	16,266,096	2.79	11	2	75,495,820	4.64
13	0.72	0.72	62,654,400	16,615,368	2.77	11	2	76,129,681	4.58
14	0.74	0.74	62,996,400	16,665,264	2.78	11	2	76,332,576	4.58
15	0.76	0.78	64,263,600	16,814,952	2.82	11	2	77,030,938	4.58
16	0.80	0.92	64,848,600	16,914,744	2.83	11	2	77,339,795	4.57
17	0.94	1.02	65,970,001	17,413,704	2.79	11	2	77,942,311	4.48
18	1.04	1.10	66,020,401	17,513,496	2.77	11	2	77,995,428	4.45
19	1.12	1.32	66,898,801	17,563,392	2.81	11	2	78,332,453	4.46
20	1.34	1.42	66,949,201	17,862,768	2.75	11	2	78,408,340	4.39
21	1.44	1.58	66,999,601	17,962,560	2.73	11	2	78,432,562	4.37
22	1.60	2.00	69,775,201	18,112,248	2.85	11	2	79,138,182	4.37

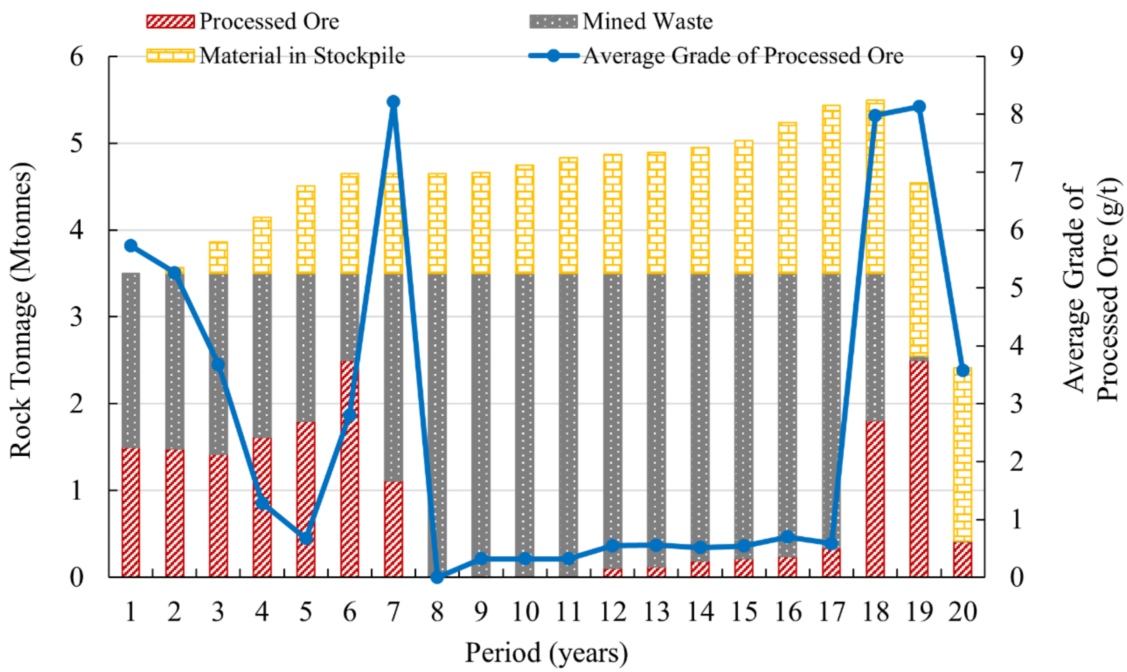


Figure 4-29: Rock material schedule and grade profile for Whittle Milawa NPV algorithm

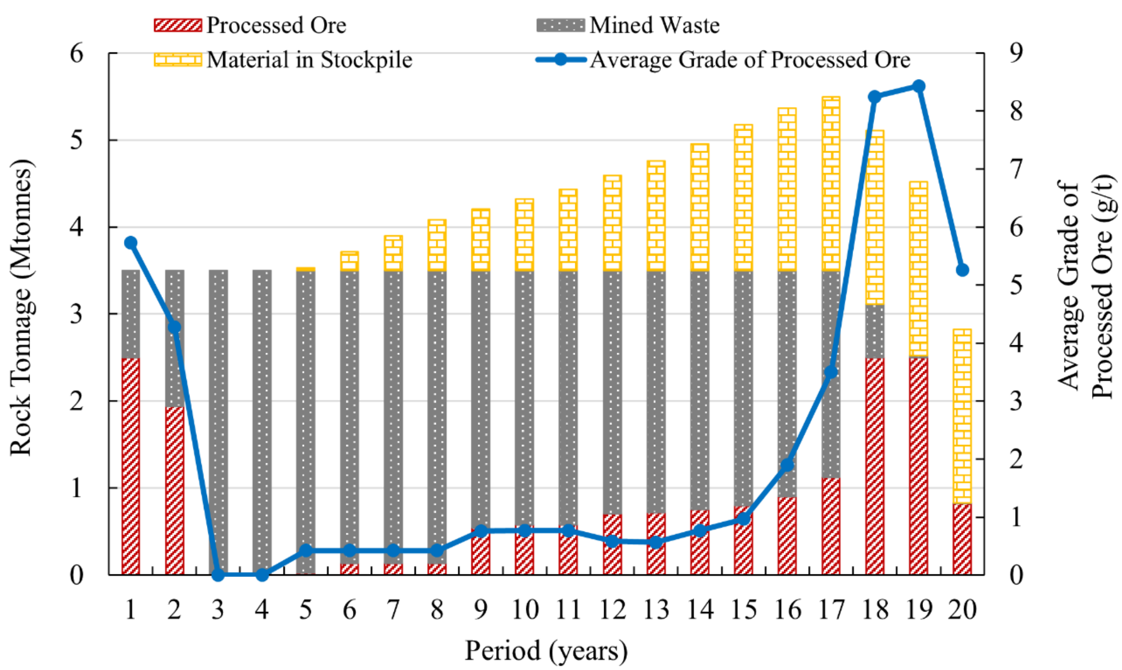


Figure 4-30: Rock material schedule and grade profile for Whittle Milawa Balanced algorithm

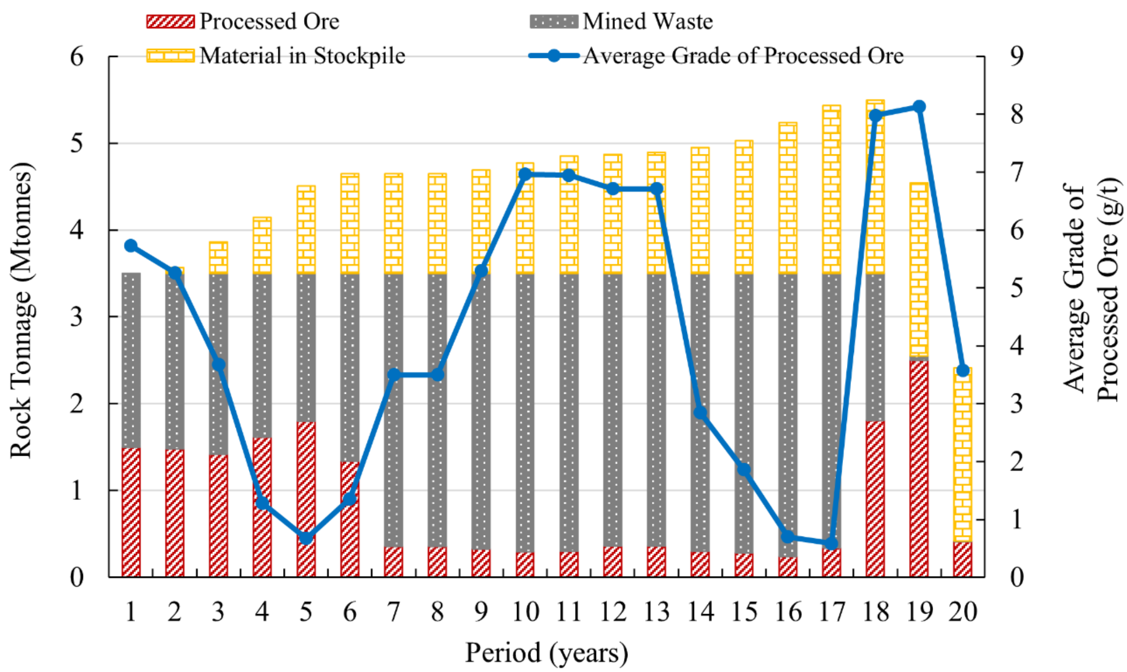


Figure 4-31: Rock material schedule and grade profile for Whittle Fixed Lead algorithm

From the Whittle optimization runs, the NPV of the OP mining project is $ 2.31 billion for Milawa NPV, $ 2.02 billion for Milawa Balanced, and $ 2.19 billion for Fixed Lead algorithm, while the NPV generated from the MILP model for combined OPUG extraction is $ 4.01 billion. This indicates a potential loss of about $ 1.70 billion (about 42%) if a proper global mining options evaluation is not undertaken prior to committing the deposit to open pit exploitation. It must be noted that the NPVs generated will be impacted by some differential costs required in setting up the two different systems of mining including the initial capital outlay, environmental and socio-economic factors. The results also show Whittle mined about 84% of the deposit in approximately 20 years through OP mining while the MILP model extracted about 78% of the deposit in 8 years through OPUG mining. This is because the MILP schedules more ore in the early part of the mine life and ignores the excessive waste stripping that is more pronounced in the Whittle results by switching the mining option from OP to UG. The stripping ratio from Whittle extraction was 2.79 and that from the MILP model was 0.34. Similarly, the results from the Whittle run indicates that some ore materials were stockpiled but not processed. This is avoided by the MILP model as it is a requirement not to stockpile any material it cannot process prior to the end of mine life. The comparison shows that the MILP framework provides a rigorous tool for strategically evaluating the mining options for a deep-seated near-surfaced deposit during prefeasibility and feasibility studies. Table 4-10 shows a summary performance comparison of the MILP model to Whittle production scheduling algorithms.

Table 4-10: Summary performance of the MILP compared to Whittle algorithms

Parameter	MILP	Whittle Algorithm		
		Milawa NPV	Milawa Balanced	Fixed Lead
NPV ($B)	4.01	2.31	2.02	2.19
Mine Life (years)	8.00	19.2	19.3	19.2
Stripping Ratio	0.34	2.79	2.79	2.79
Resource Depletion (%)	77.7	83.5	83.5	83.5

Some notable conclusions that can be drawn from the comparison of the MILP model with Whittle are:

a) The results from the MILP model includes a discounted OP limit that excludes marginal OP blocks that are more profitable through UG mining and hence extracted as such, while Whittle extracts all scheduled blocks through OP mining;

b) Whittle scheduled material to the stockpile but did not process it. The MILP model, however, do not stockpile any mineralized material that cannot be processed prior to the end of mine life;

c) The MILP model schedule has known optimality gap compared to the Whittle schedule which is based on heuristic algorithms with no guarantee of optimality;

d) The long Whittle OP mine life results in significant NPV impact from time value of money due to discounting;

e) The MILP OP outline in the combined OPUG mining scenario can be used as guidance in deciding the Whittle OP limit and corresponding revenue factor in the parametric solution, for subsequent Whittle OP production scheduling analysis.

f) The MILP model supports global optimization of resource governance towards simultaneous improved NPV and resource depletion ratio.

4.6.4 Summary findings from Case study 3

The MILP model exploited the gold deposit through combined sequential and simultaneous OPUG mining and implemented the UG ore extraction through overhand method of mining.

The MILP model was compared to Whittle optimization tool (industry standard software) for a performance assessment based on Case study 3 gold deposit. The MILP model scheduled the deposit over 8 years mine life while Whittle scheduled the deposit over 20 years mine life. The NPV generated by the MILP model was \$ 4.01 billion while the maximum NPV from Whittle (Milawa NPV) was \$ 2.31 billion; about 42% loss in financial benefits. The stripping ratio from Whittle results was 2.79 while that from the MILP model was 0.34; indicating that, the MILP model significantly avoids the excessive mining of waste to uncover mineralized material by switching from OP to UG mining option. Whittle results show that some mineralized material was stockpiled but not processed throughout the life of mine. The MILP however avoids stockpiling any material that cannot be processed in the future for profit. This MILP model implementation for mining options and transitions optimization demonstrates potential value to a mining project when the global mining options decisions are guided by a rigorous optimization process.

4.7 Computational performance of the MILP model

The computation of the three case studies used to validate and verify the MILP model were implemented on an Intel(R) Core™ i7-7700HQ CPU Dell computer @ 2.80GHz, with 32 GB RAM. At a gap tolerance of 5% between the best integer and feasible integer solutions, the characteristics and computational performance of the integrated MILP model for the case studies are summarized in Table 4-11. The relative increase in the number of blocks, scheduling periods and decision variables in the MILP formulation correspondingly increases the coefficient matrix *A* (the set of constraints equations) and the solution time of the CPU.

Table 4-11: Computational performance of the integrated MILP model for case studies (base cases)

Case study (base case)	Periods (*T*)	No. of blocks	CPU time (*hours*)	Coefficient Matrix A (*rows* x *cols*)	No. of decision variables	No. of binary variables	No. of continuous variables
1	20	605	0.014	326024 x 97400	97400	36800	60600
2	12	2598	3.476	1215264 x 312528	312528	187632	124896
3	15	2774	28.13	1937083 x 750060	750060	292080	457980

CHAPTER 5

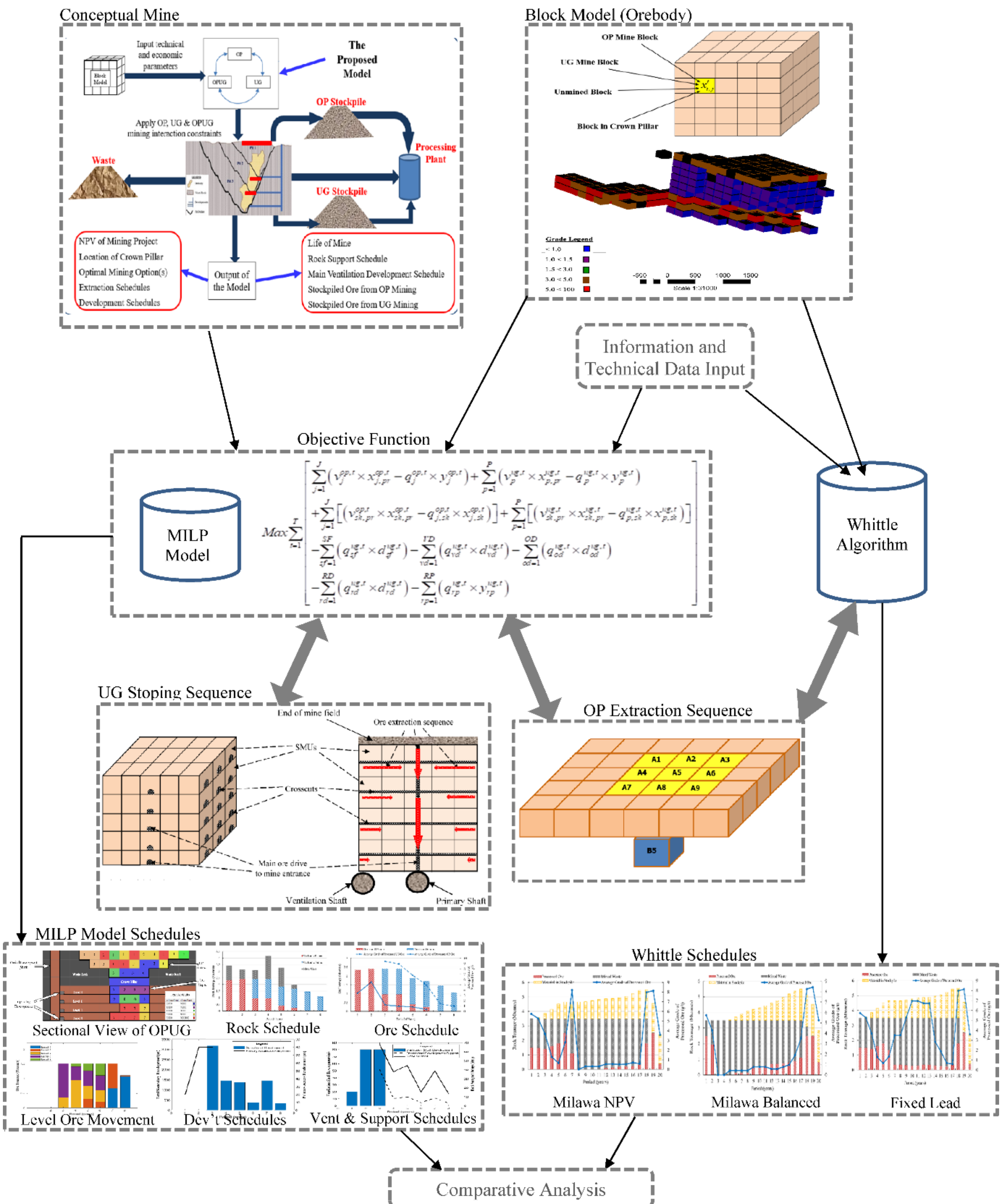
SUMMARY, CONCLUSIONS, CONTRIBUTIONS, AND RECOMMENDATIONS

5.1 Summary of the research

An unbiased mixed integer linear programming (MILP) model with a multi-objective function and sets of technical and economic constraints is formulated, implemented, and tested on three case studies for mining options and transitions optimization. The mathematical programming framework is applicable for the evaluation of a mineral resource that is complex, closer to the earth surface (shows some outcrops) and further extends to great depths. Such deposits are said to be amenable to either open pit (OP) mining, underground (UG) mining, simultaneous open pit and underground (OPUG) mining, sequential OPUG mining, or combinations of simultaneous and sequential OPUG mining. The MILP framework is based on an optimization process referred to as Competitive Economic Evaluation (CEE). The proposed CEE optimization process ensures each mining block is unbiasedly available for extraction by any of the mining options. The goal of the MILP framework is to simultaneously improve resource recovery for mineral deposits while maximizing the net present value (NPV). The multi-objective function of the MILP model determines: (a) position of the required crown pillar; (b) primary access development schedule; (c) ventilation development schedule; (d) operational development schedule; (e) rock support and reinforcement schedule for operational development; (f) rock support and reinforcement schedule for operational stopes; (g) extraction strategy and schedule for the optimal mining option(s); (h) life of mine; and (i) NPV at the pre-feasibility and feasibility stages of the mining project. The MILP framework performed better than the industry standard optimization tool (Whittle) because while the MILP model interrogates the deposit for both OP and UG extraction, Whittle which is primarily designed for OP mining, misses some mining options opportunities that has the potential to add significant value to the mining project. This MILP model provides a powerful tool for mining options and transitions evaluation and demonstrates potential value to the extraction of a deep-seated near-surface deposit when the global mining options decisions are guided by a rigorous optimization process.

A summary of the research methodology and the developed framework is presented in Figure 5-1. The mathematical framework was designed and implemented in MATLAB programming environment. The large-scale optimization solver, IBM ILOG CPLEX, based on the branch and cut algorithm was integrated into MATLAB to solve the MILP problem. The main components of the MILP framework comprise of the objective function and constraints. The inputs of the model are a geologic block model, economic and technical parameters on the mining project including processing capacity of the plant, mining capacity of the equipment fleet, on-site crown pillar requirements, on-site geotechnical considerations, and extraction sequence of UG mining. The MILP model does not require predetermined cut-off grade for the mine but allows the optimizer to decide which mineralized rock material can be processed and by which mining option. The MILP model was implemented on three case studies. The third case study was further implemented with an industry standard optimization software (Whittle) and the results compared.

Case study 1 was implemented with a laboratory prepared dataset. The proposed MILP model interrogated the synthetic copper deposit and determined the optimal extraction strategy among these mining options: (a) independent OP mining, (b) independent UG mining with crown pillar, (c) simultaneous OPUG mining with crown pillar, (d) sequential OPUG mining with crown pillar, and (e) combinations of simultaneous and sequential OPUG mining with crown pillar. The location of the 3D crown pillar together with the required capital and operational development schedules are decided by the optimization process. The results from the case study showed a combined sequential and simultaneous OPUG mining option with crown pillar as the optimal option to exploit the deposit. The ore and rock extraction schedules for both mining operations together with the operational and capital development schedules were determined for the synthetic copper project. The output of the model in mapping out the ore extraction per level in each period further provided more insight into the mining sequence of the appropriate UG mining method. Sensitivity analysis conducted on selected technical and operational parameters indicated that determination of the optimal mining option using the MILP model is most sensitive to the selling price of copper, followed by the quantity of ore processed by the UG mining operation, and the completion rate of the UG capital development (shaft).



Parameter	MILP	Whittle Algorithm		
		Milawa NPV	Milawa Balanced	Fixed Lead
NPV ($B)	4.01	2.31	2.02	2.19
Mine Life (years)	8	19.2	19.3	19.2
Stripping Ratio	0.34	2.79	2.79	2.79
Resource Depletion (%)	77.7	83.5	83.5	83.5

3

Figure 5-1: Summary of research methodology and developed framework

Case study 2 was implemented with a gold deposit. The multi-objective function of the MILP model determines: (a) position of the required crown pillar; (b) primary access development schedule; (c) ventilation development schedule; (d) operational development schedule; (e) rock support and reinforcement schedule for operational development; (f) rock support and reinforcement schedule for stopes; (g) extraction strategy and schedule for the optimal mining option(s); (h) life of mine; and (i) net present value (NPV) of the mining project. Results from the implementation indicated that the gold deposit case study (Case study 2) is optimally exploited by a combined sequential and simultaneous OPUG mining option with crown pillar. The model determined the NPV, life of mine, location of the crown pillar, and schedules for primary access development, operational development, main ventilation shaft, and rock support and reinforcement for operational development and stopes. Analysis of the results show ore extraction started through independent OP mining from Year 1 to Year 3, and after completion of the development of the main ventilation shaft, primary access (shaft) and operational openings, the mine transitioned into a simultaneous OPUG mining for Years 4 and 5. Ore extraction switched completely from simultaneous OPUG mining to independent UG mining from Year 6 to Year 12. The Net Present Value (NPV) was estimated as $ 2.52 billion at a gap tolerance of 5%. Sensitivity analysis on gold price indicated that, the optimal OPUG mining option with resource recovery ratio of about 87% and NPV of $ 2.52 billion will switch to an independent OP mine with NPV of $ 2.23 billion and a resource recovery ratio of about 99% when the gold price falls by about 11%. The NPV is proportionally sensitive to the quantity of ore being delivered from the UG mining operation. It is however only sensitive to an increase in the delays associated with providing rock support and reinforcement in the secondary or operational development openings and stopes.

Case study 3 was implemented with a gold deposit and the results from the MILP model were further compared with a similar implementation with Whittle optimization software. The MILP model exploited the gold deposit through combined sequential and simultaneous OPUG mining and implemented the UG ore extraction through overhand method of mining. The MILP model scheduled the deposit over 8 years mine life while Whittle scheduled the deposit over 20 years mine life. The NPV generated by the MILP model was $ 4.01 billion while the maximum NPV from Whittle (Milawa NPV) was $ 2.31 billion; about 42% loss in financial benefits. The stripping ratio from Whittle results was 2.79 compared 0.34 from the MILP model. Inspection of the results indicate that the MILP model significantly avoids the mining of excessive waste to uncover mineralized material by switching from OP to UG mining option. Whittle results show that some mineralized material was stockpiled but not processed throughout the life of mine. The MILP model however avoids stockpiling material it cannot process prior to the end of mine life. This MILP model implementation for mining options and transitions optimization demonstrates potential value to a mining project when the global mining options decisions are guided by a rigorous optimization process.

5.2 Conclusions

The literature review conducted as part of this research established limitations in the current body of knowledge in mining options and transitions optimization. The literature showed that there has never been any previous attempt to formulate an unbiased global mathematical programming framework with the capacity to determine whether a mineral resource should be exploited by: (a) independent open pit (OP) mining, (b) independent underground (UG) mining with crown pillar, (c) simultaneous open pit and underground (OPUG) mining with crown pillar, (d) sequential OPUG mining with crown pillar, and (e) combinations of simultaneous and sequential OPUG mining with crown pillar. The literature also showed that there has not been any previous attempt to integrate ore quality (grade) blending, three-dimensional (3-D) crown pillar positioning, primary access development, secondary access (operational) development, ventilation requirement, and rock support and reinforcement requirement in the mining options and transitions optimization framework. This research therefore pioneers the effort to employ a mathematical programming model in the form of mixed integer linear programming to contribute to the body of knowledge and provide a novel understanding in the area of mining options and transitions optimization at the prefeasibility stage of a mining project.

An unbiased mathematical programming framework for the strategic evaluation of a mineral resource amenable to OP and UG mining operations has been developed. The optimization framework is implemented and tested on three case studies (copper and gold deposits). The objectives of this research outlined in Chapter 1 have been achieved within the research scope and the following conclusions were drawn:

1. A Competitive Economic Evaluation (CEE) optimization technique is implemented as the backbone of the open pit-underground mining options and transitions planning process.

2. The unbiased multi-objective MILP optimization framework maximizes the NPV of the mining project while generating a strategic extraction schedule for OP and/or UG materials.
3. The MILP framework deploys the CEE process to select the optimal mining option(s) for exploiting a mineral deposit, including: (a) independent open pit (OP) mining, (b) independent underground (UG) mining with crown pillar, (c) simultaneous open pit and underground (OPUG) mining with crown pillar, (d) sequential OPUG mining with crown pillar, and (e) combinations of simultaneous and sequential OPUG mining with crown pillar.
4. The MILP model simultaneously determines the: (a) position of the required 3-D crown pillar; (b) primary access development schedule; (c) main ventilation development schedule; (d) operational development schedule; (e) rock support and reinforcement schedule for operational development; (f) rock support and reinforcement schedule for operational stopes; (g) extraction strategy and schedule for the optimal mining option(s); (h) life of mine; and (i) net present value (NPV) of the mining operation.
5. The MILP framework provides a systemic workflow for simultaneously promoting improved resource recovery and maximum investment return on mining projects.

The comparative analysis of the outputs generated by the MILP model and Whittle optimization software concludes with the following:

1. The MILP model generated a production schedule with a significantly higher NPV compared to the NPV from Whittle Milawa NPV, Milawa Balanced and Fixed Lead algorithms which are standard industry tools.
2. The MILP model generated a schedule with shorter mine life (8 years) than Whittle Milawa Balanced, Milawa NPV and Fixed Lead algorithms (20 years).
3. The MILP model extracted the orebody by combined simultaneous and sequential open pit and underground (OPUG) mining options while Whittle interrogated the orebody for open pit mining only. It must be noted that Whittle is primarily designed for optimizing OP mining operations only.
4. The MILP scheduler provided a steadier flow of ore to the processing plant than the Whittle scheduling algorithms.

5. These results proved that the MILP framework provides a more powerful and rigorous tool for optimizing a deep-seated near-surface mineral resource amenable to both open pit and underground mining. The comparative analysis provides insight into potential pitfalls that may occur during mining options analysis in prefeasibility studies.

5.3 Contributions of PhD research

This research has developed a mathematical programming framework that deploys the CEE optimization technique based on a multi-objective Mixed Integer Linear Programming (MILP) model for evaluating mineral resources amenable to both open pit (OP) and underground (UG) mining. The major contributions of this research are as follows:

1. The research has proposed a novel optimization methodology referred to as Competitive Economic Evaluation (CEE). This technique ensures an unbiased global optimization solution is obtained from the mathematical program.
2. This is a pioneering effort in developing an integrated mathematical programming model based on MILP optimization framework for evaluating mineral resources amenable to both OP and UG mining options. This research contributes significantly to the body of knowledge on mining options and transitions optimization and creates the basis and justification for developing such specialized mine planning software modules.
3. This is a novel endeavor to formulate a MILP framework that has the capability to select the optimal mining options for exploiting a mineral deposit, including: (a) independent open pit (OP) mining, (b) independent underground (UG) mining with crown pillar, (c) simultaneous open pit and underground (OPUG) mining with crown pillar, (d) sequential OPUG mining with crown pillar, and (e) combinations of simultaneous and sequential OPUG mining with crown pillar.
4. The MILP model enables step-changes in evaluating extraction of mineral resources amenable to both OP and UG mining. It provides a mathematical programming framework which simultaneously integrates the following in the mining options and transitions optimization formulation:
   a) 3-D crown pillar positioning;
   b) Rock extraction scheduling for the optimal mining option(s);

c) Ore grade quality control;

d) Stockpile management for each mining option;

e) UG primary access development requirement;

f) UG secondary access development requirement;

g) UG ventilation development requirement;

h) UG rock supports and reinforcements requirement for operational development;

i) UG rock support and reinforcement requirement for operational stopes.

5. The research has developed a rigorous mathematical programming model and technique that expands the frontiers of integrated strategic mine planning and optimization for resource extraction evaluation of a mineral deposit by generating production schedules with improved net present value compared to a current mining industry software package.
6. The MILP framework provides a systematic workflow based on optimization techniques for mining options evaluation of mineral resource extraction during pre-feasibility and feasibility studies. The formulation is flexible in defining surface mining methods such as open pit mining and quarrying, and underground mining methods such as sublevel stoping, open stoping, vertical crater retreat (VCR), vein mining, longwall caving, sublevel caving, block caving, and top slicing.

5.4 Recommendations for further research

Although the mixed integer linear programming framework pioneered a mathematical optimization tool for mining options and transitions planning, there is the need for continual investigation into using optimization techniques for mineral resource extraction evaluation in the mining industry. The following recommendations could improve and add to the body of knowledge in this research area:

1. The MILP framework can be extended to investigate room and pillar stoping, and artificially supported stoping methods in mining options and transitions planning optimization. The model will then be applicable to underground stoping methods including cut and fill, square set, stull stoping, shrinkage, and resuing. This will improve the exhaustiveness of the model and its application.

2. The MILP model assumes that data from the geologic block models are deterministic values, and future cost and price data used for the economic block models are constant. The MILP framework should be extended to include stochastic parameters. Stochastic modelling of the mining options and transitions planning problem will evaluate the impact of uncertainty associated with variables such as grade, mineral price, and mining cost during optimization.
3. To enhance computational tractability, the solution efficiency of the MILP model needs to be improved by reducing the CPU runtime to make the framework more user friendly for mine planners. Other modeling approaches such as genetic algorithm could be explored in efficiently solving the OP-UG mining options and transitions planning problem.
4. At the prefeasibility stage of a mine, decision criteria are based on several performance indicators. The MILP framework should be improved by integrating real value options in determining the size and life of mine.
5. The current MILP framework should be extended to handle different SMUs for each mining options. This will provide flexibility for the optimizer to generate improved NPV and a practical mining strategy.
6. Additional underground activities to control the impact of water on crown pillar and stope stability should be costed and included in the framework.

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APPENDICES

APPENDIX 1:

Afum, B. O., Ben-Awuah, E., & Askari-Nasab, H. (2019). A mixed integer linear programming framework for optimising the extraction strategy of open pit – underground mining options and transitions. *International Journal of Mining, Reclamation and Environment*, *34* (10), 700-724.

APPENDIX 2:

Afum, B. O., & Ben-Awuah, E. (2019). *Open pit and underground mining transitions planning: A MILP framework for optimal resource extraction evaluation*. in Proceedings of Application of Computers and Operations Research (APCOM) 2019: Mining Goes Digital, Taylor & Francis Group, Politechnika Wroclawska, Wroclaw, Poland, pp. 144-157.

APPENDIX 3:

Afum, B. O. and Ben-Awuah, E. (2021). A review of models and algorithms for open pit and underground mining options and transitions optimization: Some lessons learnt and the way forward*, Mining*, 1 (1), 112-134.

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A mixed integer linear programming framework for optimising the extraction strategy of open pit – underground mining options and transitions

Bright Oppong Afum[a], Eugene Ben-Awuah[a] and Hooman Askari-Nasab[b]

[a]Mining Optimization Laboratory (MOL), Bharti School of Engineering, Laurentian University, Sudbury, Ontario, Canada; [b]Mining Optimization Laboratory (MOL), School of Mining and Petroleum Engineering, Department of Civil and Environmental Engineering, University of Alberta, Edmonton, Alberta, Canada

ABSTRACT

The decision to exploit a mineral deposit that can be extracted by open pit and/or underground mining involves consideration of the following options: (a) independent open pit extraction; (b) independent underground extraction; (c) simultaneous open pit and underground (OPUG) extraction; (d) sequential OPUG extraction; and (e) combinations of (c) and (d). This paper investigates the extraction strategy for deposits using Mixed Integer Linear Programming (MILP) optimisation framework to maximise the net present value and determine the schedules for mining, processing, underground capital and operational developments, and 3D crown pillar position. The MILP framework is implemented for a gold deposit. The results showed a combined sequential and simultaneous open pit and underground (OPUG) mining with crown pillar as the optimal extraction option generating NPV that is 11% and 13% better than independent OP or UG mining, respectively.

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Mixed integer linear programming; open pit and underground mine planning; mining options; mine optimisation; transition depth; crown pillar

1. Introduction

At the prefeasibility stage of a mining project, several decisions are made before the commencement of the project. The correctness and accuracy of these decisions provide confidence to both mine planners and investors. One key decision is the suitable mining option(s) required to exploit the mineral deposit. The decision on an optimal extraction option becomes complicated when the deposit extends from the surface to great depth. A deep-seated ore body showing significant outcrops could be exploited using different mining options. The term mining options refer to the extraction of a deposit by either surface mining methods, underground mining methods, or both [1]. Similarly, mining options optimisation discusses the initiatives undertaken in the extractive industry to expand, change, defer, abandon or adopt strategies for a mining method(s) and sometimes investment opportunities; based on economics, technology or market conditions [1–15].

Mining methods for surface extraction include open pit mining, open cast or strip mining, quarrying, and highwall mining while underground mining (referred to as underground stoping) include open stoping, room and pillar stoping, longwall stoping, cut-and-fill stoping, sublevel stoping, shrinkage stoping, backfill stoping, cave mining, sublevel caving and block caving [16]. In this paper, surface mining method mainly focuses on open pit mining while underground mining method focuses on open stoping. Optimisation studies for each specific mining option, especially surface or open-pit mining is common. However, optimisation studies that sort to find

CONTACT Eugene Ben-Awuah ebenawuah@laurentian.ca

the suitable mining option for any deposit involving both open pit and underground extraction is uncommon. The optimal mining option for extracting a near-surface deposit may include: (a) independent open pit (OP) mining; (b) independent underground (UG) mining; (c) simultaneous open pit and underground (OPUG) mining; (d) sequential OPUG mining; and (e) combinations of sequential and simultaneous OPUG mining.

Exploiting a deposit solely by an option where all the operations are exposed to the atmosphere is classified as independent OP mining option. For an independent UG mining option, the extraction of the deposit is solely conducted within the bosom of the earth crust. Simultaneous OPUG mining option refers to the concurrent extraction of a deposit by both OP and UG mining options. However, in sequential OPUG mining option, the deposit is first exploited by OP mining option and after closure of the OP mining operations, UG mining option follows or vice versa. Combinations of sequential and simultaneous OPUG mining options refer generally to the extraction of a deposit by sequential OPUG mining whereas simultaneous OPUG mining occurs during the transition. Usually, where OPUG mining is involved, a crown pillar is required. A crown pillar is the layer of earth (rock mass) between the first stope of an underground mine and the surface of the earth or an open pit to ensure stability of the ground above. The crown pillar provides support for the hanging wall and aid with the overall stability of the stopes, but at the same time may be uneconomical to extract from the mineral reserve point of view [8,17–19].

Traditionally, to generate early revenue, the portion of the deposit closer to the surface is often exploited by open pit (OP) mining option while the deeper portions are exploited with underground (UG) mining option. In OP mining, the incremental stripping ratio and overall mining cost with depth makes UG mining more profitable beyond a certain depth. This depth has been referred to by several authors as the transition depth or point [1–4,6,7,9,11–13,20–35]. Bakhtavar [2] attempted to solve the problem of OPUG mining transition by implementing methodologies in either a simultaneous extraction mode or non-simultaneous extraction mode. In the simultaneous mode, OP and UG extraction occurs simultaneously while in the non-simultaneous mode, UG operations usually follow OP operations or vice versa. These implementation strategies for the transition problem could lead to sub-optimal solutions due to potential biasedness resulting from setting the problem up in two separate modes. Similarly, financial benefits could be missed if a deposit that can be best extracted by combinations of simultaneous and non-simultaneous OP and UG mining is implemented in either of the optimisation modes suggested by E. Bakhtavar [2].

The choice of the most economic mining option(s) can be implemented through a global optimisation approach. The technique presented in this paper allows the optimiser to select the 'best extraction strategy' suitable for the deposit under consideration devoid of a pre-selective extraction mode. The optimisation process takes into consideration the transition point which acts as the unmined crown pillar together with the integration of both capital and operational developments. This leads to several variations of mining option(s) including: (a) independent OP mining; (b) independent UG mining with crown pillar; (c) simultaneous open pit and underground (OPUG) mining with crown pillar; (d) sequential OPUG mining with crown pillar; and (e) combinations of (c) and (d). The decision to adopt any of these mining option strategies will primarily depend on the project economics and the geology of the mining area.

This paper presents a Mixed Integer Linear Programming (MILP) framework for optimal resource extraction evaluation in open pit – underground mining options and transitions planning. The model selects the most suitable mining option for extracting a deposit that is potentially amenable to either or both open pit and underground mining. The mining option selection is made in the presence of a suitable crown pillar, capital development (shaft/decline) and operational developments (level, ore drives and crosscuts). In the case when the model selects UG mining option as the preferred suitable extraction option or part of the option, the crown pillar position and schedules for the capital and operational developments are further determined. Integrating three-dimensional (3D) crown pillar positioning into the optimisation process allows the OP and UG mining options the fair opportunity to economically compete for selection. The integration of the geotechnical properties of the crown pillar including thickness and rock strength in the mathematical programming framework was not part of this study. A gold deposit is used as a case study to implement the model for evaluation.

The next section of this research paper covers a summarised literature review on open pit and underground mining transition optimisation with highlights on research gaps. Section 3 provides an overview of the mining options problem and the conceptual mining strategy employed in this research paper. Section 4 discusses the assumptions and notations used in the proposed MILP model. Section 5 introduces and explains the proposed integrated MILP model for open pit and underground mining transitions planning. Section 6 documents the implementation of the MILP model for a gold deposit while Section 7 outlines the research conclusions and identified future research work to improve on the model.

2. Summary of literature review

Strategic open pit and underground mining interface optimisation models have been developed based on determining the transition depth between open pit and underground mining. These existing models focus on investigating how an underground mining operation can be exploited after the open-pit mine life and/or finding the transition depth. Acknowledging notable challenges and shortfalls, several researchers have employed techniques, algorithms and/or models to determine the transition depth [1–4,6,7,9,11–13,20–35] and the ore block extraction strategy [7,21,24,36,37].

Optimising the location of a crown pillar is a key factor in the optimal resource extraction evaluation process for deposits amenable to both open pit and underground mining. Finding the most suitable location of the crown pillar in a combined OPUG mining operation is an interesting challenge for mining engineers today [3]. The transition from open pit to underground mining involves a complicated geomechanical process. Recent formulations of the OPUG mining transition complexes produce near optimal solutions at minimal level of confidence, and do not integrate the positioning of a 3D crown pillar, capital and operational developments in the optimisation process.

R. Kurppa and E. Erkkilä [22] assessed the simultaneous extraction between open pit and underground (OPUG) mining during the operations of the Pyhasalmi mine. They indicated that, simultaneous mining was possible due to the geometry of the orebody being worked. J. Luxford [23] argued that, cost usually drives the decision to make the transition because as the open pit waste stripping cost keeps increasing with depth, there comes a time when the underground mining cost will be less than the open pit mining cost. E. Ben-Awuah et al. [7] investigated the strategy of mining options for an orebody using a mathematical programming model. The research evaluated the financial impacts of applying different mining options separately or concurrently to extract a given orebody. The MILP formulation maximises the NPV of the reserve when extracted with: (1) open pit mining, (2) underground mining, and (3) concurrent open pit and underground mining. The location of a crown pillar together with capital and operational development requirements were not incorporated into this model.

B. King et al. [21] incorporated crown and sill pillar placement into their OPUG transition studies to separate the open pit from the underground mine. In their model, the location of the crown pillar was simulated, and preselected to divide the deposit into OP and UG mining zones before running an optimisation for the OP mining zone and planning for the UG mining zone. J.A.L. MacNeil and R. Dimitrakopoulos [24] investigated the transition decision at an operating open-pit mine within the context of a mining complex comprising five producing pits, four stockpiles and one processing plant. The proposed method improves upon previous developments related to the OPUG transition problem by incorporating geological uncertainty into the decision-making process while providing a transition depth described in three-dimensions (3D). In their research, J.A.L. MacNeil and R. Dimitrakopoulos [24] priori identified the crown pillar envelope for a gold deposit and evaluated four crown pillar locations within this envelope leading to four distinct candidate transition depths. Decomposing the OPUG optimisation process into scenarios has the tendency to compromise the global optimal solution.

A mathematical model that solves the transition problem was developed by D. Whittle et al. [37] after modifying the normal pit optimisation model based on the maximum graph closure algorithm [38]. The modifications allow the algorithm to account for the underground mining value of a block, and the requirement for a specified 2D crown pillar location above the underground mine. The algorithm of D. Whittle et al. [37] is based on what they called the 'opportunity cost approach', thus, if a given block is mined by open pit method, its open pit value is gained while its value that would have been obtained by extracting it using underground mining methods is lost. According to D. Whittle et al. [37], the optimisation approach does not control the mining sequence with time and produces a near optimal value for the mining project. Future works were therefore recommended to improve on this model.

As highlighted by B.O. Afum and E. Ben-Awuah [39], current models and algorithms for the open pit – underground mining options and transitions planning problem are primarily based on the automatic scenario analysis and opportunity cost approaches. The automatic scenario analysis approach compromises the optimal solution because the transition point is preselected before the optimisation process while the opportunity cost approach does not consider the sequence of block extraction as part of the global optimisation problem.

In this research study, we have developed, implemented, and tested a MILP optimisation framework for evaluating the extraction strategy for a deposit. The MILP model maximises the NPV of the resource and determines the optimal extraction strategy for a given ore body amenable to different mining options. The MILP framework is based on an optimisation approach referred to as Competitive Economic Evaluation (CEE). The technique allows the optimiser to select the most suitable extraction strategy, including independent OP, independent UG, simultaneous OPUG, sequential OPUG, or combinations of simultaneous and sequential OPUG, suitable to exploit any deposit under consideration. The CEE process evaluates each block of the mineral deposit and economically decides: (a) blocks suitable for OP mining, (b) blocks suitable for UG mining, (c) unmined blocks and (d) unmined crown pillar. The CEE optimisation strategy is an unbiased approach that provides fair opportunity to each mining block for selection by a mining option. In summary, the strength of the MILP optimisation framework includes:

(1) An unbiased optimisation approach – Competitive Economic Evaluation (CEE);
(2) Inclusion of crown pillar positioning in the optimisation process;
(3) Integrated production scheduling for mining option;
(4) Consideration of major UG mining developments (capital and operational).

3. Defining the mining options problem and conceptual mining strategy

A deep-seated orebody showing significant outcrop is amenable to both open pit and underground (OPUG) mining. The depth and economic outline of the open pit (OP) mine, crown pillar location, and the underground (UG) mine impact the net present value (NPV) of the mineral extraction process. In general, the UG mine is based on open stoping mining method and accessed by either a shaft or decline system. The production schedule for a combined OPUG mining requires that both mining options compete for the same reserve during optimisation [1,2,7,29,40].

The problem presented here involves: (a) defining the final pit limit outline; (b) scheduling of K mining blocks within the final pit limit over T different periods of extraction – OP mining, (c) defining the economic UG mining boundaries; (d) scheduling of K mining blocks within the economic UG mining boundaries over T different periods of extraction – UG mining, and (e) defining and scheduling K mining blocks within the combined final pit limit and economic UG mining boundaries over T different periods of extraction – simultaneous and/or sequential OPUG

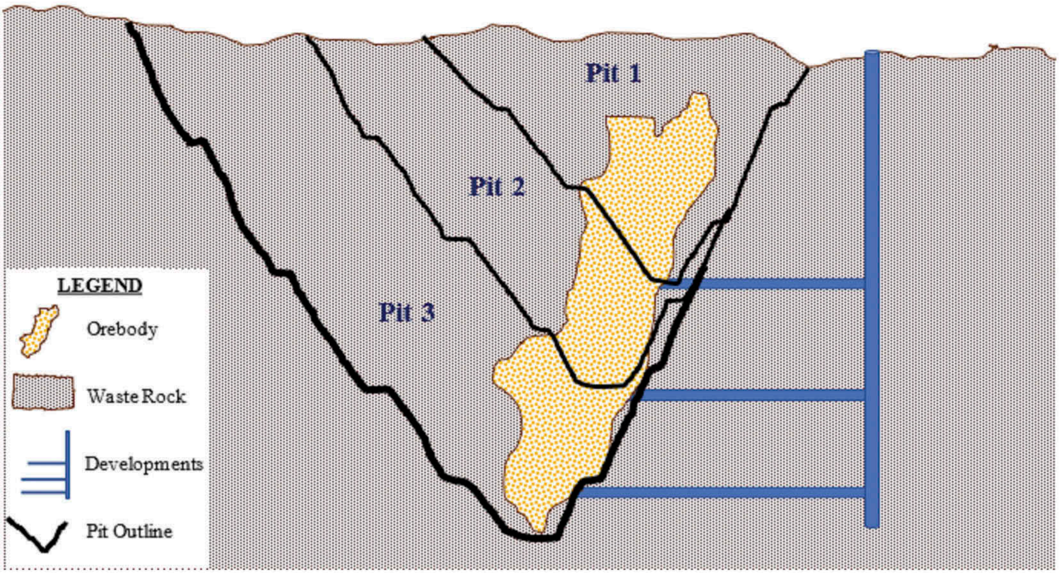

Figure 1. Schematic illustration of the open pit and underground mining transitions problem.

mining. The schedule should maximise the NPV of the operation subject to a variety of physical, technical and economic constraints. Figure 1 is a schematic representation of the mining options problem showing the interactions of open pit and underground mining for a deposit extending from surface to significant depth. A MILP formulation was developed for this strategic mining options optimisation study and applied to a gold deposit case study.

The mine planning optimisation strategy is designed to ensure open pit (OP) and underground (UG) mining options compete for a given orebody. For OP mining operation, ore is exploited from the top to the bottom following the overall open pit slope of the mine defined by a set of precedences among blocks [41]. For UG mining, the block model of the deposit is prepared while citing the location of the main capital development (shaft or decline). The locations of the operational developments (levels, ore drives and crosscuts) are positioned through each block on a level as per the design requirements. The level development links the shaft or decline from the main entrance to the ore drives through the centroid of each block. The crosscut developments extend from the ore drives to the ends of the minefield through each block, acting as stope drives. At this stage, the extraction method (retreating or advancing) is established for the ore body. Figure 2 is a schematic representation of the block model showing the underground operating developments. The arrows show a retreating mining method for the ore extraction sequence on a typical level.

4. Assumptions and notations

It is assumed that the Selective Mining Units (SMUs) for open pit mining is equivalent to the stope sizes for underground mining. In the MILP framework, the SMUs are represented by mining blocks in general, or mining-cuts in specific relation to open pit mining and mining-stope for underground mining. The location of each SMU is represented by the coordinates of the centroid. It is assumed that a crown pillar is required for the exploitation of the ore body by underground mining. The size of the crown pillar is estimated to be one vertical length of a stope or bench; thus, one bench or level in the block model will represent the crown pillar. The determination of crown pillar thickness is not part of the focus of this research but the positioning. For underground mining, ore extraction is achieved by open stoping and includes either

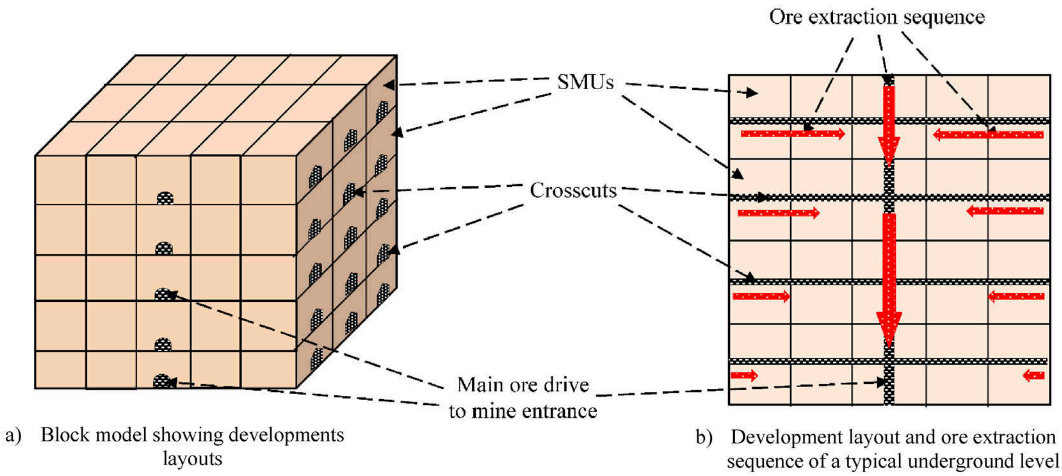

Figure 2. Isometric view of the block model and development layout of an underground level.

retreating or advancement methods. The notations for sets, indices, parameters and decision variables are as follows:

4.1. Sets

$\boldsymbol{J} = \{1, \ldots, J\}$	set of all open pit mining-cuts in the model.
$\boldsymbol{J_s} = \{1, \ldots, J_s\}$	set of all open pit mining-cuts on a level in the model.
$\boldsymbol{P} = \{1, \ldots, P\}$	set of all underground mining-stopes in the model.
$\boldsymbol{P_s} = \{1, \ldots, P_s\}$	set of all underground mining-stopes on a level in the model.
$\boldsymbol{C} = \{1, \ldots, C\}$	set of all levels (crown pillars) in the model.
$\boldsymbol{SF} = \{1, \ldots, SF\}$	set of all underground capital developments in the model.
$\boldsymbol{D} = \{1, \ldots, D\}$	set of all underground operational developments in the model.
$\boldsymbol{D_s} = \{1, \ldots, D_s\}$	set of all underground operational developments on a level in the model.
$O_j(S)$	for each open pit mining-cut (j), there is a set $O_j(S) \subset \boldsymbol{J}$, defining the immediate predecessor mining-cut (j) that must be extracted prior to extracting mining-cut (j); where S is the total number of mining-cuts in the set $O_j(S)$.
$U_p(S)$	for each underground mining-stope (p), there is a set $U_p(S) \subset \boldsymbol{P}$, defining the immediate predecessor mining-stope (p) that must be extracted prior to extracting mining-stope (p); where S is the total number of mining-stopes in the set $U_p(S)$.
$C_j(S)$	for each level, there is a set $C_j(S) \subset \boldsymbol{J}$, defining the number of mining-cuts on that level that is available for open pit extraction, or left as unmined level, or crown pillar (c); where S is the total number of mining-cuts in the set $C_j(S)$.
$C_p(S)$	for each level, there is a set $C_p(S) \subset \boldsymbol{P}$, defining the number of mining-stopes (p) on that level that is available for underground extraction, or left as unmined level, or crown pillar (c); where S is the total number of mining-stopes in the set $C_p(S)$.
$D_d(S)$	for each level, there is a set $D_d(S) \subset \boldsymbol{D_s}$, defining the number of underground operational developments on that level that must be advanced before a mining-stope (p) is extracted; where S is the total number of operational developments on a level in the set $D_d(S)$.
$D_{sf}(S)$	for each level, there is a set $D_{sf}(S) \subset SF$, defining the number of underground capital development that must be advanced before operational developments

on that level can be started; where S is the total number of capital developments in set $D_{sf}(S)$.

4.2. Indices

A general parameter f can take a maximum of four indices in the format of $f_{p,d}^{op,t}$. Where:

$t \in \{1, \ldots, T\}$	index for scheduling periods.
$j \in \{1, \ldots, J\}$	index for open pit mining-cuts in the model.
$p \in \{1, \ldots, P\}$	index for underground mining-stopes in the model.
$sf \in \{1, \ldots, SF\}$	index for underground capital development in the model.
$d \in \{1, \ldots, D\}$	index for underground operational developments in the model.
$c \in \{1, \ldots, C\}$	index for crown pillars in the model
op	index for open pit mining option
ug	index for underground mining option
$opug$	index for combined open pit and underground mining option

4.3. Parameters

$v_j^{op,t}$	the open pit (op) discounted revenue generated by selling the final product within mining-cut j in period t minus the discounted extra cost of extracting mining-cut j as ore and processing it.
$v_p^{ug,t}$	the underground (ug) discounted revenue generated by selling the final product within mining-stope p in period t minus the discounted extra cost of extracting mining-stope p as ore and processing it.
$q_j^{op,t}$	the open pit (op) discounted cost of mining all the material in mining-cut j in period t as waste.
$q_p^{ug,t}$	the underground (ug) discounted cost of mining all the material in mining-stope p in period t as waste.
$q_{sf}^{ug,t}$	the underground (ug) discounted capital development cost.
$q_d^{ug,t}$	the underground (ug) discounted operational development cost.
g_j	average grade of element in ore portion of mining-cut j.
g_p	average grade of element in ore portion of mining-stope p.
o_j	ore tonnage in mining-cut j (mineralised material).
o_p	ore tonnage in mining-stope p (mineralised material).
w_j	waste tonnage in mining-cut j (non-mineralised material).
w_p	waste tonnage in mining-stope p (non-mineralised material).
d_{cd}	capital development length cd.
d_{od}	operational development length od.
$g_{lb}^{op,t}$	lower bound on acceptable average grade of element for open pit mining (op) in period t.
$g_{lb}^{ug,t}$	lower bound on acceptable average grade of element for underground mining (ug) in period t.
$g_{ub}^{op,t}$	upper bound on acceptable average grade of element for open pit mining (op) in period t.
$g_{ub}^{ug,t}$	upper bound on acceptable average grade of element for underground mining (ug) in period t.
$T_{pr,lb}^{op,t}$	lower bound on ore processing capacity requirement from open pitmining in period t.
$T_{pr,lb}^{ug,t}$	lower bound on ore processing capacity requirement from underground mining in period t.

$T_{pr,ub}^{op,t}$	upper bound on ore processing capacity requirement from open pit mining in period t.
$T_{pr,ub}^{ug,t}$	upper bound on ore processing capacity requirement from underground mining in period t.
$T_{m,lb}^{op,t}$	lower bound on available open pit mining capacity in period t.
$T_{m,lb}^{ug,t}$	lower bound on available underground mining capacity in period t.
$T_{m,ub}^{op,t}$	upper bound on available open pit mining capacity in period t.
$T_{m,ub}^{ug,t}$	upper bound on available underground mining capacity in period t.
$T_{pr,lb}^{opug,t}$	lower bound on ore processing capacity requirement from both open pit and underground mining in period t.
$T_{pr,ub}^{opug,t}$	upper bound on ore processing capacity requirement from both open pit and underground mining in period t.
$T_{sf,lb}^{ug,t}$	lower bound on capital development (shaft) for underground mining in period t.
$T_{sf,ub}^{ug,t}$	upper bound on capital development (shaft) for underground mining in period t.
$T_{d,lb}^{ug,t}$	lower bound on operational development for underground mining in period t.
$T_{d,ub}^{ug,t}$	upper bound on operational development for underground mining in period t.
r	processing recovery; the proportion of mineral commodity recovered.
sp^t	selling price of mineral commodity in present value terms.
sc^t	selling cost of mineral commodity in present value terms.
pc^t	extra cost in present value terms per tonne of ore for mining and processing in period t.
cm_l^t	cost per bench or level l in present value terms of mining a tonne of rock material by open pit mining in period t.
cm^t	cost in present value terms of mining a tonne of rock material by underground mining in period t.
ccd^t	cost in present value terms of constructing a unit length of capital development during underground mining in period t.
cod^t	cost in present value terms of constructing a unit length of operational development during underground mining in period t.

4.4. Decision variables

$x_j^{op,t} \in \{0,1\}$	continuous variable, representing the portion of mining-cut j to be extracted as ore and processed in period t through open pit mining.
$x_p^{ug,t} \in \{0,1\}$	continuous variable, representing the portion of mining-stope p to be extracted as ore and processed in period t from underground mining.
$d_{sf}^{ug,t} \in \{0,1\}$	continuous variable, representing the portion of capital development sf to be advanced in period t for underground mining.
$d_d^{ug,t} \in \{0,1\}$	continuous variable, representing the portion of operational development d to be advanced in period t for underground mining.
$y_j^{op,t} \in \{0,1\}$	continuous variable, representing the portion of mining-cut j to be mined in period t through open pit mining; fraction of y characterises both ore and waste included in the mining-cut.
$y_p^{ug,t} \in \{0,1\}$	continuous variable, representing the portion of mining-stope p to be mined in period t through underground mining; fraction of y characterises both ore and waste included in the mining-stope.
$y_c^t \in \{0,1\}$	binary integer variable; equal to one if a level c is left as crown pillar in period t, otherwise zero.

$y_j^t \in \{0, 1\}$	binary integer variable; equal to one if a mining-cut j or all mining-cuts J_s on a level are extracted through open pit mining in period t, otherwise zero.
$y_p^t \in \{0, 1\}$	binary integer variable; equal to one if a mining-stope p or all mining-stopes P_s on a level are extracted through underground mining in period t, otherwise zero.
$y_d^t \in \{0, 1\}$	binary integer variable; equal to one if operational development $d_d^{ug,t}$ on a level is advanced in period t, otherwise zero.
$b_j^t \in \{0, 1\}$	binary integer variable controlling the precedence of extraction of mining-cut for open pit mining. b_j^t is equal to one if extraction of mining-cut j has started by or in period t, otherwise it is zero.
$b_p^t \in \{0, 1\}$	binary integer variable controlling the precedence of extraction of mining-stope for underground mining. b_p^t is equal to one if extraction of mining-stope p has started by or in period t, otherwise it is zero.
$b_{sf}^t \in \{0, 1\}$	binary integer variable controlling the precedence of capital developments for underground mining. b_{sf}^t is equal to one if capital development sf has started by or in period t, otherwise it is zero.
$b_d^t \in \{0, 1\}$	binary integer variable controlling the precedence of operational developments for underground mining. b_d^t is equal to one if operational development d has started by or in period t, otherwise it is zero.

5. The integrated MILP model

An integrated MILP model is formulated to determine the time and sequence of extraction of ore and waste blocks over the mine life for open pit and/or underground mining. The proposed MILP model evaluates the orebody and determines the best mining option that produces an optimal extraction sequence to maximise the Net Present Value (NPV) of the mining project. The mining options could either be open pit mining or underground mining or a combination of both open pit and underground mining. The model further determines the capital and operational development schedules required to extract the ore body by underground mining. The NPV of the extraction strategy is maximised in the presence of technical, geotechnical, geological, and economic constraints to enforce the mining sequence, grade blending requirements, capital and operational developments, and mining and processing capacities.

The objective function of the MILP model maximises the NPV of the mining project for both open pit and underground mining. The quantity of ore processed is controlled by the continuous decision variables $x_j^{op,t}$ and $x_p^{ug,t}$ for open pit and underground mining, respectively. Similarly, the quantity of rock material extracted by open pit and underground mining are also controlled by the continuous decision variables $y_j^{op,t}$ and $y_p^{ug,t}$ respectively. The continuous variables ensure fractional extraction and processing of the mining-cut or mining-stope in different periods of the mine life.

The constraints of the proposed MILP model control the mining, processing, plant head grade, geotechnical slope, crown pillar, capital development (decline, shaft), operational development (level, ore drives, crosscuts), and precedence relations for open pit mining, underground mining and developments. Acceptable upper and lower targets have been defined for the mining, processing, developments, and grade inequality constraints.

The mining capacity is a function of the ore reserve (mineralised material) and the designed processing capacity which is also based on the available mining fleet acquisition for the operation. The processing capacity constraints define the ore production schedule in each period and further ensures the run-of-mine (ROM) material satisfies the quantity specification for the processing plant. Based on the head grade of the plant, limiting grade requirements for ore blending from each mining option are defined within a lower and upper grade targets for the operation.

5.1. Modelling the economic block values and stope development costs

The economic block values are defined based on the SMUs, thus, mining-cuts for open pit mining and mining-stopes for underground mining. The value of a block is a function of the recovered quantity of mineral present in the block (processing recovery), the discounted revenue from selling the commodity, and the discounted mining, processing and selling costs depending on the mining option. Basically, each SMU has discounted economic block values; $EBV_j^{op,t}$ for when it is extracted by open pit mining, and $EBV_p^{ug,t}$ for when it is extracted by underground mining. These are defined by Equations (1) and (2).

The discounted revenues generated by selling the final product within the block being extracted in period t by open pit mining, $v_j^{op,t}$ and underground mining, $v_p^{ug,t}$ are, respectively, given in Equations (3) and (4). Similarly, the discounted costs of mining all the material within the block being extracted in period t by open pit mining, $q_j^{op,t}$ and underground mining, $q_p^{ug,t}$ are, respectively, given in Equations (5) and (6). Equations (7) and (8) define, respectively, the additional discounted capital and operational development costs incurred for the underground mining operations.

$$EBV_j^{op,t} = v_j^{op,t} - q_j^{op,t} \tag{1}$$

$$EBV_p^{ug,t} = v_p^{ug,t} - q_p^{ug,t} \tag{2}$$

$$v_j^{op,t} = \sum_{j=1}^{J} o_j \cdot g_j \cdot r \times (sp - sc) - \sum_{j=1}^{J} o_j \cdot pc \tag{3}$$

$$v_p^{ug,t} = \sum_{p=1}^{P} o_p \cdot g_p \cdot r \cdot (sp - sc) - \sum_{p=1}^{P} o_p \cdot pc \tag{4}$$

$$q_j^{op,t} = \left(o_j + w_j\right) \cdot cm_l^t \tag{5}$$

$$q_p^{ug,t} = \left(o_p + w_p\right) \cdot cm^t \tag{6}$$

$$q_{sf}^{ug,t} = d_{cd} \cdot ccd^t \tag{7}$$

$$q_d^{ug,t} = d_{od} \cdot cod^t \tag{8}$$

5.2. Objective function

The objective function of the MILP model shown in Equation (9) maximises the NPV of the mining project for combined open pit and underground mining options. During optimisation, if OP only is selected, the part of the objective function associated with the unselected UG mining option, capital developments and operational developments becomes zero.

$$Max\left[\sum_{t=1}^{T}\left(\begin{array}{l}\sum_{j=1}^{J}\left(v_j^{op,t}\cdot x_j^{op,t} - q_j^{op,t}\cdot y_j^{op,t}\right) + \sum_{p=1}^{P}\left(v_p^{ug,t}\cdot x_p^{ug,t} - q_p^{ug,t}\cdot y_p^{ug,t}\right)\\ -\sum_{sf=1}^{SF}\left(q_{sf}^{ug,t}\cdot d_{sf}^{ug,t}\right) - \sum_{d=1}^{D}\left(q_d^{ug,t}\cdot d_d^{ug,t}\right)\end{array}\right)\right] \tag{9}$$

5.3. Constraints

The constraints of the MILP model are given in Equations (10)–(50). The constraints have been presented in seven categories namely: (a) OP mining constraints; (b) UG mining constraints; (c) interaction of OP mining with UG mining constraints; (d) crown pillar constraints; (e) capital development constraints; f) operational development constraints; and (g) non-negativity constraints. These constraints work together simultaneously to implement the conceptual mining options strategy.

5.3.1. Open pit mining constraints

$$T^{op,t}_{m,lb} \leq \sum_{j=1}^{J} \left[\left(o_j + w_j \right) \cdot y_j^{op,t} \right] \leq T^{op,t}_{m,ub} \qquad \forall t \in \{1, \ldots, T\} \tag{10}$$

$$T^{op,t}_{pr,lb} \leq \sum_{j=1}^{J} \left(o_j \times x_j^{op,t} \right) \leq T^{op,t}_{pr,ub} \qquad \forall t \in \{1, \ldots, T\} \tag{11}$$

$$g^{op,t}_{lb} \leq \left[\frac{\sum_{j=1}^{J} \left(g_j \cdot o_j \cdot x_j^{op,t} \right)}{\sum_{j=1}^{J} \left(o_j \cdot x_j^{op,t} \right)} \right] \leq g^{op,t}_{ub} \qquad \forall t \in \{t, \ldots, T\} \tag{12}$$

$$x_j^{op,t} - y_j^{op,t} \leq 0 \qquad \forall t \in \{1, \ldots, T\}, j \in \{1, \ldots, J\} \tag{13}$$

$$b_j^t - \sum_{t=1}^{T} y_s^{op,t} \leq 0 \qquad s \in O_j(S), \forall j \in \{1, \ldots, J\}; \tag{14}$$

$$\sum_{t=1}^{T} y_j^{op,t} - b_j^t \leq 0 \qquad \forall j \in \{1, \ldots, J\} \tag{15}$$

$$b_j^t - b_j^{t+1} \leq 0 \qquad \forall t \in \{1, \ldots, T-1\}, j \in \{1, \ldots, J\} \tag{16}$$

$$\sum_{t=1}^{T} y_j^{op,t} \leq 1 \qquad \forall j \in \{1, \ldots, J\} \tag{17}$$

$$\left(\sum_{js=1}^{Js} b_j^t \cdot \frac{1}{Js} \right) - y_j^t \leq 0 \qquad \forall t \in \{1, \ldots, T\}, j \in \{1, \ldots, J\} \tag{18}$$

$$y_j^t - y_{j-1}^t \leq 0 \qquad \forall t \in \{1, \ldots, T\}, j \in \{1, \ldots, J\}; \tag{19}$$

The OP mining constraints (10)–(19) control the extraction dynamics of the open pit production schedule. Equation (10) is the mining capacity constraint for the open pit mining operation. The open pit mining is controlled by the continuous decision variable $y_j^{op,t}$. This inequality ensures the total tonnage of rock material mined in each period is within the acceptable lower and upper limits of the total available equipment capacity for the open-pit mining operation.

Equation (11) is the processing capacity constraint that controls the quantity of mill feed from the open pit mining option. The processing constraint is controlled by the continuous decision variable $x_j^{op,t}$. This inequality ensures that uniform ore is fed to the processing plant throughout the mine life within acceptable lower and upper targets of ore being delivered from the OP mining option in each period.

Equation (12) represents the grade blending constraint for open pit mining. This inequality ensures that quality ore is delivered to the processing plant in each period. The ore grade schedule in each period ensures the run-of-mine (ROM) material satisfies the ore quality specification of the processing plant. Based on the cut-off grades or head grade of the plant, limiting grade requirements for appropriate ore blending from each mining option are defined within a lower and upper grade targets for the mining operation.

Equation (13) defines the relationship between the ore and mining-cut tonnages controlling the open pit mining and processing decisions. Thus, the continuous variable $x_j^{op,t}$ is always smaller than or equal to the continuous variable $y_j^{op,t}$. Equations (14)–(16) control the vertical precedence relationship of mining-cut extraction and the appropriate geotechnical mining slope for the open pit mining option. For open pit mining-cut y extraction, nine overlying mining-cuts, $s \in O_j(S)$ should be extracted before the underlying mining-cut is removed.

Equation (17) ensures that a block is extracted once in the life of the open pit mine. Equation (18) ensures that, a level belongs to an open pit mining option when one or more blocks on that level is extracted by open pit mining while Equation (19) ensures that when a level is considered for open pit mining, the immediate level below it is available to be considered for open pit mining option.

5.3.2. Underground mining constraints

$$T_{m,lb}^{ug,t} \le \sum_{p=1}^{P} \left[\left(o_p + w_p \right) \cdot y_p^{ug,t} \right] \le T_{m,ub}^{ug,t} \qquad \forall t \in \{1, \ldots, T\} \tag{20}$$

$$T_{pr,lb}^{ug,t} \le \sum_{p=1}^{P} \left(o_p \cdot x_p^{ug,t} \right) \le T_{pr,ub}^{ug,t} \qquad \forall t \in \{1, \ldots, T\} \tag{21}$$

$$g_{lb}^{ug,t} \le \left[\frac{\sum_{p=1}^{P} \left(g_p \cdot o_p \cdot x_p^{ug,t} \right)}{\sum_{p=1}^{P} \left(o_p \cdot x_p^{ug,t} \right)} \right] \le g_{ub}^{ug,t} \qquad \forall t \in \{t, \ldots, T\} \tag{22}$$

$$x_p^{ug,t} - y_p^{ug,t} \le 0 \qquad \forall t \in \{t, \ldots, T\}, p \in \{1, \ldots, P\} \tag{23}$$

$$b_p^t - \sum_{t=1}^{T} y_s^{ug,t} \le 0 \qquad s \in U_p(S), \forall p \in \{1, \ldots, P\} \tag{24}$$

$$\sum_{t=1}^{T} y_p^{ug,t} - b_p^t \le 0 \qquad \forall p \in \{1, \ldots, P\} \tag{25}$$

$$b_p^t - b_p^{t+1} \le 0 \qquad \forall t \in \{1, \ldots, T-1\}, p \in \{1, \ldots, P\} \tag{26}$$

$$\sum_{t=1}^{T} x_p^{ug,t} \leq 1 \qquad \forall p \in \{1, \ldots, P\} \tag{27}$$

$$\left(\sum_{ps=1}^{Ps} b_p^t \cdot \frac{1}{Ps} \right) - y_p^t \leq 0 \qquad \forall t \in \{1, \ldots, T\}, p \in \{1, \ldots, P\} \tag{28}$$

Equations (20)–(28) are deployed to control the underground mining operations. Equation (20) is the mining capacity constraint for the underground mining operation. The underground mining is controlled by the continuous decision variable $y_p^{ug,t}$. This inequality ensures the total tonnage of rock material mined in each period is within the acceptable lower and upper limits of the total available equipment capacity for the underground mining operation. Equation (21) is the processing capacity constraint that controls the quantity of mill feed from the underground mining option. Equation (21) ensures the contribution of ore production from the underground mine to the overall OPUG processing capacity does not exceed a pre-defined limit.

Equation (22) represents the grade blending constraint for underground mining. The ore grade schedule in each period ensures the run-of-mine (ROM) material satisfies the ore quality specification of the processing plant. Based on the cut-off grades or head grade of the plant, limiting grade requirements for appropriate ore blending from each mining option are defined within a lower and upper grade targets for the mining operation.

Equation (23) defines the relationship between the ore and mining-stope tonnages controlling the underground mining and processing decisions. Thus, the continuous variable $x_p^{ug,t}$ is always smaller than or equal to the continuous variable $y_p^{ug,t}$. Equations (24)–(26) control the lateral precedence relationship of mining-stope extraction on each level for open stoping underground mining option. For underground mining, ore extraction sequence is implemented in a retreating manner towards the main entrance of the mine for each underground level (Figure 2).

Equation (27) defines the reserve constraint for underground mining. This inequality ensures that each stope $x_p^{ug,t}$ on a level y_p^t is extracted once in the life of the underground mine. Equation (28) ensures that a level belongs to underground mining option when one or more blocks on that level is extracted by underground mining.

5.3.3. Open pit and underground mining interactions constraints

$$b_j^t + b_p^t \leq 1 \qquad \forall t \in \{1, \ldots, T\}, j \in \{1, \ldots, J\}, p \in \{1, \ldots, P\} \tag{29}$$

$$T_{pr,lb}^{opug,t} \leq \left[\sum_{j=1}^{J} \left(o_j \cdot x_j^{op,t} \right) + \sum_{p=1}^{P} \left(o_p \cdot x_p^{ug,t} \right) \right] \leq T_{pr,ub}^{opug,t} \qquad \forall t \in \{1, \ldots, T\} \tag{30}$$

$$y_j^t + y_c^t + y_p^t \leq 1 \qquad \forall t \in \{1, \ldots, T\}, j \in \{1, \ldots, J\}, p \in \{1, \ldots, P\}, c \in \{1, \ldots, C\} \tag{31}$$

Equations (29)–(31) manage the interaction between the open pit and underground mining operations. Equation (29) represents the interaction of open pit mining-cuts with underground mining-stopes. The inequality ensures that each mining block is extracted by only one mining option or left as unmined block in the crown pillar. Equation (30) is the combined processing capacity constraint that controls the overall mill feed being delivered from both the open pit mining option and the underground mining option. This inequality represents the contribution of ore production from both open pit and underground mining options to the processing plant. Equation (31) ensures that, a level or bench is either considered for open pit mining, or underground mining, or left as crown pillar, or unmined level. Equation (31) further ensures that, one level or bench cannot simultaneously represent the open-pit mine, underground mine and crown pillar.

5.3.4. Crown pillar constraints

$$y_c^t - y_{j-1}^t \leq 0 \qquad \forall t \in \{1, \ldots, T\}, j \in \{1, \ldots, J\}, c \in \{1, \ldots, C\} \tag{32}$$

$$y_{c-1}^t - y_p^t \leq 0 \qquad \forall t \in \{1, \ldots, T\}, p \in \{1, \ldots, P\}, c \in \{1, \ldots, C\} \tag{33}$$

$$y_c^t - y_c^{t+1} \leq 0 \qquad \forall t \in \{1, \ldots, T\}, c \in \{1, \ldots, C\} \tag{34}$$

$$\sum_{c=1}^{C} y_c^t = 1 \qquad \forall t \in \{1, \ldots, T\} \tag{35}$$

Equations (32)–(35) control the positioning of the required crown pillar and its relationship to the location of the open pit and underground mining operations. Equation (32) ensures that the crown pillar is located at the bottom of the open pit mine while Equation (33) ensures the crown pillar is always located above the level being considered for underground mining. Equation (34) ensures that a level acting as the crown pillar stays the same throughout the life of the mining operation while Equation (35) ensures that one level always acts as the unmined crown pillar.

5.3.5. Capital development constraints

$$T_{sf,lb}^{ug,t} \leq \sum_{sf=1}^{SF} \left(d_{cd} \cdot d_{sf}^{ug,t} \right) \leq T_{sf,ub}^{ug,t} \qquad \forall t \in \{1, \ldots, T\} \tag{36}$$

$$b_{sf}^t - \sum_{t=1}^{T} d_{sf,s}^{ug,t} \leq 0 \qquad \mathrm{s} \in \mathrm{D}_{sf}(\mathrm{S}), \forall sf \in \{1, \ldots, SF\} \tag{37}$$

$$\sum_{t=1}^{T} d_{sf}^{ug,t} - b_{sf}^t \leq 0 \qquad \forall sf \in \{1, \ldots, SF\} \tag{38}$$

$$b_{sf}^t - b_{sf}^{t+1} \leq 0 \qquad \forall t \in \{1, \ldots, T-1\}, sf \in \{1, \ldots, SF\} \tag{39}$$

$$d_{d,s}^{ug,t} - \sum_{t=1}^{T} d_{sf}^{ug,t} \leq 0 \qquad s \in D_d(S), \forall sf \in \{1, \ldots, SF\},\ d \in \{1, \ldots, D\} \tag{40}$$

$$\sum_{t=1}^{T} d_{sf}^{ug,t} \leq 1 \qquad \forall sf \in \{1, \ldots, SF\} \tag{41}$$

The capital development constraints ensure that the required capital development is considered when underground mining is the preferred extraction option. Equation (36) defines the capital development capacity constraints for underground mining. This inequality ensures that the total length of capital developments required in each period is within the acceptable lower and upper limits of the total available equipment capacity for developing the underground mine. Equations (37)–(40) control the precedence relationships between the sections of capital development leading to each level and the operational developments on each level.

Equations (37)–(39) ensure that sets of capital developments representing a section above a level must be completed before the capital development b_{sf}^t of that level commences. Equation (40) ensures that the development of the operational level $d_{d,s}^{ug,t}$ linking the capital development could only commence after completion of a set of required capital developments $\sum_{t=1}^{T} d_{sf}^{ug,t}$ above and on that level. Equation (41) ensures that each section of the capital development (shaft or decline) is extracted ones in the life of the underground mine.

5.3.6. Operational development constraints

$$T_{d,lb}^{ug,t} \leq \sum_{d=1}^{D} \left(d_{od} \cdot d_{d}^{ug,t}\right) \leq T_{d,ub}^{ug,t} \qquad \forall t \in \{1, \ldots, T\} \qquad (42)$$

$$b_{d}^{t} - \sum_{t=1}^{T} d_{d,s}^{ug,t} \leq 0 \qquad s \in D_d(S), \forall d \in \{1, \ldots, D\} \qquad (43)$$

$$\sum_{t=1}^{T} d_{d}^{ug,t} - b_{d}^{t} \leq 0 \qquad \forall d \in \{1, \ldots, D\} \qquad (44)$$

$$b_{d}^{t} - b_{d}^{t+1} \leq 0 \qquad \forall t \in \{1, \ldots, T-1\}, d \in \{1, \ldots, D\} \qquad (45)$$

$$x_{p}^{ug,t} - \sum_{t=1}^{T} d_{d,s}^{ug,t} \leq 0 \qquad s \in D_d(S), \forall d \in \{1, \ldots, D\}, p \in \{1,\ldots, P\} \qquad (46)$$

$$\left(\sum_{d=1}^{D_s} b_{d,s}^{t} \cdot \frac{1}{D_s}\right) - y_{d}^{t} \leq 0 \qquad s \in D_d(S), \forall t \in \{1, \ldots, T\}, d \in \{1, \ldots, D\} \qquad (47)$$

$$y_{c}^{t} + y_{d}^{t} \leq 1 \qquad \forall t \in \{1, \ldots, T\}, d \in \{1, \ldots, D\}, c \in \{1, \ldots, C\} \qquad (48)$$

$$\sum_{t=1}^{T} d_{d}^{ug,t} \leq 1 \qquad \forall d \in \{1, \ldots, D\} \qquad (49)$$

These set of constraints from Equations (42)–(49) define the operational development requirements including the type and length of each operational development (level, ore drive, crosscuts) and lateral precedence relations with the block extraction sequence. Equation (42) defines the operational development capacity constraint for underground mining, ensuring that the total length of operational development (level, ore drive, crosscuts) required in each period is within the acceptable lower and upper limits of the total available equipment capacity for developing the underground mine.

Equations (43)–(45) control the lateral precedence relationships of the underground operational developments required for exploiting the ore body. Equation (46) ensures that in any period, there is a set of operational developments $\sum_{t=1}^{T} d_{d,s}^{ug,t}$ that must be completed before exploiting a stope $x_{p}^{ug,t}$. In the case of Equation (47), an underground level is activated when operational development has commenced or completed for that level while Equation (48) ensures that a level acting as the crown pillar would not be available for operational development and vice versa. Equation (49) ensures that the operational development (level, ore drive, crosscuts) at a section of the mine is advanced ones in the life of the underground mine.

5.3.7. Non-negativity constraints

Equation (50) ensures that the decision variables for open pit and underground mining, open pit and underground processing, crown pillar, open pit mining benches, and underground mining levels are non-negative. The inequality further defines that the binary variables controlling the geotechnical and extraction sequence in the open pit and underground mining operations are non-negative and integers.

$$x_j^{op,t}, y_j^{op,t}, x_p^{ug,t}, \ y_p^{ug,t}, d_{sf}^{ug,t}, d_d^{ug,t} \geq 0 \ \ and \ \ b_j^t, b_p^t, b_{sf}^t, b_d^t, \ \ y_j^t, y_p^t, y_c^t, y_d^t \geq 0 \ \ \text{and integers} \tag{50}$$

6. Computational implementation of the MILP model

The formulated MILP model was implemented with an experimental case study. MATLAB 2018a [42] environment was used to define the modelled framework and IBM ILOG CPLEX Optimisation Studio [43] was integrated into MATLAB to solve the MILP at a gap tolerance of 5%. The model was tested on an Intel(R) Core™ i7-7700HQ CPU Dell computer @ 2.80 GHz, with 32 GB RAM. The MILP formulation is implemented for an independent OP mining option by setting the ore processing capacity of UG mining to zero, and for an independent UG mining option by setting the ore processing capacity of OP mining to zero. The NPVs and extraction strategies obtained from the independent OP and UG mining options are compared to the results from the OPUG mining option.

6.1. Case study – gold deposit

The mixed integer linear programming (MILP) model was implemented and tested on a gold deposit. The gold deposit is represented by a geologic block model, which is a 3D array of cubical blocks containing 6,656-unit blocks of sizes 30 m x 30 m x 20 m. These unit blocks represent the Selective Mining Units (SMUs) for OP and UG mining. The ore body in the block model is irregularly shaped with a total mineral resource of 19.2 Mt at an average Au grade of 4.391 g/t; minimum grade of 0.011 g/t and maximum grade of 14.783 g/t. Figure 3 is a layout of the gold deposit showing mineralised blocks.

The gold deposit shows high mineralisation at the top and bottom sections of the ore body while lower grade ores occur in the middle portions of the ore body. The outcrop nature of the deposit suggests that it could be exploited with open pit mining for earlier financial benefit. However, the incremental cost of open pit mining with depth and the higher grades occurring at the bottom sections of the deposit may support underground mining as a better option at a certain depth during the mineral exploitation process. Therefore, the essence of evaluating this deposit with the proposed MILP framework. Table 1 is a statistical description of the gold deposit.

6.1.1. Economic and mining data

The yearly processing capacities are determined based on the proposed plant capacities for the mine while the yearly mining capacities are deduced from the ore and waste proportions of the deposit. An incremental bench cost of \$ 2.0 per 10 m was used as the open pit mining variable cost as the pit extends downwards. The economic, mining and processing data used for evaluating the gold deposit as summarised in Table 2 were estimated using data from CostMine [44] and prefeasibility reports of two similar gold mining companies in Canada [45]. All economic values are reported in Canadian dollars.

6.1.2. Results and discussions

Evaluation of the gold deposit with the integrated Mixed Integer Linear Programming (MILP) model indicates that a combined sequential and simultaneous open pit and underground

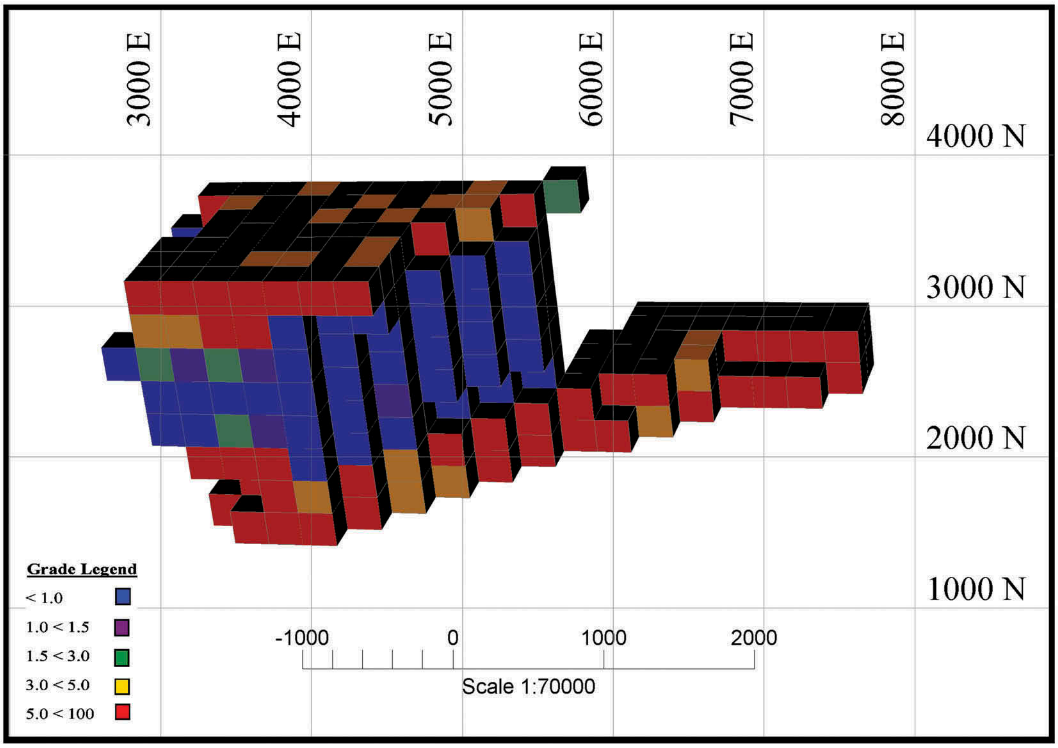

Figure 3. Layout of the gold deposit showing mineralised blocks.

Table 1. Statistical description of the gold deposit.

No.	Description	Value
1	Total mineralised material (Mt)	19.15
2	Minimum value of Au (g/t)	0.011
3	Maximum value of Au (g/t)	14.783
4	Average value of Au (g/t)	4.391
5	Variance $(g/t)^2$	16.539
6	Standard deviation (g/t)	4.067
7	Number of levels/benches	8

(OPUG) mining operation is the preferred mining option with the highest Net Present Value (NPV) of $ 2.43 billion. The integrated MILP model was modified to evaluate the ore body for an independent open pit (OP) mining option and an independent underground (UG) mining option with a required crown pillar, capital and operational developments. The open pit mining option was evaluated by setting the production requirements for underground mining operation to zero and vice versa. The NPV of the OPUG mining option is about 11% better than an independent OP mining option and about 13% better than an independent UG mining option. The summarised results are shown in Table 3. The gold deposit is therefore best exploited with a combined open pit and underground (OPUG) mining option for a mine life of 9 years with Level 4 acting as the unmined crown pillar.

A combined open pit and underground (OPUG) mining option gave the highest NPV of $ 2.43 billion with a resource depletion of 86% compared to an independent open pit mining option or underground mining option. With a total ore production of 16.5 Mt from the combined OPUG mining option, the OP mining operation contributed 7.56 Mt of ore while the UG mining operation contributed 8.91 Mt of ore. The remaining ore of about 2.68 Mt (14.0%) is left as unmined in the

Table 2. Economic, mining and processing data for evaluating the gold deposit.

No.	Parameter	Values
1	Open pit mining cost ($/t)	8.0
2	Underground mining cost ($/t)	300.0
3	Processing cost ($/t)	15.0
4	Selling cost ($/oz)	50.0
5	Selling price of gold ($/oz)	1500.0
6	Discount rate (%)	10.0
7	Processing recovery (%)	95.0
8	Max open pit (OP) processing capacity (Mt/year)	2.0
9	Min open pit (OP) processing capacity (Mt/year)	0.0
10	Max open pit (OP) mining capacity (Mt/year)	10.0
11	Min open pit (OP) mining capacity (Mt/year)	0.0
12	Max underground (UG) processing capacity (Mt/year)	1.5
13	Min underground (UG) processing capacity (Mt/year)	0.0
14	Max open pit & underground (OP-UG) processing capacity (Mt/year)	2.0
15	Min open pit & underground (OP-UG) processing capacity (Mt/year)	0.0
16	Max underground (UG) mining capacity (Mt/year)	10.0
17	Min underground (UG) mining capacity (Mt/year)	0.0
18	Incremental bench cost ($/10 m)	2.0
19	Operational development cost ($/m)	7000
20	Capital development cost ($/m)	15,000
21	Max operating development (m/year)	10,000
22	Max capital development (m/year)	60

Table 3. Results from the integrated MILP model for the gold deposit.

Mining option	Net present value ($ M)	NPV of OPUG compared (%)	Processed ore tonnage (Mt)	Resource depletion (%)
Open pit mining	2,151	−11.4	14.8	77.5
Underground mining	2,114	−12.9	17.0	88.5
Combined open pit and underground mining	2,428	-	16.5	86.0

crown pillar. During the implementation of the OPUG mining option, the crown pillar may be assessed for recovery which will contribute a further increase in the total revenue and mine life of the project.

A sectional view through the rock formation showing the OP outline, location of the required crown pillar, the UG stopes and the mineralised zone extending beyond the open pit limit is shown in Figure 4. The numbers shown on each block in Figure 4 indicate the year of extraction for the combined OPUG mine. The crown pillar is located on Level 4, thus, the first 3 levels (or benches) are extracted by open pit mining while Levels 5 to 8 are extracted by underground mining.

Figure 4. A sectional view through the rock formation showing the mine blocks, OP outline, crown pillar, UG stopes, developments and mineralised zone extending beyond the pit bottom for the case study.

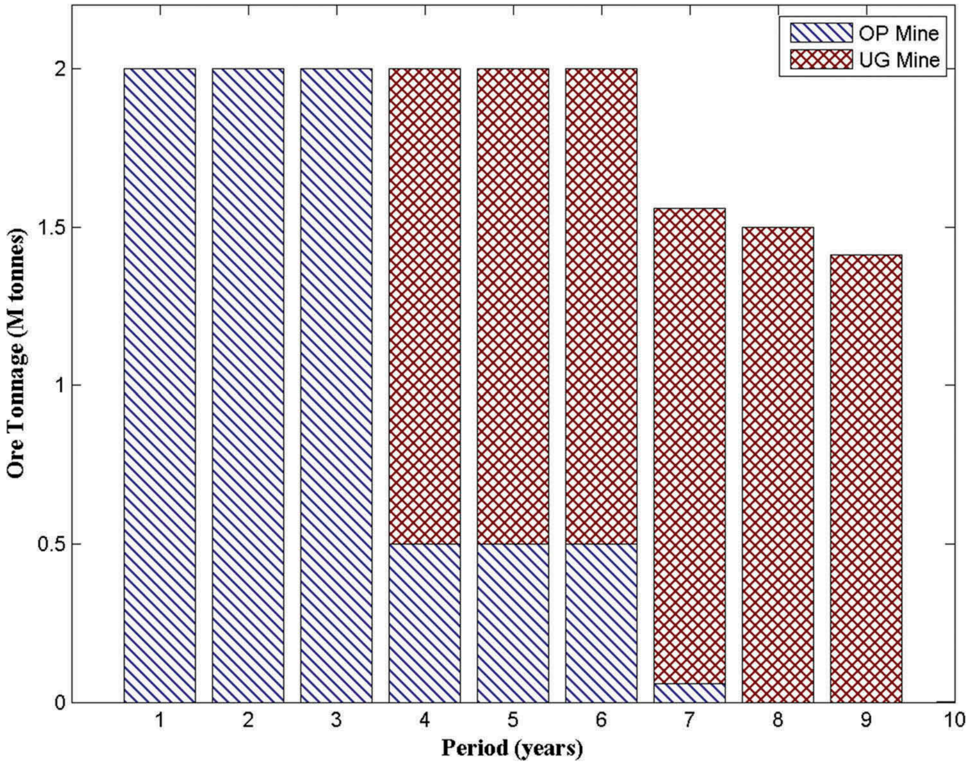

Figure 5. Yearly ore production schedule for the OPUG mining option.

The ore and rock extraction schedules for the combined open pit and underground mining (OPUG) option with a crown pillar are shown in Figures 5 and 6 respectively. The focus of the production schedule is to generate a uniform feed for the processing plant. The ore is extracted by an independent open pit mining option in the initial 3 years of the mine life before underground ore production begins in the 4th year. The ore production for the open pit mine decreases from the 4th year through to the 7th year while underground ore production maintains the required plant capacity for the remaining mine life. After year 7, the ore production completely comes from an independent underground mining operation. In summary, the ore extraction profile for the combined OPUG mining option is generally sequential over the mine life and simultaneous between years 4 and 7. Due to the near-surface mineralisation of the gold deposit, the ore extraction schedule in the earlier years of the mine life meets the full plant requirement of 2.0 Mt.

The rock extraction schedule (Figure 6) for open pit mining ranges between 2.1 Mt in year 1 and increases to 5.3 Mt in year 3. In year 4, the open pit rock extraction schedule decreases significantly as underground mining starts at the rate of 1.5 Mt per year until the end of mine life. Open pit rock extraction is completed in year 7. The peak open pit rock extraction tonnage of 5.3 Mt in year 3 is as a result of the requirements to produce 2 Mt of ore to meet the plant requirements in the midst of declining ore tonnes. Thus, the need for more waste stripping to uncover the ore.

The average grade of ore processed in each period for the combined open pit and underground mining option with a crown pillar is shown in Figure 7. The yearly average grade of gold ore processed by the underground mining operation is generally higher compared to the open pit mining operation. From Figure 7, it is convincing that the grade blending constraints of the MILP model prioritises higher grades over lower grades in exploiting the mineral deposit. Thus, the open

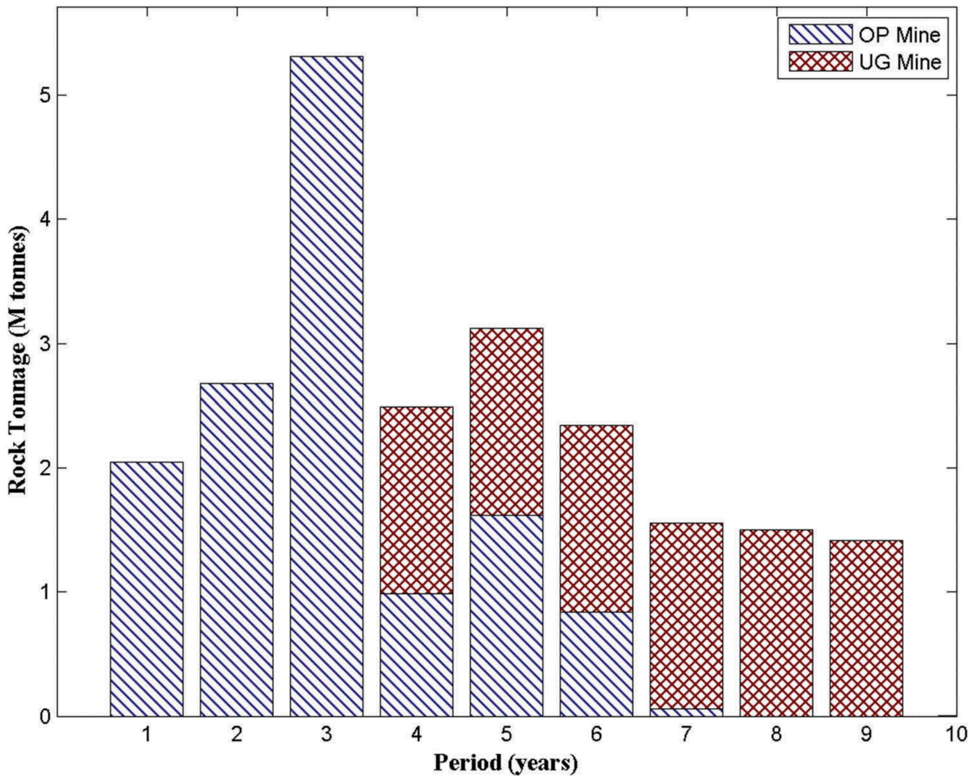

Figure 6. Yearly rock production schedule for the OPUG mining option.

pit mining operation targeted the outcropped high-grade ore while the underground mining operation targeted the deep-seated high-grade ore.

To highlight the nature of the underground mining operation and obtain further information on the proposed underground mining method, the levels contributing to the mineral extraction process are shown in Figure 8. With the crown pillar located on Level 4, ore extraction by the underground mining operation started on Levels 7 and 8 (Figure 4) from the 4th year through to the 7th year of the mine life when ore production on Level 6 starts. Ore production on Level 5 occurs in the final 2 years of the mining project. With this knowledge on the mining sequence (Figure 8) and the geotechnical properties of the rock formation, any suitable underhand mining method of ore extraction could be further evaluated and selected as the appropriate underground mining method to exploit the deep seated part of the deposit. The capital and operational development schedules for the underground mine is shown in Figure 9. With a total available operational development capacity of 10,000 m per year, level advancement starts immediately after completion of the capital development in the 4th year.

6.2. Limitations of the model

A notable limitation of the MILP optimisation framework is the assumption of similar selective mining units (SMUs) for both OP and UG mining operations. The SMU of a typical OP mine has wider lateral dimensions (X and Y directions) than its vertical dimension (Z direction). However, for UG mining, the lateral dimensions of stope sizes are usually smaller compared to the vertical. The assumption of similar SMUs for both OP and UG mining may impact the practicality of the selected mining option. At a gap tolerance of 5%, the solution time for the MILP implementation on the gold deposit with problem size of 20,832 variables (12,998 continuous variables and 7,834 binary

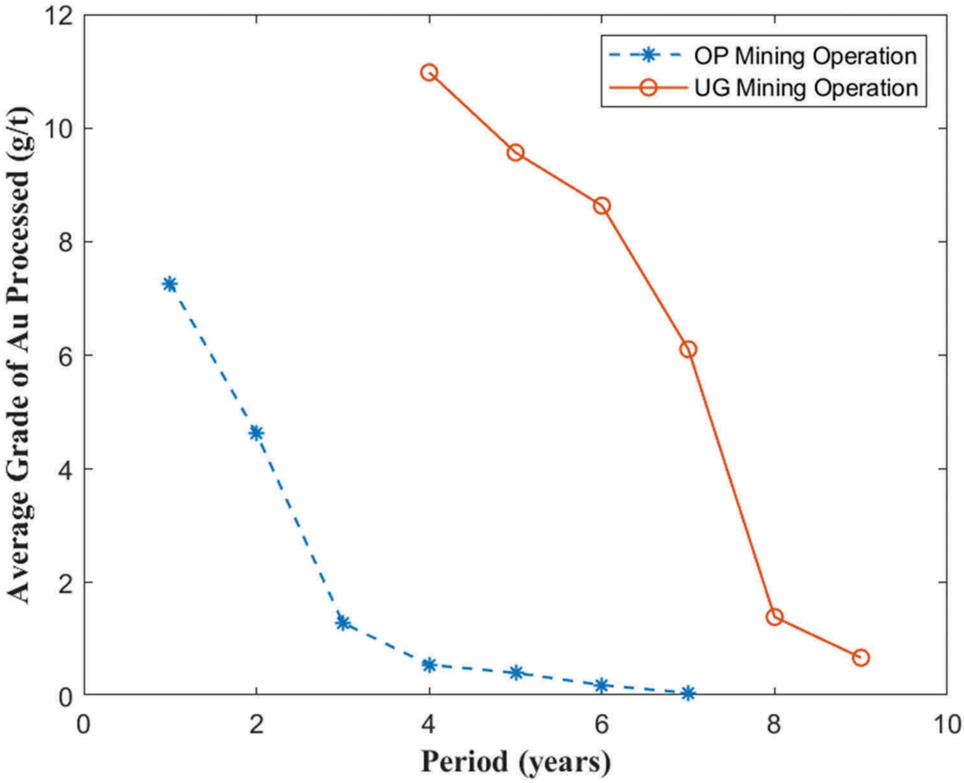

Figure 7. Average grade of ore processed in each year for OPUG mining option.

variables) is 14.2 h. Future steps of this model development will focus on implementing varying SMUs for OP and UG mining options and improving the computational efficiency of the model to solve large scale problems in a reasonable time.

7. Conclusions and further research work

In this paper, an integrated Mixed Integer Linear Programming (MILP) model for evaluating the extraction of a mineral deposit has been developed, implemented, and tested on a gold deposit. The proposed MILP model evaluates a deposit and determines the optimal ore extraction strategy using one of these mining options: (a) independent OP mining, (b) independent UG mining with crown pillar, (c) simultaneous OPUG mining with crown pillar, (d) sequential OPUG mining with crown pillar, and (e) combinations of (c) and (d). The location of the 3D crown pillar together with the capital and operational development length schedules are decided by the optimisation process. The MILP model is applicable to all types of deposits and for any preferred direction of mineral extraction sequence. The capital development can either be a shaft, decline or both and during optimisation, depending on the optimal mining option, the capital development can either commence from the surface, bottom of the open pit mine or both.

To implement the MILP model, the block model was organised by first selecting a preferred ore extraction method on a level (advancing or retreating) and siting the possible location of the underground capital and operational developments based on the philosophy and geotechnical understanding of the mine. As in practice, the MILP model requires that an incremental cost for open pit mining is defined per depth (m) for block extraction. Thus, the cost of open pit mining increases with

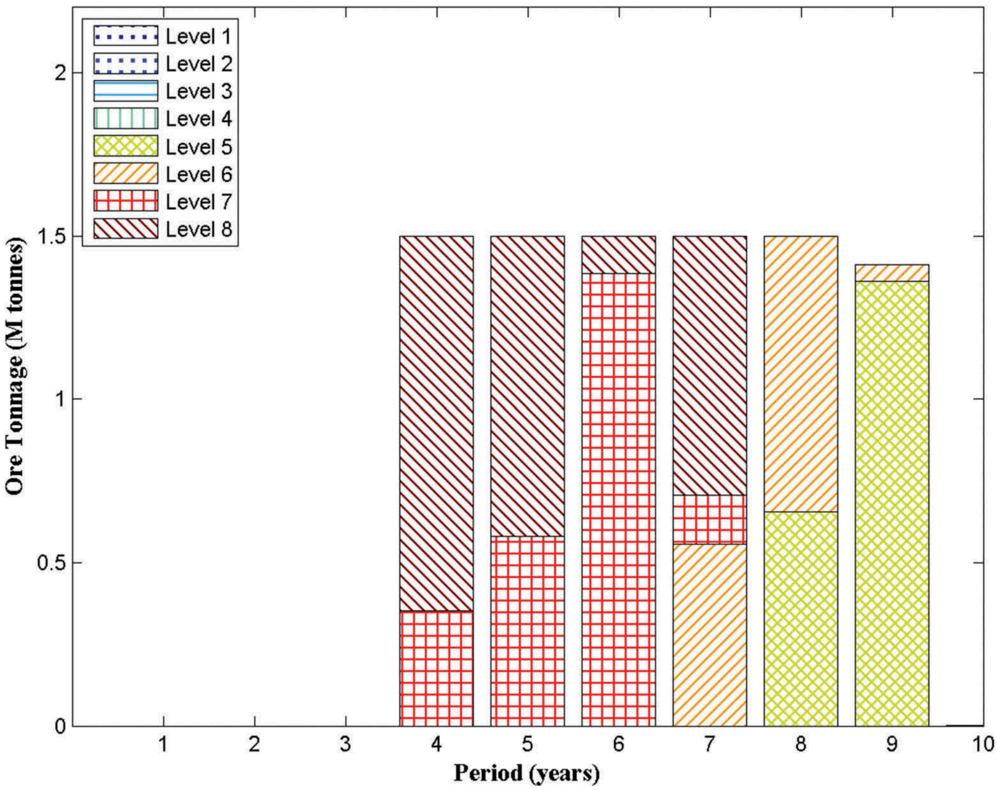

Figure 8. Ore tonnage per level extracted in each year for the underground mining operation in the OPUG mining option.

depth until underground mining is preferred to open pit mining. This concept allows the optimiser to decide when to stop the open pit mining operation, introduce a crown pillar and start the underground mining operation in the presence of both capital and operational developments. The underground capital development could commence from the bottom of the open pit or from the surface of the mine, and could be either a decline or a shaft, depending on the economics of the mining project.

The results from the case study showed a combined sequential and simultaneous open pit and underground (OPUG) mining with crown pillar was selected as the optimal extraction option to exploit the deposit. The NPV generated was $ 2.43 billion over a 9-year mine life at a gap tolerance of 5%. The NPV of the OPUG mining option was about 11% better than an independent OP mining option and about 13% better than an independent UG mining option. The ore and rock extraction schedules for the open pit and underground mining operations together with both capital and operational development schedules were determined for the gold project. The output of the model further provides more insight into the selection of the appropriate underground mining method as opposed to the traditional approach of mainly relying on the geotechnical properties of the rock formation.

Further research to improve the current model will focus on integrating geotechnical constraints, stope interactions between levels and different selective mining units (SMUs) for open pit (OP) and underground (UG) mining options. The current mixed integer linear programming (MILP) framework will also be extended to include stochastic parameters like grade and geotechnical variables.

Disclosure statement

No potential conflict of interest was reported by the authors.

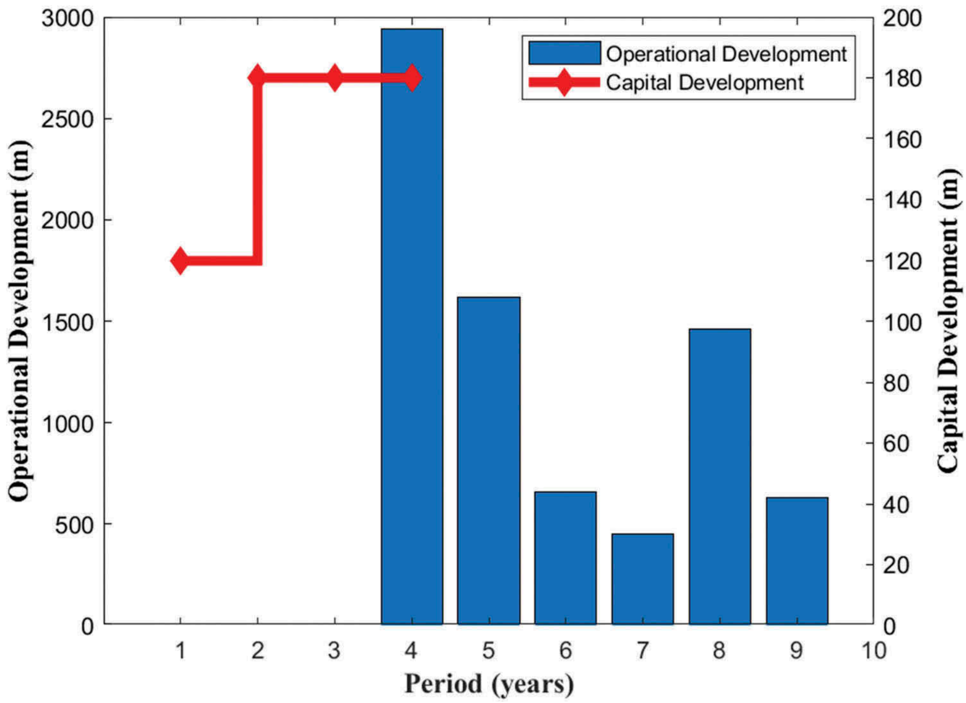

Figure 9. Capital and operational development schedules for the underground mining operation in the OPUG mining option.

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ORCID

Eugene Ben-Awuah http://orcid.org/0000-0003-2882-3853

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MILP framework for open pit and underground mining transitions evaluation

B.O. Afum
Mining Optimization Laboratory, Laurentian University, Sudbury, Canada
University of Mines and Technology (UMaT), Tarkwa, Ghana

E. Ben-Awuah
Mining Optimization Laboratory, Laurentian University, Sudbury, Canada

ABSTRACT: The strategic decisions to exploit a mineral deposit extending from the surface to great depth are essential to the financial and sustainability benefits of any mining project. Mining option strategies for resource extraction include: (a) independent open pit mining; (b) independent underground mining with crown pillar; (c) simultaneous open pit and underground mining with crown pillar; (d) sequential open pit and underground mining with crown pillar; and e) combinations of (c) and (d). This research investigates the extraction strategy that maximizes the Net Present Value (NPV) of a resource using a Mixed Integer Linear Programming (MILP) optimization framework. The MILP model determines the best extraction strategy for a given ore body and further determines the mining and processing schedule, positioning of the required crown pillar, and the schedule for underground capital and operational developments. The model is implemented for a synthetic copper case study.

1 INTRODUCTION

Decisions made at the prefeasibility stage of a mining project are essential to the commencement and sustainability of the project. Important decisions including the selection of a suitable mining option(s) for exploiting the deposit improves the confidence of mine management and investors when their correctness and accuracy are done right from the onset of the mining project. The decision on the choice of mining option(s) becomes complicated when the deposit is deep-seated and exhibits significant outcrops. The potential of such a deposit to be exploited by open pit (OP) or underground (UG) mining or both (OPUG) could lead to several variations of mining options. To generate early revenue, the portion of the deposit closer to the surface is often exploited by open pit mining option while the deeper portions are exploited with underground mining option. In OP mining, the incremental stripping ratio and overall mining cost with depth makes UG mining profitable beyond a certain depth. This depth has been referred to by several authors as the transition depth or point (Bakhtavar et al., 2009, Dagdelen and Traore, 2014, De Carli and de Lemos, 2015, King et al., 2016, MacNeil and Dimitrakopoulos, 2017, Opoku and Musingwini, 2013, Ordin and Vasil'ev, 2014, Roberts et al., 2013, Ben-Awuah et al., 2016, Ben-Awuah et al., 2015).

The choice of the most economic mining option(s) can be implemented through an optimization approach. The optimization process becomes complicated when the transition point acts as the unmined crown pillar together with the integration of both capital and operational developments. This leads to several variations of the optimal mining option(s): (a) independent OP mining; (b) independent UG mining with crown pillar; (c) simultaneous OPUG mining with crown pillar; (d) sequential OPUG mining with crown pillar; and e) combinations of (c) and (d). The decision to adopt any mining option strategy will primarily depend on the project economics and the geology of the mining area. Traditionally, the outcrop portion of the deposit is optimized for OP mining and during the OP mining operation, the deep-seated portion below the bottom of the open pit is evaluated for UG mining. This approach leads to missed financial and sustainability opportunities because it often leads to sub-optimal solutions.

A mathematical programming framework for determining the extraction strategy for any deposit that has the potential to be exploited with several variations of mining options in the presence of a suitable crown pillar, capital development (shaft/decline) and operational developments (level, ore drives and crosscuts) has been developed. Integrating three-dimensional (3D) crown pillar positioning into the optimization process allows the OP and UG mining options the fair opportunity to economically compete for selection.

This paper presents a Mixed Integer Linear Programming (MILP) framework for solving the open pit and underground mining transition problems. The model determines a suitable optimized mining option and strategy for extracting a deposit that is potentially amenable to either or both open pit and underground mining option. A synthetic copper dataset is used as a case study to implement the model for evaluation. Sensitivity analysis is further conducted to assess the influence of selected technical and economic parameters to changes in the mining options.

The next section of this research paper covers a summarized literature review on open pit to underground mining transition with highlights on research gaps. Section 3 discusses the assumptions and notations used in the proposed MILP model. Section 4 introduces and explains the proposed integrated MILP model for the open pit to underground mining transition complex. Section 5 documents the implementation of the MILP model for a synthetic copper deposit while Section 6 outlines the research conclusions and recommendations.

2 SUMMARY OF LITERATURE REVIEW

Strategic open pit and underground mining interface optimization models have been developed based on determining the transition depth between open pit and underground mining. These existing models focus on investigating how an underground mining operation can be exploited after the open pit mine life and/or finding the transition depth. Acknowledging notable challenges and shortfalls, several researchers have employed techniques, algorithms and/or models to determine the transition depth (Bakhtavar et al., 2009, Dagdelen and Traore, 2014, De Carli and de Lemos, 2015, King et al., 2016, Opoku and Musingwini, 2013, Ordin and Vasil'ev, 2014, Roberts et al., 2013, MacNeil and Dimitrakopoulos, 2017) and the ore block extraction strategy (Ben-Awuah et al., 2016, De Carli and de Lemos, 2015, King et al., 2016, MacNeil and Dimitrakopoulos, 2017, Whittle et al., 2018).

Optimizing the location of a crown pillar is a key factor in the optimal resource extraction evaluation process for deposits amenable by both open pit and underground mining. Finding the most suitable location of the crown pillar in a combined OPUG mining operations is one of the most interesting problems for mining engineers today (Bakhtavar et al., 2012). The transition from open pit to underground (OPUG) mining involves a complicated geomechanical process. Recent formulations of the OPUG mining transition complexes produces near optimal solutions at minimal level of confidence, and do not integrate the positioning of a 3D crown pillar, capital and operational developments into the optimization process.

Kurppa and Erkkilä (1967) assessed the simultaneous extraction between open pit and underground (OPUG) mining during the operations of the Pyhasalmi mine. They indicated that, simultaneous mining was possible due to the geometry of the orebody being worked. Luxford (1997) argued that, cost usually drives the decision to make the transition because as the open pit waste stripping cost keeps increasing with depth, there comes a time when the underground mining cost will be less than the open pit mining cost. Ben-Awuah et al. (2016) investigated the strategy of mining options for an orebody using a mathematical programming model. The research evaluated the financial impacts of applying different mining options separately or concurrently to extract a given orebody. The formulation maximizes the NPV of the reserve when extracted with: (1) open pit mining, (2) underground mining, and (3) concurrent open pit and underground mining (Ben-Awuah et al., 2015). The positioning of a crown pillar together with capital and operational development requirements were not incorporated into this model.

King et al. (2016) incorporated crown and sill pillar placement into their OPUG transition studies to separate the open pit from the underground mine. In their model, the location of the crown pillar was simulated, and preselected to divide the deposit into OP and UG mining zones before running an optimization for the OP mining zone and planning for the UG mining zone. MacNeil and Dimitrakopoulos (2017) investigated the transition decision at an operating open pit mine within the context of a mining complex comprising five producing pits, four stockpiles and one processing plant. In their research, MacNeil and Dimitrakopoulos (2017) priori identified the crown pillar envelope for a gold deposit and evaluated four crown pillar locations within this envelope leading to four distinct candidate transition depths. Decomposing the OPUG optimization process into scenarios has the tendency to compromise the global optimal solution.

A mathematical model that solves the transition problem was developed by Whittle et al. (2018) after modifying the normal pit optimization model based on the maximum graph closure algorithm (Lerchs and Grossman, 1965). The modifications allow the algorithm to account for the underground mining value of a block, and the requirement for a specified 2D crown pillar location above the underground mine. The algorithm of Whittle et al. (2018) is based on what they called the "opportunity cost approach", thus, if a given block is mined by open pit method, its open pit value is gained while its value that would have been obtained by extracting it using underground mining methods is lost. According to Whittle et al. (2018), the optimization approach does not control the mining sequence with time and produces a near optimal value for the mining project. Future works were therefore recommended to improve on their model.

In this research study, we have developed, implemented, and tested a MILP optimization framework for evaluating the extraction strategy for a deposit. The MILP model maximizes the Net Present Value (NPV) of the resource and determines the best extraction strategy for a given ore body amenable to different mining options. The model further determines the mining and processing schedule, the location of the required crown pillar, and the schedule for underground capital and operating developments (shaft/decline, levels, ore drives and crosscuts) for the optimal extraction option.

3 ASSUMPTIONS AND NOTATIONS

It is assumed that the size of the Selected Mining Units (SMUs) for open pit mining is equivalent to the stope sizes for underground mining. In the MILP framework, the SMUs are represented by mining blocks in general, or mining-cuts in specific relation to open pit mining or mining-stopes for underground mining. The location of each mining block or mining-cut or mining-stope is represented by the coordinates of the centroid. It is assumed that a crown pillar is required for the exploitation of the orebody by underground mining. The size of the crown pillar is estimated to be one vertical length of a stope or bench; thus, one bench or level in the block model will represent the crown pillar. For underground mining, ore extraction is achieved by a retreating method. Some of the notation of indices and parameters are as follows:

3.1 *Indices*

A general parameter f can take four indices in the format of $f_{k,l}^{a,t}$. Where:

$j \in \{1, \ldots, J\}$ index for open pit mining-cuts in the model.
$p \in \{1, \ldots, P\}$ index for underground mining-stopes in the model.

3.2 *Parameters*

$v_j^{op,t}$ the open pit (*op*) discounted revenue generated by selling the final product within mining-cut j in period t minus the discounted extra cost of extracting mining-cut j as ore and processing it.

$v_p^{ug,t}$ the underground (*ug*) discounted revenue generated by selling the final product within mining-stope p in period t minus the discounted extra cost of extracting mining-stope p as ore and processing it.

$q_j^{op,t}$ the open pit (*op*) discounted cost of mining all the material in mining-cut j in period t as waste.

$q_p^{ug,t}$ the underground (*ug*) discounted cost of mining all the material in mining-stope p in period t as waste.

g_j average grade of element in ore portion of mining-cut j.

g_p average grade of element in ore portion of mining-stope p.

o_j ore tonnage in mining-cut j.

o_p ore tonnage in mining-stope p.

r processing recovery; the proportion of mineral commodity recovered.

sp selling price of mineral commodity in present value terms.

sc selling cost of mineral commodity in present value terms.

pc extra cost in present value terms per tonne of ore for mining and processing in period t.

cm_l^t cost per bench or level l in present value terms of mining a tonne of rock material by open pit mining in period t.

cm^t cost in present value terms of mining a tonne of rock material by underground mining in period t.

4 THE INTEGRATED MILP MODEL

An integrated MILP model is formulated to determine the time and sequence of extraction of ore and waste blocks over the mine life for open pit and/or underground mining. The proposed MILP model interrogates the orebody and determines the best mining option that produces an optimal extraction sequence to maximize the Net Present Value (NPV) of the mining project. The mining options could either be open pit mining or underground mining or a combination of both open pit and underground mining. The model further determines the capital and operational development schedules required to extract the orebody by underground mining. The NPV of the extraction strategy is maximized in the presence of technical, geotechnical, geological, and economic constraints to enforce the mining sequence, grade blending requirements, capital and operational developments, and mining and processing capacities.

4.1 *Modeling the economic block value*

The economic block values are defined based on the SMUs, thus, mining-cuts for open pit mining and mining-stopes for underground mining. The value of the block is a function of the recovered quantity of mineral present in the block (processing recovery), the discounted revenue from selling the commodity, and the discounted mining, processing and selling costs.

The discounted revenues generated by selling the final product within block k being extracted in period t by open pit mining $v_j^{op,t}$ and underground mining $v_p^{ug,t}$ are respectively given in Equations (1) and (2). Similarly, the discounted costs of mining all the material within block k being extracted in period t by open pit mining $q_j^{op,t}$ and underground mining $q_p^{ug,t}$ are respectively given in Equations (3) and (4).

$$v_j^{op,t} = \sum_{j=1}^{J} o_j \times g_j \times r \times (sp - sc) - \left(\sum_{j=1}^{J} o_j \times pc \right) \tag{1}$$

$$v_p^{ug,t} = \sum_{p=1}^{p} o_p \times g_p \times r \times (sp - sc) - \left(\sum_{p=1}^{P} O_p \times pc \right) \tag{2}$$

$$q_j^{op,t} = (o_j + w_j) \times cm_l^t \quad (3)$$

$$q_p^{ug,t} = (o_p + w_p) \times cm^t \quad (4)$$

4.2 *Objective function*

The objective function of the MILP model maximizes the NPV of the mining project for combined open pit and underground mining options. During optimization, if OP only or UG only is selected, the objective function of the unselected mining option becomes zero. The objective function of the MILP model maximizes the NPV of the mining project for both open pit and underground mining. The quantity of ore processed is controlled by the continuous decision variables for open pit and underground mining respectively. Similarly, the quantity of rock material extracted by open pit and underground mining are also controlled by the continuous decision variables respectively. The continuous variables ensure fractional extraction and processing of the mining-cut or mining-stope in different periods of the mine life.

4.3 *Constraints*

The components of the constraints are grouped as follows: a) OP mining constraints; b) UG mining constraints; c) interaction of OP mining with UG mining constraints; d) crown pillar constraints; e) capital development constraints; f) operational development constraints; and g) non-negativity constraints. Acceptable upper and lower targets are defined for the mining, processing, developments, and plant head grade requirements. The mining capacity is a function of the ore reserve and the designed processing capacity which is also based on the available mining fleet acquisition for the operation. The processing capacity constraints define the ore production schedule in each period and further ensures the run-of-mine (ROM) material satisfies the quantity specification of the processing plant.

4.3.1 *OP mining constraints*

The OP mining constraints are deployed to control the open pit mining operations. The set of constraints include mining and processing capacities, grade blending, ore and mining-cut tonnages, available reserve, and vertical block precedence relations.

4.3.2 *UG mining constraints*

The UG mining constraints control the underground mining operation. These constraints include the mining and processing capacities, grade blending, ore and mining-stope tonnages, available reserve, and lateral block precedence relations.

4.3.3 *Interaction of OP mining with UG mining constraints*

The interaction of OP mining with UG mining constraints ensure that each mining block is extracted by only one mining option or left as unmined block in the crown pillar. The constraints further include a combined processing capacity that represents the contribution of ore production from both OP and UG mining operations.

4.3.4 *Crown pillar constraints*

The crown pillar constraints control the positioning of the required crown pillar. These constraints define the relationship of the location of the open pit and underground mining operations. The constraints ensure that the crown pillar is located at the bottom of the open pit mine, is always located above the underground mining, and stays unmined throughout the mine life.

4.3.5 *Capital development constraints*

These constraints ensure that the required capital development is considered when underground mining is the preferred extraction option. These set of constraints define the capital development requirements for the underground mining which involve the total length of capital development (shaft) and the relationship with the required operational developments.

4.3.6 *Operational development constraints*

These constraints ensure that the required operational developments are considered when underground mining is the preferred option for extracting the ore body. These set of constraints define the operational development requirements including the type and length of each operational development (level, ore drive, crosscuts) and lateral precedence relations with the block extraction sequence.

5 COMPUTATIONAL IMPLEMENTATION OF THE MILP MODEL

5.1 *Preparation of the model*

The formulated MILP model was implemented with an experimental case study. MATLAB 2018a (Mathworks, 2018) environment was used to define the modelled framework and IBM ILOG CPLEX Optimization Studio (ILOG, 2015) was integrated into MATLAB to solve the MILP at a gap tolerance of 5%. The model was tested on an Intel(R) Core™ i7-7700HQ CPU Dell computer @ 2.80GHz, with 32 GB RAM.

In open pit mining, the ore is exploited from the top to the bottom with a 45^{o} slope to ensure the geotechnics of the mine is controlled. A cross-section of the block model showing the precedence of block extraction for the open pit mine is shown in Figure 1. In Figure 1, to extract block 1, blocks 2, 3 and 4 needs to be extracted. However, to extract block 4, blocks 7, 8 and 9 must be priori extracted while to extract block 9, blocks 14, 15 and 16 must be priori extracted. It therefore follows that to mine block 1, all the shaded regions with blocks 1 to 16 must be afore-mined.

However, in underground mining, the block model of the deposit is prepared by citing the location of the main capital development (shaft) and defining the location of the operational developments (levels, ore drives and crosscuts) on each level. The level development links the shaft or decline to the ore drives through the centroid of each block. The crosscut developments extend from this ore drive to the ends of the minefield through each block, acting as stope drives. At this stage, the extraction method (retreating or advancing) is established for the ore body. Figure 2 is a schematic representation of the block model showing the underground operating developments. The arrows show a retreating mining method for the ore extraction sequence on a typical level.

5.2 *Case study—synthetic copper deposit*

The Mixed Integer Linear Programming (MILP) model was implemented and tested on a synthetic copper deposit. The copper dataset is represented by a geologic block model which is a 3D array of cubical blocks containing 605-unit blocks. These unit blocks represent the

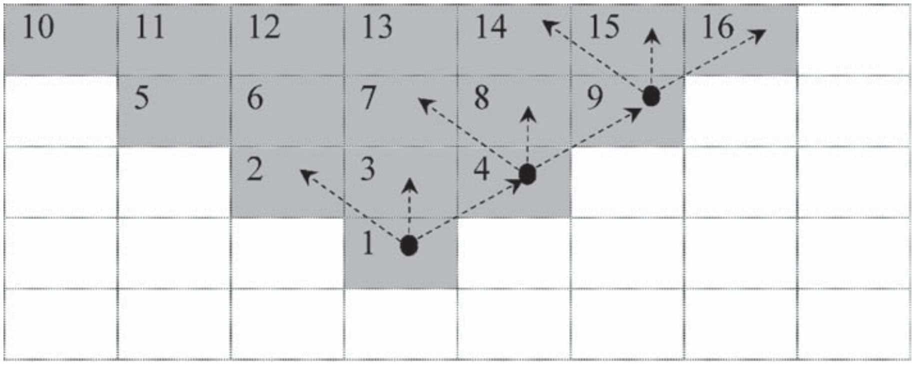

Figure 1. A cross-section of the block extraction precedence for OP mining in the MILP model.

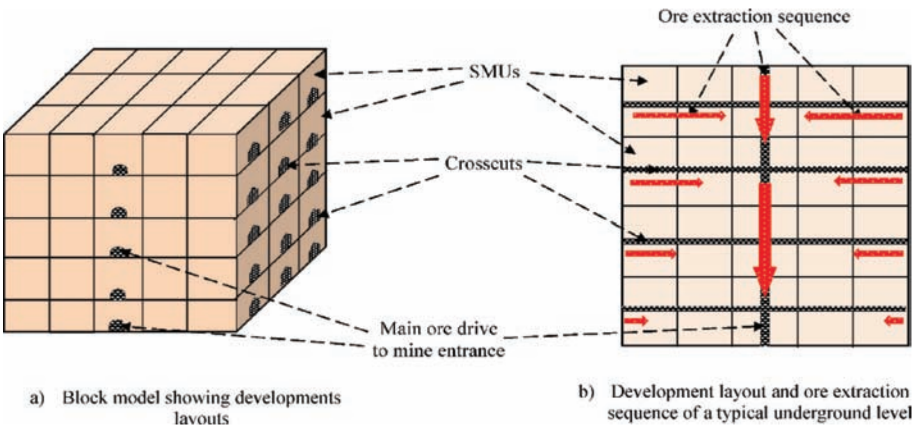

Figure 2. Isometric view of the block model and development layout of an underground level.

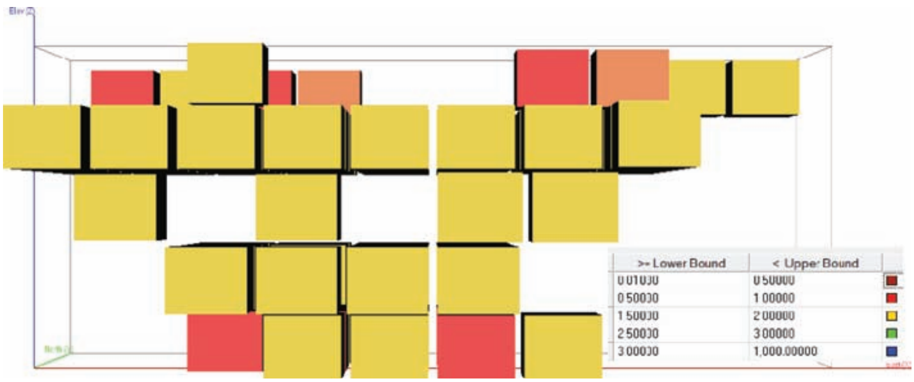

Figure 3. Layout of the synthetic copper deposit showing mineralized blocks.

Selective Mining Units (SMUs) for OP and UG mining. The ore body in the block model is irregularly shaped with a total mineral resource of 116.10 Mt at an average Cu grade of 1.05%, minimum grade of 0.72% and maximum grade of 3.0%. Figure 3 shows the layout of the synthetic copper deposit showing mineralized blocks. Table 1 and Table 2 respectively show the statistical description and distribution of the quality and quantity of metal in the deposit. The quality of the copper deposit is shown by a few blocks occurring at the top portions of the ore body with higher metal contents. The bottom portions of the deposit show a similar mineralization while low quality ore blocks occur in the middle sections. The copper distribution in the deposit suggests it could be exploited with open pit mining for earlier financial benefit. However, the incremental cost of open pit mining with depth may cause underground mining to compete as the better option at a certain depth during the mineral exploitation process. Therefore, the essence of evaluating this deposit with the proposed MILP framework.

5.2.1 *Economic and mining data*

The yearly processing capacities are determined based on the proposed plant capacities for the mine while the yearly mining capacities are deduced from the ore and waste proportions of the deposit. An incremental bench cost of \$4.0 per 15 m bench was used as the open pit mining variable cost as the pit extends downwards. The economic, mining and processing data used for evaluating the copper deposit as summarized in Table 3 were estimated using data obtained from CostMine (2016) and pre-feasibility reports of two mining companies in Canada (Centerra Gold Inc. and Premier Gold Mines Limited, 2016).

Table 1. Statistical description of the synthetic copper deposit.

No.	Description	Value
1	Total mineralized material (Mt)	116.10
2	Minimum value of Cu (%)	0.719
3	Maximum value of Cu (%)	3.000
4	Average value of Cu (%)	1.051
5	Variance ($\%^2$)	0.394
6	Standard deviation (%)	0.628
7	Number of levels/benches	5

Table 2. Distribution of the metal quality and quantity in the deposit.

Level no.	Total metal content (% × million)	Total ore tonnage (t × million)	Value
1	19.10	12.15	9
2	40.22	52.65	39
3	4.05	5.40	4
4	15.74	20.25	15
5	42.84	25.65	19

Table 3. Economic, mining and processing data for evaluating the copper deposit.

No.	Parameter	Value
1	Open pit mining cost ($/t)	8.0
2	Underground mining cost ($/t)	300.0
3	Processing cost ($/t)	15.0
4	Selling cost ($/lb)	1.5
5	Selling price of copper ($/lb)	3.5
6	Discount rate (%)	10.0
7	Processing recovery (%)	95.0
8	Max open pit (OP) processing capacity (Mt/year)	8.0
9	Min open pit (OP) processing capacity (Mt/year)	0.0
10	Max open pit (OP) mining capacity (Mt/year)	20.0
11	Min open pit (OP) mining capacity (Mt/year)	0.0
12	Max underground (UG) processing capacity (Mt/year)	3.0
13	Min underground (UG) processing capacity (Mt/year)	0.0
14	Max open pit & underground (OPUG) processing capacity (Mt/year)	8.0
15	Min open pit & underground (OPUG) processing capacity (Mt/year)	0.0
16	Max underground (UG) mining capacity (Mt/year)	3.0
17	Min underground (UG) mining capacity (Mt/year)	0.0
18	Incremental bench cost ($/15 m)	4.0
19	Operating development cost ($/m)	7000
20	Capital development cost ($/m)	15,000
21	Max operating development (m/year)	5000
22	Max capital development (m/year)	30

5.2.2 *Results and discussions*

Evaluation of the copper deposit with the integrated Mixed Integer Linear Programming (MILP) model indicates that a combined sequential and simultaneous open pit and underground (OPUG) mining operation is the preferred mining option with the highest Net Present Value (NPV) of $265.89 billion. The integrated MILP model was modified to evaluate the ore body for an independent open pit (OP) mining option and an independent underground

(UG) mining option with the required crown pillar, capital and operational developments. The NPV of the OPUG mining option is about 14.4% better than an independent OP mining option and about 24.1% better than an independent UG mining option. The evaluated results are shown in Table 4. The copper deposit is therefore best exploited with a combined open pit and underground (OPUG) mining option for a mine life of 19 years with Level 3 acting as the unmined crown pillar.

With a total ore production of 100.05 Mt from the combined OPUG mining option with crown pillar, the OP mining operation contributes 64.80 Mt of ore while the UG mining operation contributes 35.25 Mt of ore. The remaining 15.95 Mt of the available mineral resource is either left in the crown pillar (5.40 Mt) or left as unmined low-grade stopes (10.55 Mt). In a more practical underground mine, where the crown pillar is recovered ("robbed") during the operational life of mine, the 5.40 Mt of ore deposit will be mined to further improve the total net present value (NPV) of the project if the ore quality is economical.

A sectional view through the ore body showing the open pit limit, location of the required crown pillar, an unmined low-grade stope and the mineralized zone extending beyond the bottom of the open pit limit is shown in Figure 4. The crown pillar is located on Level 3; thus, the first 2 levels (or benches) are extracted by open pit mining while Levels 4 and 5 are extracted by underground mining.

The ore and rock extraction schedules for the combined open pit and underground mining (OPUG) option with a crown pillar are shown in Figure 5 and Figure 6 respectively. The ore is extracted by an independent open pit mining option in the first 2 years of the mine life before underground ore production begins simultaneously from the 3rd year through to the 12th year. The ore production schedule switches completely to an independent underground mining option in the remaining 2 years of the mine life. In summary, the ore extraction profile for the combined OPUG mining option is a blend of sequential and simultaneous open pit and underground mining options for a mine life of 14 years. Due to the outcrop of the ore

Table 4. Results from the integrated MILP model for the case study.

Mining option	Net present value ($b)	NPV of OPUG compared (%)	Processed ore tonnage (Mt)	Resource depletion (%)
Open pit mining	227.73	–14.4	116.00	100.0
Underground mining	201.72	–24.1	103.95	89.6
Combined open pit and underground mining	265.89	–	100.05	86.3

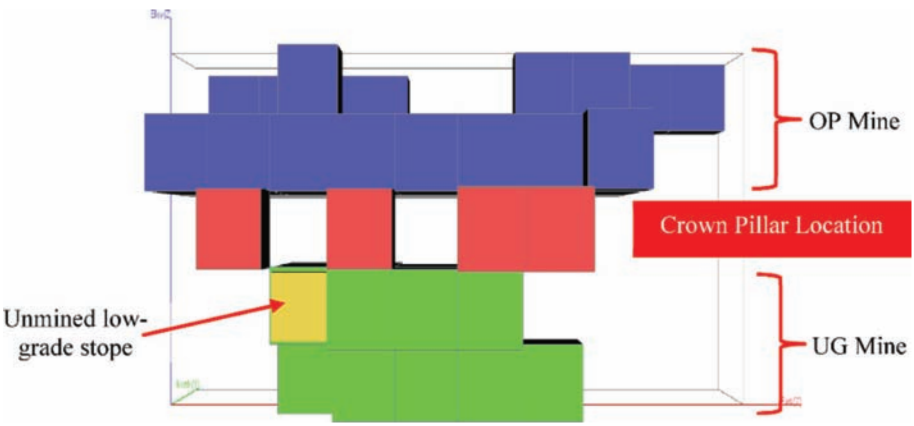

Figure 4. A sectional view through the block model showing the open pit limit, crown pillar and underground mining regions for the case study.

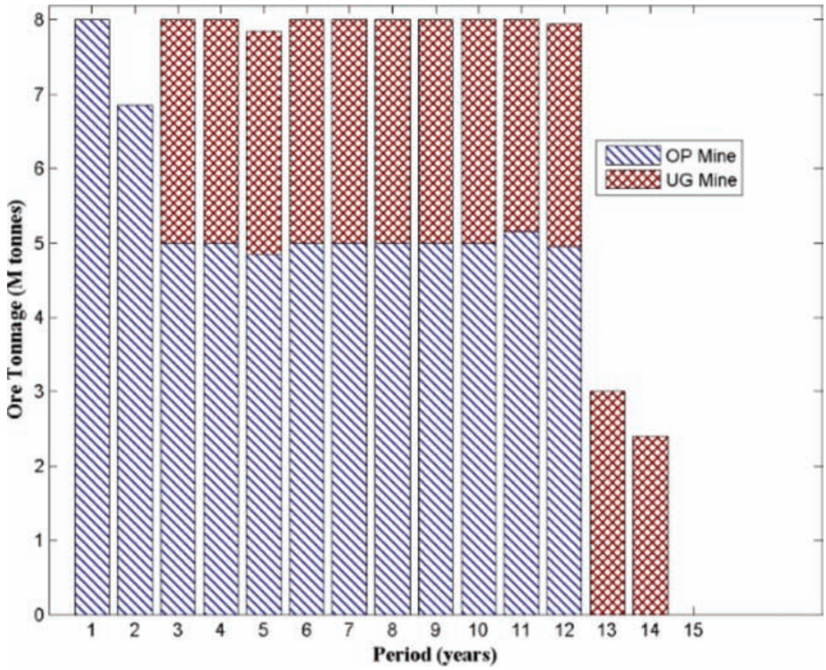

Figure 5. Yearly ore production schedule for the OPUG mining option.

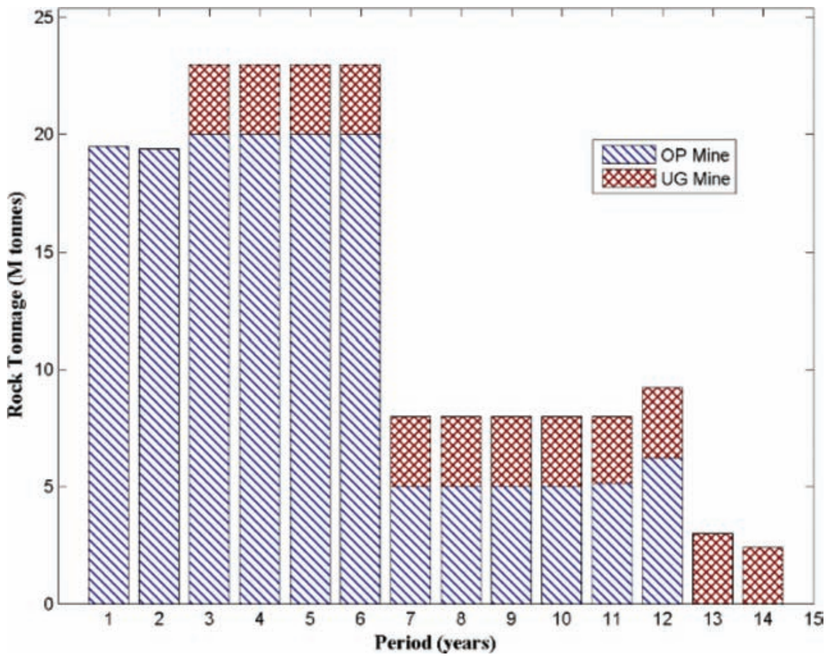

Figure 6. Yearly rock production schedule for the OPUG mining option.

body, the ore extraction schedule in the first year of the mine life satisfies the full capacity of the plant requirement of 8.0 Mt.

The average grade of Cu ore processed in each period for the combined open pit and underground mining option with a crown pillar is shown in Figure 7. In Figure 7, the yearly

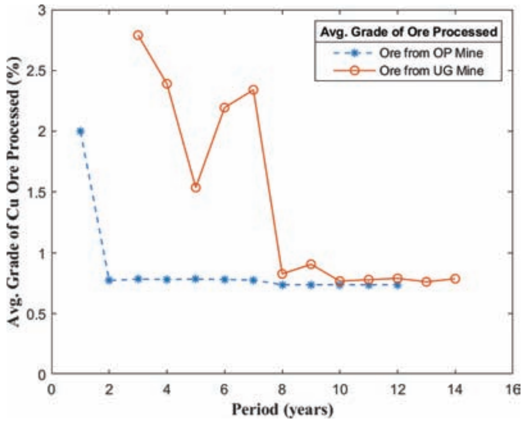

Figure 7. Average grade of ore processed in each year for OPUG mining option.

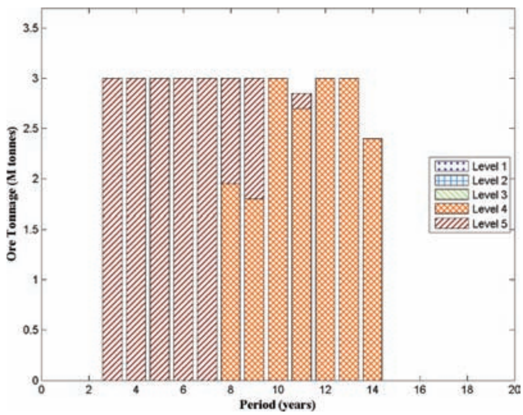

Figure 8. Ore tonnage per level extracted in each year for the underground mining operation in the OPUG mining option.

average grade of processed ore for the underground mining operation is generally higher compared to the open pit mining operation. This shows that underground mining prioritizes the extraction of high-grade ores to cover the high cost of mining by generating higher revenues. It is convincing that the grade blending constraints of the MILP model prioritizes higher grades over lower grades in exploiting the mineral deposit by any of the mining options. Thus, the open pit mining operation targeted the outcropped high-grade ore while the underground mining operation targeted the deep-seated high-grade ore.

To highlight the nature of the underground mining operation and obtain further information on the required exploitation method, the levels contributing to the mineral extraction schedule are shown in Figure 8. With the crown pillar located on Level 3, ore extraction by the underground mining operation started on the last level, Level 5, from the 3rd year through to the 7th year of the mine life before continuing with ore production from Level 4 to the end of mine life. With this knowledge on the mining sequence from each level (Figure 8) and the geotechnics of the rock formation, any suitable underhand method of ore extraction could be further evaluated and selected as the appropriate underground exploitation method to develop this deposit. The operational development schedule for the underground mine is shown in Figure 9. With a total operational development capacity of 10,000 m per year, advancement starts immediately after completion of the capital development (shaft) in the 3rd year but ends in the 6th year before commencing again from year 8 to 12.

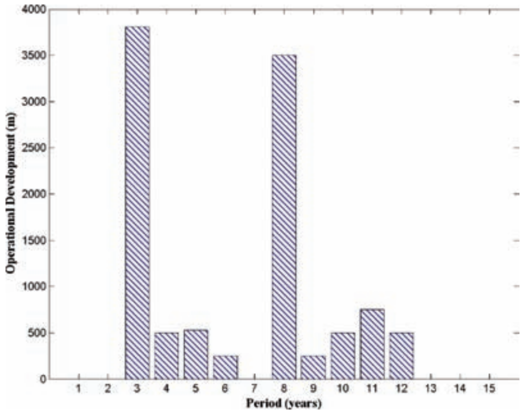

Figure 9. Operational development schedule for the underground mining operation in the OPUG mining option.

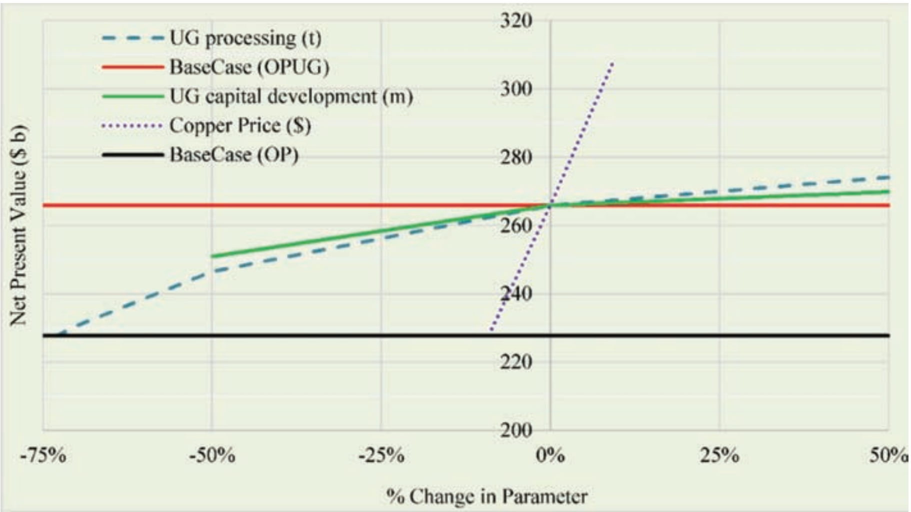

Figure 10. Sensitivity assessment of selected technical and economic parameters used in the MILP model.

5.2.3 *Sensitivity analysis*

The sensitivity of the results to gold price, underground processing capacity and underground capital development (shaft) is evaluated with the MILP model. This sensitivity analysis was conducted to examine the selected technical or economic parameters that influence the choice of mining option(s) for the deposit. The sensitivity plot is shown in Figure 10. In Figure 10, the MILP model is very sensitive to the market price of copper followed by the quantity of ore processed from the UG mining operation and the completion rate of capital development.

For this case study, about 9% decrease in the copper price (from \$3.18 to \$2.89 per lb) will change the optimal mining option from combined OPUG mining to an independent OP mining option for a reduced NPV of \$227.73. Similarly, when the underground processing capacity decreases by about 73% (0.81 Mt), the combined OPUG mining will change to an independent OP mining option. This copper deposit is however less sensitive to the completion rate of underground capital development (shaft) as it did not cause a change in mining option within the changes tested.

6 CONCLUSIONS AND RECOMMENDATIONS

An integrated Mixed Integer Linear Programming (MILP) model for evaluating the extraction of a mineral deposit has been developed, implemented, and tested on a synthetic copper dataset. The proposed MILP model interrogates a deposit and determines the optimal extraction strategy for a combination or one of the mining options: (a) independent open pit mining, (b) independent underground mining with crown pillar, (c) simultaneous open pit and underground mining with crown pillar, (d) sequential open pit and underground mining with crown pillar, and (e) combinations of simultaneous and sequential open pit and UG mining with crown pillar. The location of the 3D crown pillar together with the required capital and operational development schedules are decided by the optimization process. The MILP model is applicable to all types of deposits and for any preferred direction of mineral extraction sequence. The capital development can either be a shaft, decline or both and during optimization, depending on the optimal mining option, the capital development can either commence from the surface, bottom of the open pit mine or both.

To implement the MILP model, the block model was organized by first selecting a preferred ore extraction method on a level (advancing or retreating) and siting the possible location of the underground capital and operational developments based on the philosophy and geotechnical understanding of the mine. As in practice, the MILP model requires that an incremental cost for open pit mining is defined per depth (m) for block extraction. Thus, the cost of open pit mining increases with depth until underground mining becomes preferable to open pit mining. This concept allows the optimizer to decide when to stop the open pit mining operation, introduce a crown pillar and start the underground mining operation in the presence of both capital and operational developments.

The results from the case study showed a combined sequential and simultaneous open pit and underground mining option (OPUG) with crown pillar was selected as the optimal mining option to exploit the deposit. The ore and rock extraction schedules for the open pit and underground mining operations together with the operational and capital development schedules were determined for the synthetic copper project. The output of the model in mapping-out the ore extraction per level in each period further provides more insight into the mining sequence for selection of the appropriate underground mining method. A sensitivity analysis conducted on selected technical and operational parameters indicate that the determination of the optimal mining option using the MILP model is very sensitive to the selling price of copper, followed by the quantity of ore processed by the underground mining operation and the completion rate of the underground capital development (shaft).

The authors recommend that stockpile management and geotechnical information be incorporated into the MILP model to ensure the outputs of the model is exhaustive and realistic. Similarly, it is necessary to extend the model from its current deterministic approach to a stochastic framework to address the impact of grade uncertainty in the choice of mining option and project evaluation.

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Review

A Review of Models and Algorithms for Surface-Underground Mining Options and Transitions Optimization: Some Lessons Learnt and the Way Forward

Bright Oppong Afum and Eugene Ben-Awuah *

Mining Optimization Laboratory (MOL), Bharti School of Engineering, Laurentian University, Sudbury, ON P3E 2C6, Canada; bafum@laurentian.ca
* Correspondence: ebenawuah@laurentian.ca

Abstract: It is important that the strategic mine plan makes optimum use of available resources and provides continuous quality ore to drive sustainable mining and profitability. This requires the development of a well-integrated strategy of mining options for surface and/or underground mining and their interactions. Understanding the current tools and methodologies used in the mining industry for surface and underground mining options and transitions planning are essential to dealing with complex and deep-seated deposits that are amenable to both open pit and underground mining. In this study, extensive literature review and a gap analysis matrix are used to identify the limitations and opportunities for further research in surface-underground mining options and transitions optimization for comprehensive resource development planning.

Keywords: strategic mining options optimization; mathematical programming models; transition depth; open pit-underground mining; resource development planning

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1. Introduction

Surface mining is known to be relatively highly productive, very economic, and safer for workers compared to underground mining for most suitable deposits. However, recent evolution in environmental regulations and societal expectations may result in the development of small, high-grade deposits by shallow open pits (OP) or in the establishment of high-grade underground (UG) mines in place of extensive OP operations [1]. Optimizing the extraction of a mineral deposit in the presence of both surface mining methods and UG mining methods result in the most economic decision generated by identifying the best mining option for the deposit. In resource development planning, optimizing resource exploitation depends largely on the mining option used in the extraction. The term mining options optimization has been used by researchers and professionals to refer to the initiatives or choices undertaken in the extractive industry to expand, change, defer, abandon, or adopt strategies for a mining method(s) and sometimes investment opportunities; based on changing economics, technology, or market conditions [2–10]. For mineral deposits with orebodies that extend from near surface to several depths, such orebodies are amenable to different variations of either open pit mining, underground mining, or both.

Some studies have been conducted to solve the surface–underground mining options and transitions optimization (SUMOTO) problem. These studies have focused on determining the transition depth and the resulting production schedules for the OP and UG mining operations using simplified optimization frameworks. These models do not extensively address the multi-objective optimization nature of the SUMOTO problem, and do not formulate the problem with a complete description of the practical mining environment. Specifically, the existing models do not incorporate essential developmental infrastructure such as primary and secondary mine accesses, ventilation requirement, and geotechnical support and reinforcement in the optimization framework. Results from these models often lead to localized optimal solutions or biased solutions that are usually impractical to implement in the mining environment [2,3,11–24].

Existing optimization algorithms used in attempting the mining options problem include the Lerchs–Grossman (LG) algorithm, Seymour algorithm, floating cone technique, network flows, dynamic programming, neural network, theory of graphs, and mathematical formulations [25]. Some authors have studied the surface–underground mining options and transitions optimization problem with available commercial software packages including Surpac Vision, Datamine's NPV Scheduler, Whittle Four-X, Geovia MineSched, integrated 3D CAD systems of Datamine, Vulcan, MineScape, MineSight, Isatis, XPAC, Mineable Reserve Optimizer (MRO), Blasor pit optimization tool, COMET cut-off grade and schedule optimizer, and Datamine Studio 3 [7,9,17,26,27]. The techniques used by these authors are not generic but mostly scenario based and often lead to localized optimization solutions.

Although Bakhtavar, Shahriar, and Oraee [3] employed a heuristic algorithm to compare economic block values computed for both open pit and underground mining on a depth flow basis to solve the SUMOTO problem, results from the heuristic algorithm do not offer a measure of optimality as it is the case in mathematical programming optimization. Notable authors that used mathematical programming to solve the mining transition problem limit their model to the determination of transition depth and block extraction sequence for the open pit and underground mining operations [2,4,8,9,11–15,17–19,24,25,28,29]. Similarly, other authors have developed stochastic mathematical programming models to solve the surface–underground mining options and transitions optimization problem. They focused on determination of the transition depth in 2D environment, and do not incorporate other essential underground mining constraints such as primary and secondary development, ventilation shaft development, and geotechnical requirements for the development openings and stopes in the optimization framework [7,20,30,31]. This is because the optimization of underground mines is computationally complex [32] and integrating it with open pit mining makes it more challenging [33].

The positioning of the required crown pillar thickness in the SUMOTO problem is key to the operations of such mines. Some authors pre-selected the depth of the crown pillar (transition depth) before evaluating portions above the crown pillar for open pit mining and portions below the crown pillar for underground mining [4,18,25,27,28]. This may lead to suboptimal solutions and will require evaluating multiple crown pillar locations in a scenario-based approach. A few authors have attempted to incorporate the positioning of the crown pillar in the optimization process [14,24,30,31,34,35]. Their models were good improvements over previous works but were missing some constraints such as the ventilation requirement and rock strength properties required for practical implementation. The transition from OP to UG mining is a complicated geomechanical process which requires the consideration of rock mass properties [36,37].

Bakhtavar [12] reviewed the combined open pit with underground mining methods for the past decade and noticed that the transition problem has been implemented in either simultaneous or non-simultaneous modes. He asserts that non-simultaneous mode of combined mining is more acceptable because large-scale underground caving methods with high productivity and low costs can be used. However, in simultaneous mode, horizontal and vertical slices underhand cut and fill with cemented backfill is more feasible to be used with OP mining. Afum, Ben-Awuah, and Askari-Nasab [35] implemented a mathematical programming model that allows the optimization approach to decide whether the mineral deposit should be exploited with either simultaneous, non-simultaneous, sequential, or any of these combinations thereof.

Most existing models in general do not include the requirements of essential underground mining infrastructure such main access to the underground mine (shaft or decline or adit development), ventilation development, operational development (levels, ore and waste drives, crosscuts), and necessary vertical development (ore passes, raises). Equally, these existing models do not incorporate rock strength properties in the SUMOTO problem.

Although these essential infrastructure and geotechnical characteristics of the rock formation are significant to underground mining operations, their added complexities make it difficult to be included in the SUMOTO models. According to Bullock [38], mine planning is an iterative process that requires looking at many options and determining which, in the long run, provides the optimum results. Using such iterative process could lead to some inferior or sub-optimal solution(s) that do not constitute the global optimal solution.

In summary, this paper reviews relevant literature on algorithms and models for the SUMOTO, identifies gaps and opportunities that can be explored for further research and implementation in the mining industry, and further introduces the significance of employing mathematical programming for planning resources amenable to both options. Figure 1 is a schematic representation of the surface–underground mining options and transitions planning problem for deposits amenable to both mining options.

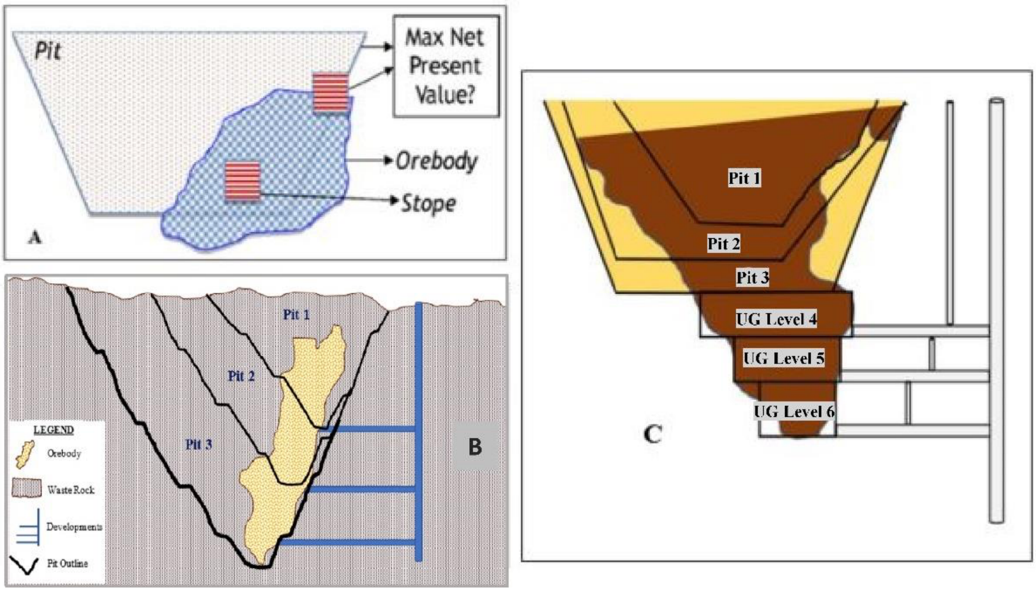

Figure 1. Schematic representation of the surface–underground mining options and transitions planning problem. (**A**) illustrates evaluation of an orebody to generate maximum net present value (NPV) depending on how each mining block is extracted; either through open pit mining, open stope extraction, or both. (**B**,**C**) demonstrate the extraction of a mineral resource by open pit (OP), underground (UG), or both open pit and underground (OPUG) mining for optimum resource development planning. ((**A**)—Ben-Awuah, Otto, Tarrant, and Yashar [4]; (**B**)—Afum, Ben-Awuah, and Askari-Nasab [34]).

1.1. Classification of Mining Methods (Mining Options)

Mining is defined as the process of exploiting a valuable mineral resource naturally occurring in the earth crust [39,40]. The extraction of mineral resources from the earth crust is classified broadly into two; surface mining and UG mining. In surface mining, all the extraction operations are exposed to the atmosphere while in UG mining, all the operations are conducted in the bosom of the earth crust. The main objective of a mineral project development is the maximization of investment returns; the "golden rule" of mining or the investor's "law of conservation" [41]. Therefore, adopting the best mining option that maximizes the project's value is a requirement to the establishment of a successful mine. Planning a surface mine is often simpler compared to an underground mine because there are broad similarities between different variations of surface mining as opposed to the variations of underground mining. Thus, planning an underground mine is necessarily complicated by the availability of many different types and variations of mining systems [37]. These surface and underground mining variations are also generally referred to as classes of mining methods. The classification of surface mining methods and underground mining methods are respectively illustrated in Figures 2 and 3.

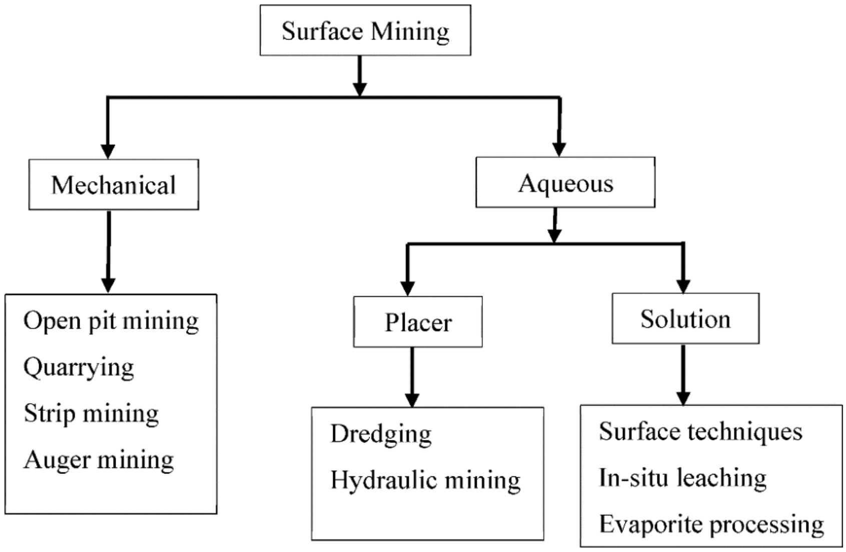

Figure 2. Classification of surface mining methods.

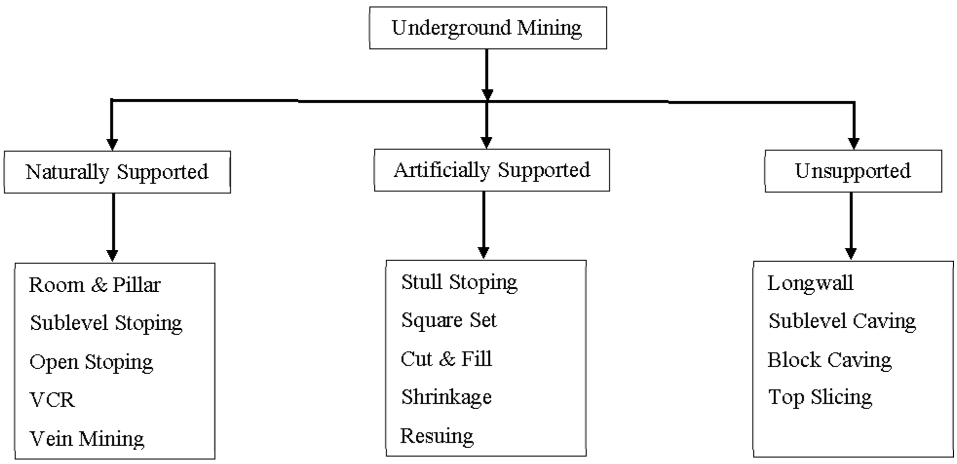

Figure 3. Classification of underground mining methods.

Surface mining methods are broadly classified into mechanical and aqueous extraction methods (Figure 2). Mechanical surface mining methods include open pit mining, quarrying, strip mining, and auger mining, while aqueous surface mining methods include placer mining and solution mining. Placer mining includes dredging and hydraulic mining while solution mining includes surface techniques such as in-situ leaching and evaporite processing. Based on the rock formation strength, UG mining methods are broadly classified into naturally supported methods, artificially supported methods, and unsupported methods (Figure 3). Naturally supported mining methods include room and pillar, sublevel stoping, open stoping, vertical crater retreat (VCR), and vein mining. Artificially supported mining methods include stull stoping, square set, cut and fill, shrinkage, and resuing while unsupported methods include longwall, sublevel caving, block caving, and top slicing. According to Nelson [1], some of the factors that must be considered when choosing between surface or underground mining methods include:

1. Extent, shape, and depth of the deposit;
2. Geological formation and geomechanical conditions;
3. Productivities and equipment capacities;

4. Availability of skilled labor;
5. Capital and operating costs requirements;
6. Ore processing recoveries and revenues;
7. Safety and injuries;
8. Environmental impacts, during and after mining;
9. Reclamation and restoration requirements and costs;
10. Societal and cultural requirements.

1.2. Mineral Projects Evaluation

When a mineral deposit is discovered, several evaluations are conducted towards the project's viability. The evaluation methods used are broadly grouped into two: positive evaluation methods and normative evaluation methods [42]. Positive evaluation methods assess the quantity and quality of the mineral project while normative evaluation methods assess the social and ethical values of the mineral project. Positive evaluation methods deal with investigations related to the geology of the formation, technology required to develop the deposit, investment decisions such as net present value (NPV), internal rate of return (IRR), and options valuations, and financial evaluation to show how funds will be raised and repaid in future. This research focuses on positive evaluation methods for mineral projects with the exception of how funds are raised and repaid for the project.

A mineral property could be described as being at early-stage or advanced-stage exploration, development, defunct, dormant, or production stage [43]. Three types of studies are undertaken according to the stage of life of the mineral project under evaluation. These studies are scoping study, prefeasibility study, and feasibility study [44,45]. Scoping study is a preliminary assessment of the technical and economic viability of a mineral property while a prefeasibility study is a comprehensive study of the viability of the mineral project subject to operational constraints at which the preferred UG mining method or OP mining arrangement is established, including an effective mineral processing method. Feasibility study, on the other hand, is a comprehensive study on the design and cost of the selected mining option for developing the mineral project. Usually, the confidence level associated with a prefeasibility study is lower than a feasibility study while the confidence level for a scoping study is also lower than a prefeasibility study.

The outcome of a prefeasibility study on a mineral property is a mineral reserve that is profitable. Thus, the mineral resource is technically and economically evaluated and if it is profitable, it becomes a mineral reserve otherwise it remains a mineral resource until prevailing factors (commonly referred to as modifying factors) become favorable [44]. The details of the evaluation studies for a mineral project depends mainly on the stage of life of the mine and the prevailing regulatory requirements of the region. These detailed evaluation considerations may include geological and geostatistical modeling, geotechnical investigation, mining optimization studies, cost-benefit analysis, equipment selection, rock transportation studies, rock stability and slope requirement assessment, crown pillar location investigation, blasting and fragmentation studies, environmental baseline studies, environmental management and impact studies, and mine closure and reclamation studies. During prefeasibility studies, typical technical and economic considerations include geological, geostatistical, and mining optimization investigations. These investigations primarily define the spatial grade distribution of the deposit, and uncovers the size, shape, depth (extent), and orientation of the deposit, and further validate the profitability associated with the mining strategy.

Identifying the preferred mining option (whether OP or UG or both) during prefeasibility studies on a mineral property involves strategic optimization analysis. This analysis ensures much value is attained during the implementation of the resource development plan. When the mineral resource is closer to the earth surface (sometimes with significant outcrop), OP mining evaluation studies are outrightly conducted. When the mineral resource is deeply buried in the earth crust (with no significant outcrop or presence near the earth surface), UG mining evaluation studies are conducted. However, when the mineral deposit is significantly closer to the earth surface and, also, deeply buried in the earth crust, the deposit becomes amenable to both OP and UG mining. In such cases, the portion of the orebody near the earth surface is evaluated to be exploited by OP mining method to produce early revenue, while the deeper portion is either outrightly evaluated for UG mining or the evaluation is deferred for later years in the future [7]. In general, OP mining methods are characterized by relatively low mining and operating costs, high stripping ratio, and extended time in accessing the mineral ore [4,46] while UG mining is characterized by high mining and operating costs, high-grade ore, and earlier times in retrieving the ore [31,47–50].

In most cases, for mineral deposits amenable to OPUG mining, the UG mining evaluation is not undertaken during prefeasibility studies but conducted in later years when the OP mine stripping ratio increases towards the critical limit. The effect of this traditional evaluation approach is an increased overall mining cost and a potential loss of financial benefits [51,52]. Similarly, where UG mining commences at the onset of the mineral project, and later converts or transitions to OP mining, significant financial loss can occur if the global mining strategy was not defined for the entire mineral deposit during prefeasibility studies. Historically, some mining companies that transitioned from OP to UG or vice versa include Telfer, Golden Grove, and Sunrise Dam in Australia [53]; Grasberg mine in Indonesia [54]; Akwaaba and Paboase mines of Kinross Chirano Gold Mine in Ghana [55]. In addition, Lac Des Iles mine located in Canada and Newmont Ahafo mine in Ghana both operate OPUG mining operations.

The extraction evaluation of such mineral resources amenable to OPUG mining is referred to in this research as mining options optimization [2,6,10,15]. The importance of rigorously assessing the economics for a mineral deposit extraction, before deciding on the mining option to adopt is therefore essential to mineral resource planning. This may ensure important decisions of canceling a major OP pushback and transitioning to UG mining or vice versa is known at the onset of the mining project [3,52].

2. Evaluation Techniques for Mining Options and Transitions Planning

The outcome of an evaluation study for a mineral deposit amenable to OPUG mining includes the optimal mining option, strategic extraction plan, and a transition depth or location. The variations of the mining option are independent OP mining, independent UG mining, concurrent OP and UG mining, OP mining followed by UG mining, and UG mining followed by OP mining. The strategic extraction plan includes the sequences of rock extraction and the determination of life of mine and transition depth. The transition depth defines the location or position of the crown pillar. The extraction strategy when OPUG mining is preferred could either be sequential mining or parallel mining or both [47]. Respectively, other researchers used the terms simultaneous or non-simultaneous or combined OPUG mining to refer to these same mining options [30].

Sequential mining is when the mineral deposit is continuously extracted by an independent OP mining method(s) until the pit limit is completely mined out before being followed by UG mining method(s), while parallel mining is when the mineral deposit is simultaneously or concurrently extracted by OP and UG mining in the same period or time. Transitioning is the main challenge for OPUG mining projects due to the complexity and implications of where and when to position the crown pillar (or identify the transition depth) [7] in the presence of various mining constraints. Over the years, five fundamental approaches have been used to determine the transition point or location of the crown pillar [24,47,56,57]. These techniques are (1) biggest economic pit, (2) incremental undiscounted cash flow, (3) automated scenario, (4) stripping ratio, and (5) opportunity cost analyses.

For the biggest economic pit approach, the mineral resource is primarily evaluated for OP mining. When the OP mining limit is obtained, the portion of the mineral resource falling outside the OP outline is evaluated for UG mining. The biggest economic pit is the simplest and most commonly used traditional approach for evaluating a mineral resource amenable to OPUG mining options. For the biggest economic pit, the pit usually terminates when the marginal cost of waste stripping outweighs the marginal revenue obtained from processing additional amounts of ore.

In the case of the incremental undiscounted cash flow approach, the marginal OP profit from the mineral project per depth is evaluated and compared to the marginal UG profit. Due to increasing cost of stripping waste per depth, there is a point where the marginal OP profit is lower than the marginal UG profit. This depth is the transition point which then acts as the crown pillar during transition. This transition depth is typically shallower compared to the largest economic pit [47]. This method assumes that UG mining profits do not depend on the depth of operation, therefore, there will be a point where the marginal profits from UG mining operation will exceed that from OP mining operation.

The automated scenario analysis approach accounts for discounting unlike the incremental undiscounted cash flow approach. It is based on the premise that, per an equivalent unit of throughput, UG mines are characterized by high cut-off grades and therefore have higher cash flow compared to OP mines for the same throughput. The approach is implemented by compiling schedules for OP and UG mining and comparing the computed NPV for each potential transition point. Thus, a set of transition points are evaluated and the OPUG mining arrangements that offers the highest NPV is selected for further analysis and design [47]. This method is time consuming and complex.

The stripping ratio analysis features the use of allowable stripping ratio (ASR) planned by mine management for the OP mine and overall stripping ratio (OSR) computed per depth for the OP mine to determine the transition depth [56,57]. The stripping ratio is expressed by this relation with emphasis on exploitation cost of 1 tonne of ore in UG mining and in OP mining, as well as, removal cost of waste in relation to 1 tonne of ore extracted by OP mining. As OP mining deepens, the stripping ratio usually increases, increasing the overall mining cost. An OSR is calculated and used to determine the breakeven point of the OP mine relative to its depth. The OP mine transitions to UG mine when OSR is equal to the ASR established by management of the mining project.

The opportunity cost technique is an extension of the LG algorithm that optimizes the OP ultimal pit while considering the value of the next best alternative UG mining option. This approach also employs the strength of the undiscounted cashflow technique and assumes that rock materal in the transition zone will be mined by UG method if not mined by OP method [24]. The methodology ensures that a minimum opportunity cost is achieved for the selected optimal mining option at the expense of the unselected mining option.

An evaluation technique that seeks to leverage the advantages of all these five fundamental approaches were recently introduced by Afum and Ben-Awuah [33]. This approach is referred to as the competitive economic evaluation (CEE) technique. The CEE process evaluates each block of the mineral deposit and economically decides: (a) blocks suitable for OP mining, (b) blocks suitable for UG mining, (c) unmined blocks, and (d) unmined crown pillar simultaneously. The CEE optimization strategy is an unbiased approach that provides fair opportunity to each mining block for selection by a mining option.

2.1. Notable Research on Mining Options and Transitions Planning Optimization

Historically, a cash flow and NPV based algorithm was introduced in 1982 to examine the open pit-underground (OP-UG) transition interface [21]. Subsequently, in 1992, Nilsson reviewed the previous 1982 algorithm and produced a new algorithm that considers the transition depth as a critical input for evaluating deposits amenable to OP and UG extraction [22]. An algorithm for determining the depth of transition was thereafter introduced by Camus [58]. This algorithm was presented based on the economic block values for OP and UG extraction methods. The technique involves the implementation of the OP extraction algorithm taking into consideration an alternate cost due to UG exploitation. In 1997, a model was developed by Shinobe [10] that enables the mine operator to determine the optimum time of conversion based on discounted cash flow (DCF) techniques, and cost estimation equations according to O'Hara and Suboleski [59].

The model assumed that the underground resources were confined, and their extraction is technically feasible. Whittle Programming Pty developed an applied approach referred to as quantified operational scenarios for interfacing OP-UG mining methods in the OP to UG transition problem [60].

In 2001 and 2003, an evaluation technique based on allowable stripping ratio with a mathematical form for the objective function was introduced by Chen, Li, Luo, and Guo [57] and Chen, Guo, and Li [56]. Volumes of ore and waste within the final pit limit were assumed to be a function of depth for determining the transition point. A heuristic algorithm based on economic block values of a two-dimensional block model that compares the total value when using OP mining for extracting a particular level to UG mining of the same level was developed for the mining options problem [15]. The algorithm was based on the fact that typical ore deposits showing significant outcrops can be potentially exploited by OP mining followed by UG mining. Thus, mining is completed at the initial levels by OP extraction methods while transitioning to UG mining as the operation deepens from the middle levels.

Until 2009, only a few available algorithms could solve the optimal transition depth problem with some limitations. Bakhtavar, Shahriar, and Oraee [2] developed a model for solving the transition depth problem by modifying Nilsson's algorithm. The model generated two different mining schedules for OPUG mining. Each mining method is employed to extract mining blocks on the same level in series. The incremental NPVs of the extraction for each level blocks are compared and if the incremental NPV of the OP mine is larger than that of the UG mine, the algorithm transcends by adding the next series of level blocks to the previously optimized mining schedule; and the incremental NPV of the OPUG mine is compared again. Evaluation results from the first level to the last level is monitored to identify the optimal transition depth (level) for the establishment of a crown pillar. The remaining portions of ore below the crown pillar were evaluated and extracted utilizing UG stoping method(s).

Another heuristic model based on the economic block values of OPUG extraction was developed by Bakhtavar, Shahriar, and Mirhassani [14]. This new model was an improvement over the previous model by factoring in the NPV achieved through the mining process. Thus, for any level of the block model, the computed NPV from OP mining operation is compared to the NPV derived from UG mining operation for the same level. Although the model solves the transition problem using some technical and economic parameters, it does not consider mining and processing capacities (equipment requirements), and uncertainties in the geological and geotechnical characteristics of the orebody. Uncertainties of ore grades were considered in subsequent mathematical programming frameworks for the transition problem [20,30]. However, the implementation of these existing models has always assumed the mining options and transitions planning scheme to be a stepwise process and hence implements their solution strategy as OP mining followed by UG mining either in parallel (simultaneous) modes or sequential (non-simultaneous) modes [4,18,20,24,30]. The challenge to this assumption also forms the basis of this research. Table 1 shows a matrix comparison of notable research on the OP-UG mining options optimization problem in the last decade.

Table 1. Notable research on the open pits (OP)-underground (UG) mining options optimization problem for the past decade.

Name of Author(s)		**Whittle et al.**	**MacNeil & Dimi-trakopoulos**	**King et al.**	**Ben-Awuah et al.**	**De Carli & de Lemos**	**Ordin & Vasil'ev**	**Dagdelen & Traore**	**Roberts et al.**	**Opoku & Musingwini**	**Bakhtavar et al.**
Year		2018	2017	2016	2016	2015	2014	2014	2013	2013	2012
Research Focus		Transition depth & production schedule	Transition Depth-Instances	Transition Depth-Instances	Assessment of Transition Problem	Transition Depth-Instances	Transition Depth-Dynamic	Transition Depth	Assessment of Transition Problem	Transition Depth-Dynamic	Transition Depth
Approach Used	Model/Algorithm	Modification of maximum graph closure method	Stochastic Integer Model	Mixed Integer Linear Programming (MILP)	Mixed Integer Linear Programming (MILP)		Dynamic Programming-lag, trend, nonlinear	MILP-OptiMine® used to optimize the transition problem	MILP-OP & UG		(0-1) Integer Programming
	Software Application				Evaluator	Studio 3 & NPV Scheduler		OptiMine, Whittle, Studio 5D and EPS	Blasor for OP optimization; COMET for OP schedule; Evaluator for UG optimization	Whittle for OP optimization; XPAC for OP schedule; Datamine's MRO for UG optimization	

Table 1. *Cont.*

Name of Author(s)		**Whittle et al.**	**MacNeil & Dimitrakopoulos**	**King et al.**	**Ben-Awuah et al.**	**De Carli & de Lemos**	**Ordin & Vasil'ev**	**Dagdelen & Traore**	**Roberts et al.**	**Opoku & Musingwini**	**Bakhtavar et al.**
Outputs/Performance/ Transition Indicators	NPV	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	IRR				Yes	Yes					
	Mining Recovery					Yes					
	Avg. ROM Grade								Yes	Yes	
	Metal Price to Cost Ratio									Yes	
	Production Rate						Yes		Yes	Yes	
	Stripping Ratio					Yes	Yes			Yes	
	Mine Life					Yes	Yes	Yes	Yes		
	Revenue						Yes				
	Mining Cost								Yes		
	Production Schedule		OP & UG		OP, UG & OPUG		Standalone for OP & UG	Standalone for OP & UG			
Commodity			Gold	Confidential	Gold-silver-copper	Gold	Coal mining–kimberlite pipe	Gold	Iron	Gold-4 different deposits	Hypothetical
Notable Remarks		Mining sequence not considered	NPV compared to deterministic model	Optimality gap not improved						Deterministic not good	

As seen in Table 1, previous research on the mining options and transitions problems mainly focuses on the determination of transition depth before optimizing the production schedule for the mining arrangement. Formulating a model that allows the optimizer to select the most suitable extraction strategy, including independent OP, independent UG, simultaneous OPUG, sequential OPUG, or combinations of simultaneous and sequential OPUG, to exploit any deposit under consideration will be a major addition to the mining industry. It is important to note that in general previous research employed NPV and feasible production schedule as major performance indicators for the developed models and algorithms.

2.2. Factors Influencing Mining Options and Transitions Planning

The factors influencing mining options and transitions planning are also referred to as transition indicators. These transition indicators mostly depend on the shape and size of the mineral deposit and quantity of high-grade ore present in the selective mining units. These indicators usually vary for different orebodies and commodities, and the philosophy of mine management. The indicators can be broadly grouped into geologic, operational, geotechnical, economic, and mine management strategy on global business economic outlook [7,61]. These transition indicators are key to the identification and development of the set of constraints required for optimizing the OP-UG mining options and transitions planning problem [4,9,25].

Some of the indicators to consider in OP-UG transition optimization are mining recovery, commodity price, mineral grade, cost of extracting ore and stripping waste, mining and processing capacities, and UG dilution [62]. Subsequent research highlighted some additional important parameters that must be considered during transition planning [19]. These include workforce requirement, shape and size of the orebody, and geotechnical properties of the rock formation. Economic parameters such as discounting rate of expected cash flows and OPUG mining costs, and primary factors including the competence of mine management, characteristics of the geology of the orebody, stripping ratio, productivity rate, and capital cost requirements for the UG mining option will affect the decision for OP-UG transition as well [9,63].

The OP-UG mining options and transitions planning problem becomes complex when critical indicators such as crown pillar positioning and other essential underground mining constraints including primary and secondary access development, ventilation development, and geotechnical requirements for the development openings and stopes are integrated into the optimization framework. These essential UG mining developments were previously not considered in the optimization process because of computational complexities and the difficulty of integrating them with open pit mining operations [32]. The importance of incorporating crown pillar positioning, geotechnical support activity sequencing, underground infrastructure development, and mine management strategy in OP-UG mining options optimization studies are essential to attaining realistic mine plans [9,30].

When investigating OP-UG mining options, it is important to model all constraints and factors that have direct and remote impact on the economic and technical feasibility of the project. The strategic production plan is generally subject to several constraints that enforce the extraction sequence, blending requirements, and mining and processing capacity requirements for practical implementation.

2.3. Crown Pillar and Rock Strength Considerations in Mining Options and Transitions Planning

A crown pillar is the horizontal part of a rock formation between the first upper stope of an underground mine and an open pit excavation. A crown pillar is often provided to prevent the inflow of water from the OP floor to the UG workings, while reducing surface subsidence and caving. Finding the most suitable location of the crown pillar in a combined mining method of OPUG operations is one of the most interesting problems for mining engineers today, especially when crown pillar must collapse [14]. It is however worthy to note that crown pillar usage in the OPUG mine is not universal as sometimes it is desirable for the crown pillar to collapse during the life of mine [24].

The positioning of the required crown pillar in the surface–underground mining options and transitions planning problem is key to the operations of such mines. Some researchers pre-selected the depth of the crown pillar (transition depth) before evaluating portions above the crown pillar for OP mining and portions below the crown pillar for underground mining [4,18,25,27,28]. This may lead to suboptimal solutions and will require evaluating multiple crown pillar locations in a scenario-based approach. A few authors have attempted to incorporate the positioning of the crown pillar in the optimization process [24,33,34]. Their models were good improvements over previous works but were missing some constraints such as the ventilation requirement and rock strength properties required for practical implementation. The transition from open pit to underground mining is a complicated geomechanical process which requires the consideration of rock mass properties [35,36].

In OP to UG transition, the challenge of deformation, displacement, and rock formation stability need to be meticulously investigated. These will ensure their effect on production, worker and equipment safety, and the working environment of the UG operation are highly secured [64]. When the crown pillar thickness is large, considerable quantity of mineral deposit is lost but when the pillar is undersized and thin, the probability of pillar failure and instability of the mine is eminent [65]. According to Ma, Zhao, Zhang, Guo, Wei, Wu, and Zhang [35], ground movement and deformation study of OP mines such as geometrical, geomechanical, and analytical analyses are important in the transition study.

Optimizing the crown pillar dimensions and positioning is particularly significant in UG mining operations. Estimation of the optimum crown pillar thickness is a complicated study which involves the experience of the engineer and the use of numerical and empirical techniques [65]. Several parameters influence crown pillar stability. These parameters are broadly grouped into mining and geological [5]. The mining parameters include the crown pillar geometry; stope surroundings; supporting methods employed (including backfilling); sequence of mining operations and stress redistribution resulting from material extraction. The geological parameters include the strength and deformation characteristics, and inclination of the hanging wall, footwall, and orebody in general; geometry of the mineral deposits; virgin stress conditions and properties of the contact regions between the ore and country rock.

Crown pillar placement invariably defines the interface of the OP to UG transition. Appropriately defining a suitable location of the crown pillar is fundamental to the mining options optimization problem. Leaving an appropriate crown pillar thickness will minimize the destructive interference between the OP and UG mining operations, while maximizing ore recovery. The assumption of a uniform crown pillar of known height below the optimized pit bottom is common [14]. Subsequently, an ad-hoc branch-and-bound technique to exhaustively search the appropriate locations for crown and sill pillar placements before computing the relaxed linear programming (LP) model for the OPUG mining problem was developed by King, Goycoolea, and Newman [18]. A rounding heuristic was used to transform the relaxed LP solution into integer programming (IP) solution using satisfactory values of the objective function. The IP solution was used to terminate the several potential crown and sill pillar placements and hence decrease the computational time required. The researchers further concluded that, only 40 out of the over 3500 crown and sill pillar placement options had a relaxed LP objective function value greater than the best-known IP objective function value.

In a recent research, MacNeil and Dimitrakopoulos [20] pre-determined four possible crown pillar positions while evaluating a gold deposit for OPUG mining. Their approach resulted in four distinct transition points acting as potential candidates for a crown pillar. The size of the crown pillar however remained the same for each candidate, as the location is varied. This assumption does not support the geotechnical variability of the rock formation. According to King, Goycoolea, and Newman [18], the crown pillar is usually positioned by industrial experts based on: (1) the optimal OP mining limit, or (2) the largest undiscounted profit resulting from the extraction technique for each 3-D discretization of the orebody and country rock. This is conducted after a detailed geotechnical assessment of the rock formation in the immediate vicinity of the crown pillar.

The stability and placement of the crown pillar or transition interface significantly affects the NPV of a mining project. Numerical simulation and machine learning are among the most effective techniques for studying the crown pillar stability [66–68]. The Fast Lagrangian Analysis of Continua (FLAC) software based on numerical modeling and a hybrid support vector regression (v-SVR) analysis based on supervised machine learning algorithm are used to examine deformation characteristics of surrounding rocks in complex conditions of OP to UG mining transition. These analyses can provide approximation of rock strength properties to be incorporated in the transitions planning optimization problem.

2.4. Existing Evaluation Tools for Mining Options and Transitions Planning Crown

Most of the existing optimization models and algorithms for evaluating OP-UG mining options and transitions planning have been incorporated into software packages for easy implementation. These are mostly based on the fundamental evaluation techniques discussed in Section 2.3. Modeling techniques for optimization problems have been discussed in Appendix A. Some of the models and algorithms are Lerchs–Grossman algorithm, Seymour algorithm, floating cone technique, dynamic programming, neural network, theory of graphs, and network flow algorithm. These models and algorithms are used in software packages including NPV Scheduler, Whittle Four-X, MineSched, Vulcan, MineScape, and MineSight [25,26].

GEOVIA Whittle® software which is based on the Lerchs–Grossman algorithm [69] is commonly used to optimize the OP limit prior to assessing the remaining mineral resource outside the pit boundary for underground extraction [7]. Similarly, Blasor pit optimization software which is developed based on a combinatorial mathematical programming model is also used to identify the independent OP mining outline before UG extraction evaluation for remaining mineral resources [9]. Mathematical programming frameworks based on IP, mixed integer linear programming (MILP), and dynamic programming models have been formulated and implemented with commercial optimization solvers for the transitions planning problem [4,17,20,30].

After obtaining the OP outline and thereafter knowing the extent of the UG mining limits, production schedules are generated for each mining option. Software packages based on heuristics and mathematical programming models including GEOVIA Whittle, OptiMine®, and COMET® have been used to produce strategic production schedules for the OP portion of the mine [4,7,17]. Similarly, software packages including Snowden's Evaluator, XPAC®, and OptiMine® have been used to produce the strategic production schedule for the UG portion of the mine [4,7,9,17,18,30].

2.5. Limitations of Current Models and Algorithms for OP-UG Mining Options and Transitions Planning

Primary challenges to the mining options optimization problem include the optimization approach used, the exhaustive consideration of contributing variables to the models and algorithms, and geotechnical considerations in defining the transition interface and its contribution to the mining operation. Progressively integrating geotechnical models in strategic mine plans at the prefeasibility stage similar to how geologic models are incorporated will improve the reliability of the mine plan [70,71]. However, the information required to produce a detailed geotechnical model at the prefeasibility stage is limited and therefore difficult to model. Some of the limitations with current models and algorithms for OP-UG mining options optimization include one or more of the following:

1. Consideration of rock support and reinforcement in the optimization process;
2. Consideration of essential infrastructural development in the optimization process;

3. Consideration of stochastic variables;
4. Comprehensiveness and efficiency of models;
5. Optimality assessment.

2.5.1. Consideration of Rock Support and Reinforcement in the Optimization Process

Rock support and reinforcement are essential to the stability of openings during the development of an underground mine. The term support generally refers to the various types of geotechnical rock support used to protect underground workers and may include steel mesh, shotcrete, fibrecrete, and a variety of types of steel straps. Reinforcement on other hand refers to the various types of rock reinforcement to help prevent rock movement and may include a variety of types of rockbolts, cablebolts, rebar, and dowel. Cablebolting in stope development can particularly be very costly and may introduce considerable time delays. Most cablebolts need 30 days for the Portland cement grout to properly set for the cablebolts to be fully functional.

Several authors have acknowledged the importance of incorporating geotechnical constraints to the OP-UG transition problem [9,14,15,20,25,47,72]. To verify the impact of geotechnical constraints on the optimal solution, Roberts, Elkington, van Olden, and Maulen [9] recommended that, such constraints need to be incorporated in subsequent studies. Moving beyond determining transition depth and incorporating geotechnical parameters in locating and sizing the transition zone will have direct impact on the feasibility and sustainability of the OP-UG mining project.

These existing models do not integrate the geomechanical classification of the rock formation in the surface–underground mining option and transition problem. Although the geotechnical characteristics of the rock formation are significant to underground mining operations, their added complexities make it difficult to be included in the surface–underground mining option and transition optimization models. According to Bullock [37], mine planning is an iterative process that requires looking at many options and determining which, in the long run, provide the optimum results. Using such iterative process could lead to some inferior solution(s) or sub optimal solution(s) that do not constitute the global optimal solution.

Rock support and reinforcement in the development openings and stopes will increase the operational costs and time (delay the mine life), and further affect the quantity and sequence of rock material extracted from the stopes to the processing plants. To incorporate the rock formation's strength into the formulation, rock mass classification systems [73,74] such as the Geological Strength Index (GSI), Rock Structure Rating (RSR), Rock Mass Rating (RMR), and Q system are determined to characterize the rock formation, and then Kriging applied to populate the block model. According to Abbas and Konietzky [73], these classification systems could be grouped as qualitative or descriptive (e.g., GSI) and quantitative (e.g., Q system, RMR, and RSR) with RMR being more applicable to tunnels and mines. According to Kaiser and Cai [74], data obtained from the geology and geomechanics of the rock formation are fundamental to mine planning and development designs. This is because the behavior of the rock formation varies in the mine and therefore rock mass domaining based on stress data, and geology and geometric data becomes essential. Knowledge of the strength of the rock mass and its behavior are important for the engineering design of all kinds of support for underground excavations [75,76].

2.5.2. Consideration of Essential Infrastructural Development in the Optimization Process

The typical UG mine is interspersed with important infrastructure development that ensures the facilitations of the UG mining operations. These infrastructures include but not limited to primary access development, secondary access development, ventilation development, ore pass development, sumps, maintenance and refuge chambers, and fuel station bays. Primary access developments are usually the main development that links the entire UG mine to the surface and could be vertical or inclined shaft(s), decline(s) or adit(s), or tunnel(s). The shaft(s) are often equipped with facilities to transport humans (workers) and materials for the UG operations. Secondary access development includes the construction of lateral openings such as levels, ore and waste drives, or crosscuts, to link the UG operational activities to the primary access(es) while ventilation development often incorporates a series of bored raises and lateral drive development, and the construction of ventilation controls. Ventilation controls are a range of objects such as regulators, doors, and walls which need considerable time and money to create. Ore pass development entails the construction of vertical or near vertical openings to link the various UG levels to the main ore hoisting station. Similarly, sumps, maintenance and refuge chambers, and fuel station bays are constructed to enable the removal of water being used for the UG operations and the provision of several essential services to operating equipment UG rather than being transported to the surface for such services.

2.5.3. Consideration of Stochastic Variables

In recent studies, the industrial practice has been that the production for OP mine and UG mine are independently scheduled and merged into one for the OPUG mine. According to King, Goycoolea, and Newman [18], this approach creates a myopic solution. However, the discussions of the approach were limited to open stoping and extraction sequence. No stochastic variables such as grade and price uncertainty, were included in their model. King, Goycoolea, and Newman [18] further acknowledged that, their methodology in handling the transition problem require further work to handle the applicability, accuracy, and reduction of the optimality gap. Grade uncertainty has been identified to have a significant impact on the determination of the transition point [16].

Although MacNeil and Dimitrakopoulos [20] incorporated grade uncertainty in their work, they further identified some important notable geological uncertainties such as the rock formation, metal content, and relevant rock properties and their impact on the strategic long term planning of a mining project. MacNeil and Dimitrakopoulos [20] further recommended that, financial uncertainty should be incorporated into future studies to improve on their solution method. Ben-Awuah, Otto, Tarrant, and Yashar [4] did not consider uncertainty in their model formulation and further recommended that pre-production capital expenditure (CAPEX) and geological uncertainties should be added to the mining options evaluation. By considering stochastic variables in a risk-based mining options optimization framework, mine plans that can stand the test of time can be generated.

2.5.4. Comprehensiveness and Efficiency of Models

Comprehensiveness and efficiency of the models relates to the ability of the optimization framework to exhaustively formulate various scenarios of the OP-UG mining options problem and deliver practical results in a reasonable time frame. Such models are exhaustive and are applicable in a wide range of mining systems. Stacey and Terbrugge [77] indicated that a complete model for handling the transition problem remained a challenge they further reckoned that the transition study for a mine should start at the onset of the mine life and not be deferred to latter years since the planning and implementation could sometimes take up to 20 years for completion. Shinobe [10] developed a software based on a mathematical programming model for this challenge but assumed that the existence of underground reserves have been confined and that their extraction is technically feasible. He later recommended that the results of the program should be viewed only as a preliminary level indication of the economics of underground conversion. No final decision to proceed with the conversion should be taken, solely based on the program's output.

Majority of the current work on the transition problem lack some constraints and solution to a more general problem. This includes consideration of the design capacities and depth as primary indicators for transition in the joint evaluation problem [25]. The approach however was limited in scope in relation to a more generic framework. NPV curves and transition depth for OP to UG mining for the Botuobinskaya pipe deposit were generated to solve the transition problem. From their results, the total NPV of the OPUG mining operation was higher than the standalone NPVs of OP and UG mines for the same mining depth. In subsequent research, re-handling cost proved to be insignificant when incorporated into the transition model [18]. Additionally, it was observed that there exist undesirable fluctuations in the OPUG production schedules that must be smoothened to achieve a more practical solution. In their work, MacNeil and Dimitrakopoulos [20] incorporated the constraints for mining, processing, metal content, and precedence relationships in their model. According to the constraints affecting the transition problem identified in the works of Opoku and Musingwini [7], those constraints considered are not exhaustive since UG mining capital expenditures were not considered in the model application.

2.5.5. Optimality Assessment

Optimality assessment is a real challenge to current heuristic and meta-heuristic models and algorithms for OP-UG mining options optimization. The underlying optimization approach fundamentally affects the optimality of the resource evaluation. Unless additional steps are put in place to define an upper bound, the extent of optimality of the solution from these models is unknown. Some of the current models can solve the transition problem, usually producing near optimal solutions [14,47]. These existing models and algorithms assume open pit mining operations must surely be an option in the evaluation process and usually precedes the underground mining. Similarly, few models assess the resource with the assumption that underground mining could precede open pit mining. However, a resource assessment that is devoid of the traditional arrangement of the mining options but allows the optimization process to decide the choice and sequence of the mining option(s) is key to achieving global optimal solution. Bakhtavar, Shahriar, and Oraee [2] noted that, few methods (algorithms) have some disadvantages and deficiencies in finding the optimal transition depth. According to Askari-Nasab et al. [78], heuristic algorithms may not produce optimal solutions. This could lead to loss of huge financial benefits resulting from implementing sub-optimal mine plans.

Finch [47] also highlighted that, the effort of producing OPUG mining schedules for the possible candidates of transition interface for all the various mining and processing capacities could be time consuming and costly. The process commonly leads to the generation of sub-optimal solutions since the problem is not thoroughly investigated. According to Richard and Stefan [79], designs optimized for deterministic cases are often sub-optimal when uncertainties are recognized and their effects understood. By applying mathematical programming models with current high computing resources [31], optimal or near optimal solutions with known optimality gap can be obtained for the OP-UG mining options optimization problem in a practical time frame and at an acceptable computational cost.

3. Summary and Conclusions

The problem of optimizing resource exploitation depends largely on the mining option used in the extraction. Some mineral deposits extend from the near surface to several meters in depth. Such deposits can be amenable to both surface mining and/or underground mining, and this leads to the surface–underground mining options and transitions optimization. Relevant literature review on the SUMOTO problem has been conducted and documented. Research works in this area have primarily focused on different variations of determining the transition depth between open pit and underground mines, and the subsequent optimization of the strategic schedule for each mining option. Heuristics and exact solution methods have both been used in the past to attempt the SUMOTO problem. An algorithm or model that comprehensively and simultaneously determines an optimized open pit mine, transition interface and an underground mine for any orebody by both surface and underground mining methods in a single run will significantly add value to the mining industry. A matrix showing the various approaches adopted by researchers in tackling the OP-UG transition problem in the last decade has been developed (Table 1).

Shortfalls on previous research for mining options optimization have been discussed and opportunities for further studies identified. Notable limitations of current models and algorithms for the SUMOTO problem include: (1) consideration of rock support and reinforcement in the optimization process; (2) consideration of essential infrastructural development in the optimization process; (3) consideration of stochastic variables; (4) comprehensiveness and efficiency of models; and (5) optimality assessment of model solution. These identified limitations may often lead to sub-optimal global solution to the SUMOTO problem thereby affecting the viability of the mining project. It is therefore essential to develop a rigorous optimization framework that attempts to address some of these deficiencies.

Although the main sources of uncertainties in mining options studies have been found to include financial, technical, and geological, research on strategic mining options have handled these uncertainties independently. Incorporation of geological uncertainties in current strategic mining options studies have been applied in different forms, including, grade and tonnage uncertainties, probability indices, and the use of algorithms to further define these uncertainties. The incorporation of financial uncertainties together with geological uncertainties is limited in current research on mining options studies. As uncertainties cannot be eliminated in the mining options problem, the best strategy is to quantify uncertainty, reduce uncertainty and manage the associated risk during the production scheduling process.

In the last decade, different variations of mathematical programming models have been used by researchers to solve challenges associated with the surface–underground mining options problem. The main variations are either deterministic (linear programming and integer and mixed-integer programming) or stochastic (dynamic programming and stochastic programming) or combination of both.

4. Recommendations

The authors conclude by proposing further research into the formulation of an integrated stochastic programming model for the mining options and transitions optimization problem for all deposits including base and critical minerals. Figure 4 is a representation of the recommended research considerations to solve the complexities associated with the strategic surface–underground mining options and transitions optimization problem.

To add significant value to the mining industry, the proposed research approach in Figure 4 will result in a stochastic mining options and transitions optimization model that has the capacity to generate strategic mining options including: OP mining, UG mining, simultaneous OPUG mining, OP mining followed by UG mining, and UG mining followed by OP mining. The proposed model should be applicable at the prefeasibility stage of mining project to guide mine planners and investors in making important decisions. Some of the performance indicators of the model should include net present value, internal rate of return, discounted cashflow, price to cost ratio, blending ratio, production smoothness, stripping ratio, mine life, mining recovery, and average run-of-mine (ROM) grade. The proposed model should have the features and capacity to:

(a) Take in simulated block models as inputs in a risk-based or stochastic framework that considers grade, price, and cost uncertainty;
(b) Evaluate strategic mining options with exhaustive constraints for large-scale mining projects through efficient numerical modeling and computational techniques. In addition to standard mining and technical constraints, other notable constraints include controls for safety, geotechnical, geological, and hydrogeological conditions of the mining area;
(c) Integrate waste management constraints and synergies wherein waste material and tailings from open pit mining can be used for underground backfilling; as well as characterizing mineralized waste material as future resource;
(d) Leverage resource governance and synergies whereby open pit low grade ore can be blended with underground high-grade ore to improve processing recovery and extend mine life;
(e) Determine the size and capacity of the mining project using real value options approach.

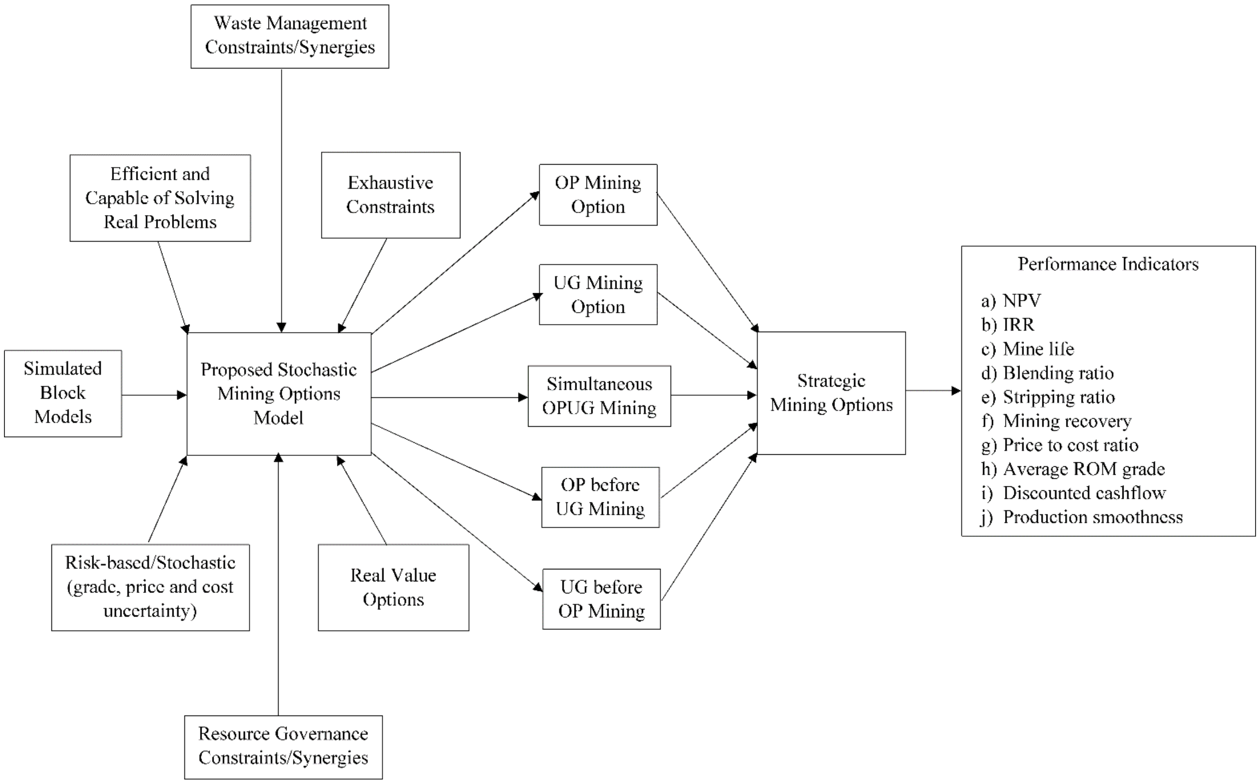

Figure 4. Schematic representation of the proposed research approach for strategic OP-UG mining options optimization.

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Appendix A. Modeling Techniques for Optimization Problems

The literature review conducted on the surface–underground mining options and transitions optimization (SUMOTO) problems indicate that mining options and transitions optimization planning are modeled and solved using different evaluation techniques. Understanding the various classifications of optimization models and their advantages are essential to the development of an optimization framework capable of handling the notable limitations and gaps associated with the current models and algorithms for SUMOTO problems. These optimization models may be grouped into four broad types according to how well they are able to define a given problem: (1) operational exercise models, (2) gaming models, (3) simulation models, and (4) analytical or mathematical programming models. Simulation and analytical models are widely used for mining optimization problems due to their practicality, robustness, and efficiency [80,81]. Table A1 shows classification of certainty and uncertainty models.

Table A1. Classification of certainty and uncertainty models [23].

Class	Strategy Evaluation	Strategy Generation
Certainty	Deterministic simulation Econometric models Systems of simultaneous equations Input-output models	Linear programming Network models Integer and mixed integer programming Nonlinear programming Control theory
Uncertainty	Monte Carlo simulation Econometric models Stochastic processes Queueing theory Reliability theory	Decision theory Dynamic programming Inventory theory Stochastic programming Stochastic control theory

Appendix A.1. Operational Exercise Models

Operational exercise is a modeling method that relates variables to the actual environment where the study decision will be applied [23]. This method has the highest degree of realism when compared to other techniques of modeling. It is usually prohibitive, expensive to implement, and associated with extreme challenge when alternatives must be assessed. This then leads to sub-optimization of the final solution. A human decision-maker forms part of the modeling processes of operational exercises.

Appendix A.2. Gaming Models

Gaming is a modeling approach constructed to represent more simple or abstract entities in the real world [23]. This method gives the mine planner the opportunity to try several variations of the decisions to make. A human decision-maker is part of the modeling processes for gaming models.

Appendix A.3. Simulation Models

Simulation models provide several ways to assess the performances of alternatives outlined by the planner. They do not allow much interferences from human interactions during the computational analyses stage of implementation [23]. Simulation models could sometimes be compared to gaming models. However, they involve the application of logical arithmetic performed in a particular sequence usually by computer programs. If the problem is exclusively defined in an analytical form, much flexibility and realistic results are attained. This is essential if uncertainties are critical components of the decisions being made. A human decision-maker is external or not part of the modeling processes of simulation and analytical models.

Appendix A.4. Analytical or Mathematical Programming Models

The fourth model category is analytical models. These models represent the problem completely in mathematical forms, usually by means of a criterion or objective subject to series of constraints that impact on the decision being made [23]. The mathematical formulation helps to determine the optimal solution in the presence of the set of constraints. Although mathematical models highly simplify the problem, they are less expensive and easy to develop.

Mathematical programming is a significant method when decision variables must be quantified for planning. The problem is defined with mathematical expressions in a well-defined structure to find an optimal solution based on performance evaluation criteria including cost, profit, and time. The optimal solution is obtained from a feasible region of alternative results. The expressions are parameters (input data) and variables which represent the optimization results or outcome. When multiple criteria decisions are required for any complex problem, a multi-objective mathematical programming model involving more than one objective function is deployed simultaneously. The main advantages of mathematical programming models (MPMs) are: (a) comparatively simple with high approximations of complicated problems, and (b) ability to search the feasible solution spaces among competing variables and alternatives [82]. Common MP techniques are stochastic programming, deterministic programming, dynamic programming, LP, nonlinear programming, IP, and MILP.

Appendix A.4.1. Stochastic Programming

Stochastic programming is a special case of programming in which some of the constraints or parameters depend on random variables. This type of programming is used to solve problems that involve uncertainty and allow stochastic variables to be accounted for [82].

Appendix A.4.2. Deterministic Programming

Deterministic programming is a form of MP that is rigorous and solves problems in finite time. The optimization model is deterministic when the parameters considered in the model are known constants [23]. The method is useful when global solution is required and is extremely difficult to find a feasible solution.

Appendix A.4.3. Dynamic Programming

Dynamic programming is a form of mathematical programming used to solve multistage complicated problems [82]. A large-scale problem is simplified by breaking it down into simpler sub-problems in a nested recursive manner as opposed to deterministic programming.

Appendix A.4.4. Linear Programming

This is a distinct case of convex programming where the model objective and the equality and inequality constraints are expressed as linear functions. The feasible solution set is usually a polytope or polyhedron with connected sets of polygonal faces and convex. The optimal solution could be a single vertex, edge or face, or even sometimes the entire feasible region when dealing with high dimension problems. Typical LP solutions could arrive as infeasible or unbounded [83].

Appendix A.4.5. Nonlinear Programming

Nonlinear programming is a special case of programming which could be convex or non-convex with nonlinear objective functions or nonlinear constraints or [83]. Some assumptions are often made on the shape and function behavior when solving nonlinear programming problems [23]. Nonparametric and simulation-based optimization techniques have been used to explicitly solve models with nonlinear constraints. However, computational feasibility have been a major disadvantage in the development and implementation of MPMs [82].

Appendix A.4.6. Integer Programming

Integer programming (IP) is a special type of LP where some or of all the decision variables are constrained to take on integers and therefore not continuous. If all the variables are discrete or integers or binaries, the model is referred to as pure integer programming. Essentially, IP problems are hard problems because they are difficult to solve and therefore referred to as combinatorial analysis than LP [23].

Appendix A.4.7. Mixed Integer Linear Programming

This is a special form of IP in which some decision variables are constrained to take on integers and others continuous. Continuous variables indicate that the variables could take on fractions as part of the solution regime. Due to the strength of MILP formulations, it is proposed as the model for solving the SUMOTO problem in this research. The MILP formulation structure is well defined with objective function, and a set of constraints to achieve the mining decisions that are usually made up of continuous variables and integers.

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