Incorporating Grade Uncertainty in Oil Sands Mine Planning and Waste Management Using Stochastic Programming

by

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ACCESSIBILITY CLAUSE AND PERMISSION TO USE

This thesis is proudly dedicated to:

My beloved parents, Frederick and Caroline, my siblings

who have always supported my goals and,

believed in my ability to attain greater heights in life.

ABSTRACT

The primary purpose of oil sands mine planning and waste management is to provide ore from the mine pit to the processing plant while containing the tailings in an efficient manner in-pit. Incorporating waste management in the mine plan is essential in maximizing the economic potential of the mineral resource and minimizing waste management costs. However, spatial variability such as grade uncertainty results in ore tonnage variations, which leads to fluctuations in the quantity of ore to be processed and waste to be managed. If grade uncertainty is not incorporated in oil sands mine planning, it may lead to under- or over-design of the waste management system required to support the mining operation, resulting in lost opportunities.

Grade uncertainties have profound impact on the Net Present Value (NPV) of the mining project as it may induce large differences between the actual and expected production targets. Thus, the primary research objectives are to develop, implement and verify an integrated oil sands mine planning optimization framework using Stochastic Mixed Integer Linear Programming (SMILP) to integrate the related domains of bitumen grade uncertainty and waste management. The SMILP model determines the order and time of extraction of ore, dyke material and waste that maximizes the Net Present Value (NPV), minimizes waste management cost, and minimizes the geological risk cost of the mining operation. Sequential Gaussian Simulation (SGS) is employed to quantitatively model the spatial variability of bitumen grade in the oil sands deposit. Multiple simulated orebody models are used as inputs for the SMILP model to generate optimal mine plans in the presence of grade uncertainty. MATLAB programming platform was chosen for the SMILP framework implementation. A large-scale optimization solver, IBM CPLEX, is used for this research.

To validate the SMILP model, an oil sands case study was implemented based on SGS realizations block models to generate a stochastic integrated production schedule (SMILP schedule), and the results compared with a conventional production scheduling approach based on Ordinary Kriging block model (OK schedule) and E-type block model (E-type schedule). The E-type block model is the average block model of the SGS realizations. In comparison, the SMILP schedule which was based on the developed SMILP framework generated an uncertainty-based integrated production schedule and waste management plan with better financial profitability compared to

the OK and E-type schedules. Additional numerical experiments and analysis was done by applying the three schedule results to each of three randomly selected realizations. The corresponding SMILP schedules generated from the realizations were consistently uniform and smooth compared to similar OK and E-type realization schedules. The SMILP framework accounts for geological risk by placing higher penalties for ore grade and ore tonnage deviations from production targets in the early years of mine life to defer production deviations to later years when more geological information becomes available to update the block model and mine plan. By deferring geological risk to later years, the risk of not reaching production targets in the earlier years is minimized, thus creating a smoother and stable production schedule. The results demonstrate that the SMILP schedule generates 14% and 17% improvements in NPV compared to the E-type and OK schedules respectively. These results prove that the SMILP model is a useful tool for optimizing stochastic integrated oil sands production schedules whilst taking into account grade uncertainty.

Keywords

Uncertainty-based oil sands mine planning, production scheduling optimization, stochastic programming, sequential Gaussian simulation, waste management, grade uncertainty, ordinary kriging.

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Obinna Mbadozie
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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

AER	Alberta Energy Regulator
COG	Cut-off Grade
ETF	External Tailings Facility
EBV	Economic Block Value
GDR	Geologic Discount Rate
GP	Goal Programming
IB	Interburden
LG	Lerchs Grossman algorithm
LP	Linear Programming
LTPP	Long-Term Production Planning
MILGP	Mixed Integer Linear Goal Programming
MILP	Mixed Integer Linear Programming
MMF	McMurray Formation
MPM	Mathematical Programming Model
NPV	Net Present Value
OB	Overburden
OK	Ordinary Kriging
SIP	Stochastic Integer Programming
SGS	Sequential Gaussian Simulation
SMILP	Stochastic Mixed Integer Linear Programming
TCS	Tailings Coarse Sand
UPL	Ultimate Pit Limit

CHAPTER 1
INTRODUCTION

1.1 Background

Mine planning describes the source, destination and sequence of extraction of ore and waste over the life of mine. The outcome of mine planning is a production schedule that defines the tonnage of ore and waste and the input grade to the processing plant in each period of time. This production schedule has a significant influence on the economics of the mine due to time value of money. Improving production scheduling is essential as the mining industry considers more marginal resources. The natural complexity of mineral deposits makes mine planning more difficult. Moreover, the production schedule must follow technical constraints for a practical mining environment, and meet the target capacity of the processing plant. Optimization methods are applied in mine planning to maximize the overall profit of the project and minimize deviations from production targets. In traditional long- term mine planning, a geological block model is used as the main input to maximize the net present value (NPV) of the project. The geological block model is a quantitative definition of the available resource. Data from drill holes are used to construct the geological block model using geostatistical techniques such as Ordinary Kriging and Inverse Distance Weighting methods. Uncertainty in the generated block model data is inevitable with relatively widely spaced drill holes. The optimality of the open pit production schedule will be affected by this uncertainty. Recent research initiatives have attempted to consider the effect of grade uncertainty on production schedules using mathematical programming. Mathematical programming have the advantage of generating production schedules with a measure of the extent of optimality (Johnson, 1969; Gershon, 1983; Dagdelen and Johnson, 1986; Akaike and Dagdelen, 1999; Caccetta and Hill, 2003). A major challenge in open pit production scheduling with mathematical programming is the size of the optimization problem. The mathematical programming formulation of realistic long-term open pit production schedules often exceeds the capacity of current hardware and optimization software (Badiozamani and Askari-Nasab, 2014) and requires innovative modeling approaches.

This research focuses on oil sands mining of the McMurray Formation (MMF) in Alberta (Hepler and Hsi, 1989). Strategic oil sands mine production planning consists of timely provision of ore material from the mine pit to the processing plant while effectively containing the tailings in-pit

(Figure 1-1). In terms of waste management, oil sands waste disposal planning is currently handled as a post-production scheduling optimization activity (Fauquier et al., 2009). Dyke materials which are used to construct the dykes in-pit are part of the waste management scheme, since most of the materials used in dyke construction and reclamation are solid waste produced as part of the mining and processing operations (Ben-Awuah and Askari-Nasab, 2013). However, spatial variability such as grade uncertainty results in variations in ore tonnages during oil sands mining which leads to variations in the quantity of waste to be managed. This may lead to over- or under-design of the waste management system required to support the mining operation. Considering the environmental challenges related to oil sands mining, grade uncertainty and waste management should be integrated into the long-term mine planning optimization framework to provide sustainable production planning for oil sands mining operations.

In this research, grade uncertainty is taken into account in an integrated uncertainty-based oil sands long term production scheduling and waste management optimization framework. A stochastic mixed integer linear programming (SMILP) model has been developed for the oil sands mining scheme incorporating grade uncertainty. As part of the implementation, instead of using the conventional approach of only one block model as input, a number of simulated realizations that are representative of grade uncertainty are used in the optimization process. In addition to maximization of the NPV of the project, secondary and tertiary objectives of minimizing waste management cost, and the cost of uncertainty associated with ore blending and ore production is modeled. Figure 1-1 shows a schematic view of the conceptual oil sands mine operation layout.

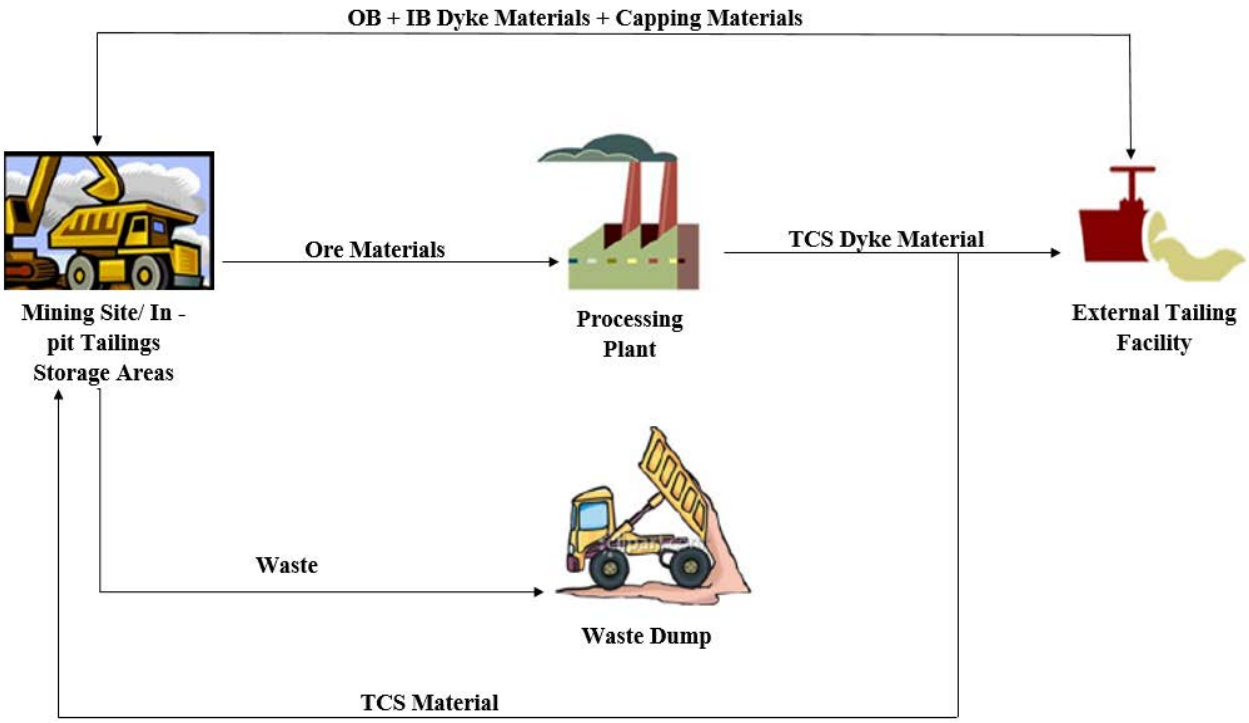

Figure 1-1: Conceptual oil sands mine operation layout modified after Badiozamani and Askari-Nasab (2014)

1.2 Statement of the problem

Production scheduling is a challenge faced by mining companies in both open pit and underground mining operations because it is a key factor in determining the return on investment of the mining project. In mine production scheduling, the mineral deposit is represented as a three-dimensional array of blocks. Each block has a weight and grade content estimated using information obtained from drilling. The block is first extracted from the earth and then processed in a mill in order to recover the contained mineral. These operations are termed mining and processing, respectively.

Mining blocks are classified as either ore or waste based on their mineral composition. Ore blocks are those that have a selling revenue greater than their processing costs, while waste blocks have a selling revenue less than the processing costs. In order to gain access to all the ore blocks, both ore and waste blocks must be extracted. Any block that must be mined in order to reach another block is called the predecessor of the second block. Decisions on block scheduling is subject to various types of constraints. The production schedule must not only respect the limits on extraction capacity (mining constraints) and the capacity of the processor (processing constraints) in each period of the life of mine, it must also take into consideration the order in which blocks are

removed from the orebody to ensure that a block is not mined before any of its predecessors (slope constraints). In addition, any block can be mined only once (reserve constraints).

Conventional approaches to optimizing open pit mine production schedules are based on a single estimated orebody model which does not account for grade uncertainty. Grade uncertainties can have a profound impact on the NPV and waste management strategy of the mining project as it may result in large differences between the actual and expected production targets, especially in the early years of mine life. Most methods developed to solve the mine production scheduling problem either ignore grade uncertainty or do not evaluate its impact on the waste management strategy. The complexity of mine production scheduling problems is increased by uncertainties due to the sparse variability of geologic data. Godoy and Dimitrakopoulos (2004) classified the uncertainties in mine planning to include: (1) in-situ grade uncertainty and material type distribution; (2) technical mining specifications uncertainties such as extraction capacities and pit slope considerations; and (3) economic uncertainties including capital and operating costs. Osanloo et al. (2008) mentions several authors such as Ravenscroft (1992), Dimitrakopoulos and Ramazan (2004), and Leite and Dimitrakopoulos (2007) who considered the uncertainty of ore grade in long-term production planning. They report that grade uncertainties result in grade accounting challenges since the grade content of the blocks are not known precisely at the time decisions are made but inferred from limited drilling information.

In addition to the impact of grade uncertainty on the long-term mine plan, oil sands waste management practices are strictly monitored by the public and lawmakers. Limitations in lease areas require the development of effective and efficient waste disposal planning systems. These systems must be fully integrated into the long-term mine plan in an optimization framework such that it generates value and supports a sustainable operation. In oil sands mining operations, the pit phase advancement occurs concurrently with the construction of in-pit dykes in the mined-out areas of the pit, and ex-pit dykes in designated areas outside the pit. These dykes are constructed to hold tailings that are produced during the processing of the oil sands ore. The materials used in constructing these dykes come from the oil sands mining operation. The dyke materials comprise of overburden (OB), interburden (IB) and tailings coarse sand (TCS). The ore material that is sent to the processing plant must have a specified minimum amount of bitumen and percentage fines, while the dyke material that is sent for dyke construction must meet the fines requirement for the dyke construction location. Any other material that does not qualify as either ore or dyke material

is sent to the waste dump. Figure 1-2 illustrates a schematic representation of the problem definition for oil sands production planning and waste management considering grade uncertainty.

In conventional oil sands strategic mine planning, a single estimated geological orebody model is used to optimize the mine production schedule. Due to the limited number of sample data during exploration, building a single geological block model results in estimating the grades of the deposit at locations previously not sampled, thus leading to a significant degree of grade uncertainty. Similarly, in current oil sands waste management planning, tailings facilities and waste dump capacities are designed with no direct consideration of grade uncertainty. This has an overall impact on the profitability of the mining project resulting from missed opportunities that arises from over- or under-estimation of plant capacity design as well as an efficient waste management strategy. This may lead to financial losses, environmental challenges and unsustainable mining operation. There is therefore a strong need to integrate grade uncertainty and waste management in oil sands mine planning.

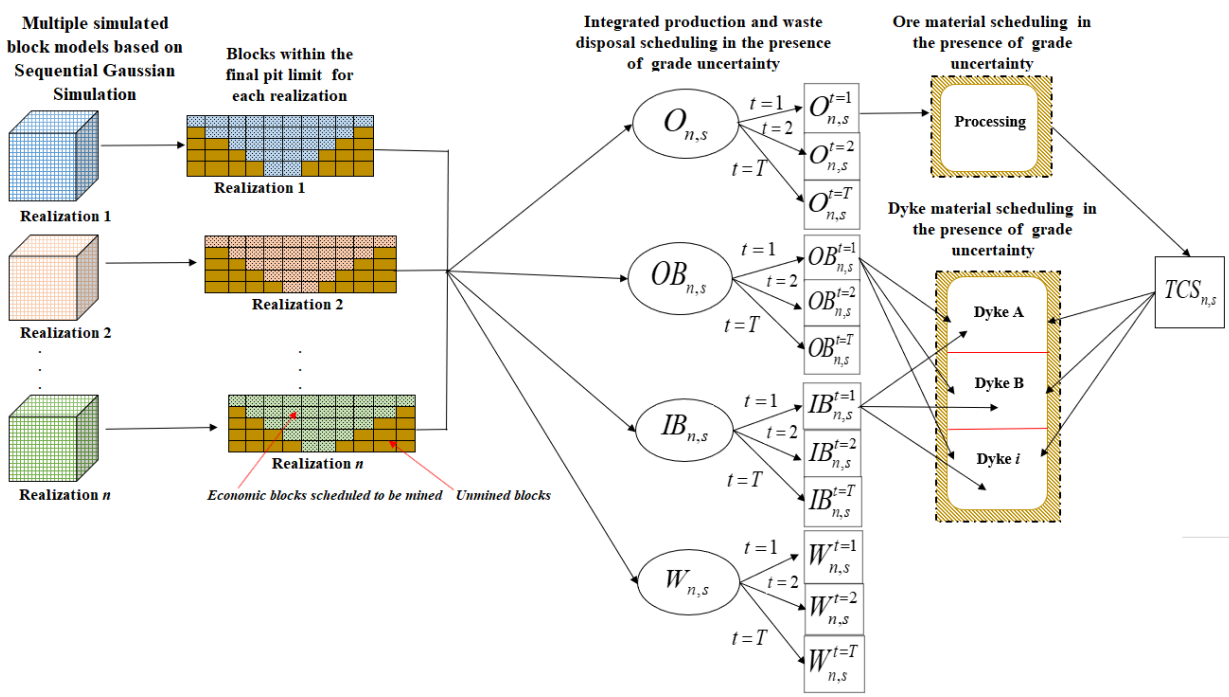

Figure 1-2: Schematic representation of the problem definition for integrated uncertainty-based oil sands production planning and waste management considering grade uncertainty

The aim of this research is to understand how uncertainty due to intrinsic variability in oil sands ore grade could impact the ore and waste tonnages as well as the overall net present value during integrated uncertainty-based mine planning and waste management. This led to the development

of a stochastic programming framework to generate risk-based mine plans for an oil sands case study. The following research question drives this investigation:

How can the spatial variability of bitumen grade in oil sands deposit be incorporated in the strategic production scheduling of ore, dyke and waste materials to maximize the net present value and minimize waste management cost for a stochastic integrated long-term mine production plan?

1.3 Summary of literature review

The use of mathematical programming optimization techniques has shown to be capable of solving long-term mine production planning problems. Mathematical programming techniques such as Linear Programming (LP), Mixed Integer Linear Programming (MILP) and Goal Programming (GP) are used in solving mine production scheduling problems using exact solution methods with known limits of optimality. As a result, the solutions provided by these methods are by definition optimal compared to heuristic optimization approaches whose solution methods may be sub-optimal. However, when the mathematical programming model is applied to a large deposit, it results in a large-scale optimization problem with numerous binary and continuous variables which becomes difficult to solve due to the current state of computer hardware and software.

Another drawback of LP, MILP and GP models in solving long-term production planning (LTPP) problems is that, they do not account for grade uncertainty during optimization. The geostatistical simulation technique known as Sequential Gaussian Simulation (SGS) is used to generate realizations that are representative of the grade uncertainty in an orebody. Each realization is an alternative image of the orebody and is equally probable. Any conventional method can be used to solve the LTPP problem using the realizations one at a time. Ravenscroft (1992) used stochastic orebody models sequentially with traditional optimization methods to evaluate the range of different mine plans generated. Ravenscroft (1992) further discussed risk analysis in mine production scheduling. He used simulated ore bodies to show the impact of grade uncertainty on production scheduling. He concluded that conventional mathematical programming models cannot accommodate risk quantification.

Dimitrakopoulos (1998) showed that there are substantial conceptual and economic differences between risk-based frameworks and traditional approaches. Dimitrakopoulos and Ramazan (2004) proposed a probabilistic method for long-term mine planning based on LP. This method uses probabilities of being above or below a cut-off to account for grade uncertainty. A LP model is

used to minimize the deviation from target production. This method does not directly and explicitly account for orebody uncertainty and also does not maximize NPV. Leite and Dimitrakopoulos (2007) also presented a risk inclusive LTPP approach based on simulated annealing. A multi-stage heuristic framework was presented to generate a final schedule, which considered geological uncertainty so as to minimize the risk of deviations from production targets. A basic input to this framework is a set of realizations. They reported significant improvement on NPV in the presence of uncertainty. However, the proposed method had a few drawbacks: (1) the method did not consider grade blending; (2) it did not control the risk distribution for the production targets; (3) the optimality of the solution could not be guaranteed; and (4) the simulated annealing process required tuning of parameters which made it difficult to apply.

Osanloo et al. (2008) mentioned several authors who consider the uncertainty of ore grade in LTPP problems and it has been reported that grade uncertainty may cause some shortfalls at the mine production stage and inconsistencies between planning expectations and actual production yields. The impact of grade uncertainty on mine production plans has led to extensive research on the application of stochastic programming models that consider grade uncertainty in solving the LTPP problem. Dimitrakopoulos and Ramazan (2008) presented a stochastic integer programming (SIP) model to generate the optimal production schedule using multiple realizations as input. The authors introduced a penalty function applied as the cost of deviation from production targets. The function was calculated from a geological discount rate (GDR) applied as a discounted unit cost of deviation from a production target. A SIP model was used to maximize NPV and minimize grade uncertainty risk that impacts the NPV of the mining project. It was concluded by the authors that the generated production schedule is the optimum solution that can produce the maximum achievable discounted total value for the project, given the available orebody uncertainty described through a set of stochastically simulated orebody models. The proposed scheduling approach considers multiple simulated orebody models without increasing the required number of binary variables and thus computational complexity. One of the shortfalls of this model was that, no explanation was provided on how the GDR parameter was determined.

In recent times, the application of stochastic programming in mine production planning have been investigated to efficiently improve the decision-making process compared to the conventional approaches of mine planning that are still currently used in the mining industry. Stochastic integer programming models offer a framework to incorporate uncertainty in key inputs of mine

production schedules, in areas such as grade and rock type uncertainties. Hence, its application in mine planning allows the incorporation of uncertainties associated with ore grades and tonnages, and waste disposal planning. In oil sands mining, the application of stochastic programming models so far lack the framework to incorporate grade uncertainty on an integrated production scheduling and waste management disposal plan since oil sands mining profitability depends on a carefully planned and integrated production scheduling and waste management strategy that is sustainable and generates value by maximizing NPV and minimizing waste disposal cost, through creation of timely tailings storage areas with less environmental footprints (Ben-Awuah et al., 2012).

This research will introduce an SMILP framework for integrated stochastic production scheduling and waste management for open pit mines, specifically oil sands mining, to generate maximum value and to support the sustainable development of oil sands resources.

1.4 Objectives of the study

In formulating a stochastic programming model to incorporate grade and tonnage uncertainties into oil sands mine planning and waste management, the primary research objectives are:

1. Develop a risk-based integrated oil sands mine planning and waste management optimization framework based on a Stochastic Mixed Integer Linear Programming (SMILP) model in the presence of ore grade and ore tonnage uncertainties, and waste management planning.
2. Determine the order and time of extraction of ore, dyke material, and waste to be removed from a predefined final pit limit over the mine life that maximizes the Net Present Value (NPV), minimizes waste management cost, and minimizes the cost of uncertainty associated with the targeted ore grade and ore tonnage.
3. Evaluate the risk profile associated with the mine plan and its impact on plant processing, dyke construction and waste management.

1.5 Context and scope of work

This research concentrates on the development and implementation of a Stochastic Mixed Integer Linear Programming (SMILP) model for long-term open pit production scheduling and waste management optimization in oil sands mining while taking into account the variability of ore

bitumen grades. The transfer of grade uncertainty into mine planning is taken into account through multiple simulated orebody realizations used as suitable inputs for the SMILP model. The optimization with the SMILP model is done on a block-by-block basis due to the consistent variations of ore bitumen grades from block to block in each realization. The main focus of this study consists of the following:

1. Developing a single geologic orebody model using Ordinary Kriging to estimate bitumen and fines content in the oil sands deposit. Cross validation for the kriged estimates is conducted for the accuracy of the modelled results (Geovia Dassault Systems, 2018).
2. Generating multiple realizations using Sequential Gaussian Simulation (SGS) which are based on Simple Kriging estimates. For validation purposes, the simulated results are compared with the Ordinary Kriging results from Step 1 (Deutsch and Journel, 1992).
3. Building multiple economic block models for the oil sands deposit and using the LG algorithm (Lerchs and Grossmann, 1965) to generate the ultimate pit limits for OK block model, E-type block model and stochastic block models (SGS realizations block models).
4. Generating a risk-based strategic production and waste management schedule for ore, dyke materials and waste using the SMILP model.
5. Evaluating the impact of using the SMILP formulation for integrated production scheduling and waste disposal planning optimization compared to the conventional mine planning approach.

For this research, the following assumptions were made in developing the SMILP model: (a) the geotechnical properties of the dyke construction material were not evaluated; (b) ore fines content uncertainties was not considered; and (c) input parameters such as future costs and selling price of the commodity used for the economic block models were kept constant. Therefore, any change in one of these input parameters would require re-optimization in order to capture the variation. Thus, a limitation of the SMILP model is the sensitivity of the price of the commodity which could impact the optimization results if there is a significant price change.

1.6 Research methodology

Grade uncertainty in long-term production planning has been reported to affect the NPV of mining projects due to the differences between the actual and expected production targets especially in the early years of production (Osanloo et al., 2008). Hence, to address the impact of grade uncertainty, instead of using a single estimated block model for production scheduling, simulated equally

probable block model realizations that are representative of grade variability were used as input to the SMILP model. The set of simulated block model realizations are generated using SGS and are referred to as stochastic block models. A conventional block model which was based on Ordinary Kriging (OK) estimation was considered as the base case block model. Results generated from the OK block model was compared with that from the stochastic block models and E-type block model. The E-type block model is the average block model of the realizations. The comparison demonstrated the impact of grade uncertainty on the integrated mine production scheduling and waste management plan. A detailed summary of the research methodology is illustrated in Figure 1-3.

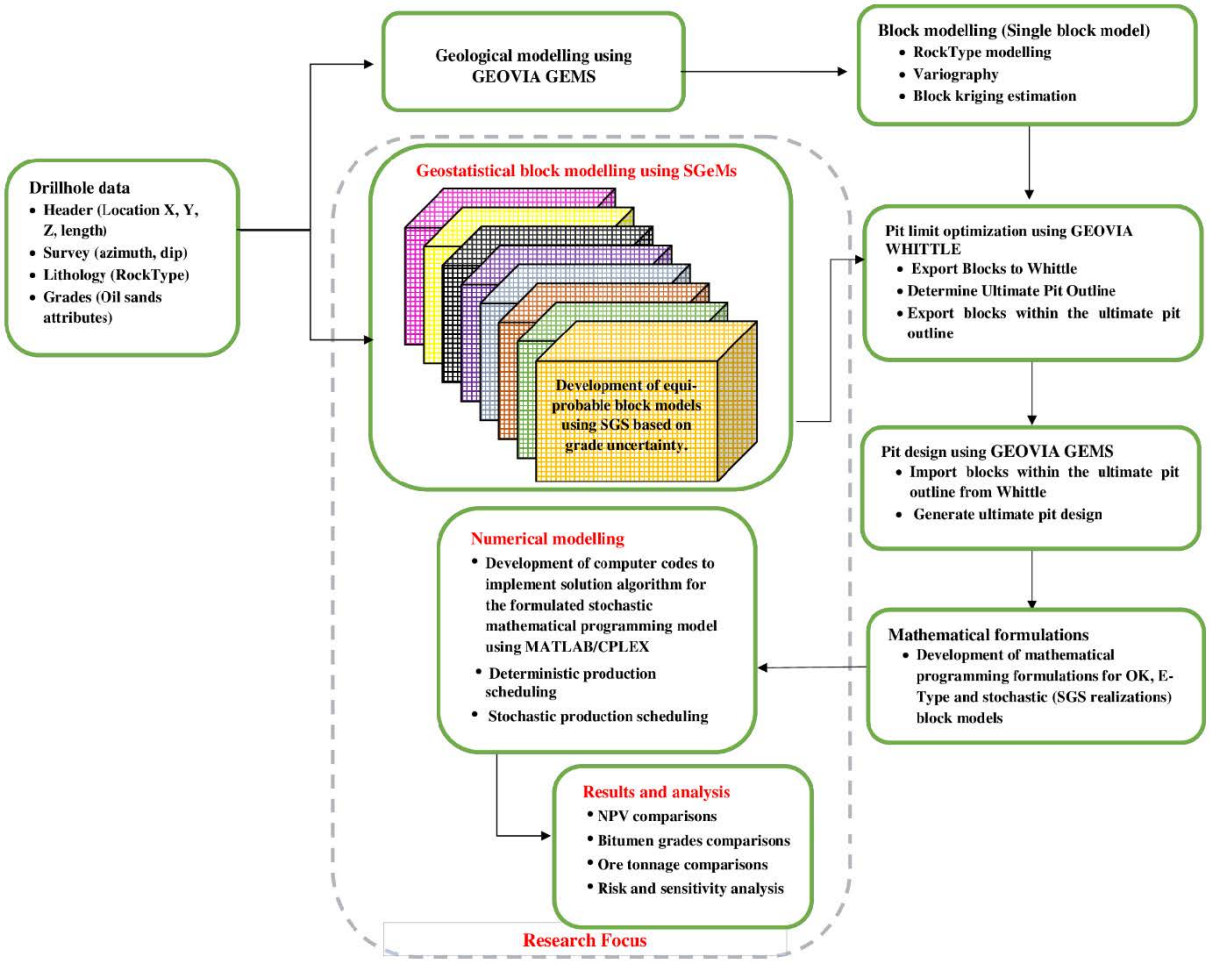

Figure 1-3: Summary of research methodology

The workflow used in this research to achieve the objectives of the study are divided into two parts. The first part includes exploratory data analysis and geostatistical modelling of the oil sands drillhole datasets. Geostatistical Software Library also known as GSLIB (Deutsch and Journel, 1998) and Geovia GEMS (Geovia Dassault Systems, 2018) were used for geostatistical modelling.

The following steps are implemented as part of the research methodology and verified with an oil sands case study:

1. Perform declustering by assigning each datum weight based on closeness of the sample data to the surrounding drillhole data so as to get representative statistics of the entire area of interest.
2. Perform multivariate statistical analysis to determine the correlation between the multivariate data.
3. Determine the principal direction of continuity for calculating variograms.
4. Perform Ordinary Kriging estimation with the data.
5. Transform data to standard Gaussian units.
6. Define parameters that will be used to construct the grid for simulation. The parameters include: (i) grid dimensions which consist of the number of blocks in each X, Y and Z directions; (ii) block size which consists of the dimensions of a single block; and (iii) origin coordinates.
7. Generate *n* realizations block models using Sequential Gaussian Simulation (SGS) and calculate the E-type block model which is the average block model of the realizations.

In the second part of the research, the main focus is to incorporate grade uncertainty into integrated oil sand mine planning and waste management based on stochastic mixed integer linear programming (SMILP). Appropriate mining concepts, mathematical and numerical models were formulated to define the inputs and outputs of the SMILP mine planning model. Specifically, this part of the research focuses on the development, analysis and implementation of three main components of the SMILP model: (i) maximizing the net present value of the mining operation, (ii) minimizing the waste management cost, and (iii) minimizing the cost of uncertainty associated with targeted ore grade and tonnage. Appropriate numerical modeling and solution techniques were deployed to convert the formulations into accomplishing the research objectives. MATLAB (2018) application was used as the programming platform to define the SMILP model and implemented for an oil sands dataset. The following additional steps are ensued in order to achieve the research objectives:

1. Generate optimal final pit limits based on the following block models: OK block model, E-type block model and *n*-realizations block models, and design the final pit outline based on the reference block model; OK block model.
2. Propose and develop a theoretical framework for a SMILP model to be used for integrated stochastic oil sands long-term production and waste disposal planning. This includes scheduling for both ore and dyke material in the presence of grade uncertainty.
3. Implement the SMILP model for the conventional case based on the OK block model and E-type block model.
4. Implement the SMILP model for the stochastic case based on the stochastic block models (SGS realizations block models).
5. Solve the SMILP models using IBM/CPLEX (ILOG, 2019) which uses a branch-and-cut algorithm in solving the optimization problem; generating an optimal solution if the algorithm is run to completion for a feasible problem. A gap tolerance (EPGAP) is used as an optimization termination criterion. This is an absolute tolerance between the gap of the best integer objective and the objective of the remaining best node.
6. Interpret the results and quantify the impact of using the SMILP formulation and workflow for integrated stochastic oil sands mine planning with respect to NPV and practicality of the generated production schedules.
7. Undertake sensitivity analysis and evaluate the risk profile associated with the mine plan and its impact on plant processing, dyke construction and waste management.
8. Provide documentation on the work flow and parameter calibration for the formulations and deployments.

1.7 Scientific contributions and industrial significance of the research

The main contributions of this research are the development of an SMILP framework that schedules for multiple material types, multiple elements, and different destinations and mining locations for integrated oil sands mine planning and waste management in the presence of grade uncertainty. The research contributes to the current literature on oil sands mine planning and waste management by:

1. Developing an integrated stochastic programming model based on grade uncertainty to simultaneously improve material handling and scheduling for the processing plant and

dyke construction, thereby expanding further the boundaries of integrated risk-based mine planning and waste management.

2. Providing a workflow for evaluating the effect of grade uncertainty on oil sands production scheduling and waste management.
3. Evaluating the range of uncertainty associated with a mine plan through risk and sensitivity analysis thereby promoting sustainable oil sands mining with improved resource governance.

The industrial significance of this research is the introduction of a SMILP framework and workflow that enables the oil sands mining industry to generate a risk-based strategic production schedule for ore and dyke materials in the presence of grade uncertainty. This is in accordance with the requirements of Directive 085 issued by the Alberta Energy Regulator (AER) (Regulator, 2016) on waste management performance and criteria for oil sands mining schemes (McFadyen, 2009). The formulation and workflow seek to optimize the oil sands mining operation by maximizing the NPV of the operation, minimizing dyke construction cost and minimizing the cost of uncertainty associated with ore blending and ore production for practical and realistic mining operation results.

1.8 Organization of the thesis

Chapter 1 of this thesis provides an overview of the research. It discusses the background of the study followed by a description of the problem statement, the objectives of the research, the context and scope of work, the proposed research methodology, and the scientific and industrial contributions of this research.

Chapter 2 documents a detailed literature review about the properties, compositions and classifications of oil sands deposits. This chapter further provides a review of open pit optimization and production scheduling methods that have been implemented in the past. The benefits and drawbacks of the reviewed production scheduling methods are also highlighted. The chapter concludes with a review on geostatistical approaches for block modeling such as kriging and conditional simulation techniques used for ore reserve estimations.

Chapter 3 contains the theoretical framework for the Stochastic Mixed Integer Linear Programming (SMILP) mine planning model. This chapter describes the mathematical programming formulation for the SMILP model for stochastic oil sands mine planning and how it can be setup in MATLAB

programming environment (Mathworks, 2018) with CPLEX optimization solver (Holmström et al., 2009; ILOG, 2019). This includes modelling of the objective functions and constraints that control materials movement during stochastic production scheduling optimization.

Chapter 4 highlights the application of the methodologies and mathematical formulations of the SMILP model that were discussed in the previous chapter. These are implemented in an oil sands case study. Relevant discussions on the performance of the SMILP model, analysis and conclusions are given for the case study.

Lastly, Chapter 5 is the concluding chapter which contains the summary and conclusions of this research. The scientific and industrial contributions of the research and recommendations for further study are highlighted.

CHAPTER 2

LITERATURE REVIEW

2.1 Background

In this chapter, a brief historical background of oil sands deposits is provided in Section 2.2. A review of the different properties and composition of oil sands material and how the various compositions play vital roles in bitumen grade recovery is given in Section 2.3 to Section 2.5. Evaluations of past and recent techniques for optimizations and scheduling of open pit mining operations are discussed in Section 2.6 to Section 2.8. Geostatistical approaches such as Ordinary Kriging and Sequential Gaussian Simulation techniques in ore reserve estimation are reviewed in Section 2.9. A comparison between Condition Simulation technique and Ordinary Kriging is reviewed in Section 2.10. The impacts of grade uncertainty in mine planning optimization is highlighted in Section 2.11. Lastly, Section 2.12 provides a concluding summary of the literature review.

2.2 Brief history of oil sands deposits

Canada's oil sands are concentrated mostly in Alberta and extends slightly into Saskatchewan's border. Oil sands deposits are localized in three regions: the Athabasca Basin, the Peace River Basin, and the Cold Lake Basin. The Athabasca Basin is by far the largest, spanning an area of about 40,000 km² (Conly et al., 2002). All mineable oil sands are located north of Fort McMurray, Alberta within the Athabasca Basin, where the deposit can be found very close to the surface (Mossop, 1980).

The mineral deposit considered in this study is located in the Lower Cretaceous McMurray Formation, which consists of a dominant continental sequence of uncemented sands and shales overlying an unconforming surface of Devonian limestone. The McMurray formation is approximately 38 m thick, and it is overlain by about 128 m of strata (Hepler and Hsi, 1989). The Athabasca Basin consists of the following stratigraphic formations: Muskeg, Pleistocene unit, Clearwater Formation, McMurray Formation and Devonian carbonates (Masliyah et al., 2011). The topmost layer of the strata is known as muskeg; it is the wet topmost layer of overburden material that contains the seeds and roots of native plants and is used for the topmost layer of the reclaimed land. Prior to mining, this layer is removed, stockpiled and then used later for

reclamation (Masliyah et al., 2011). The Pleistocene unit overlies the Clearwater Formation, and both are considered as waste rocks sitting above the bitumen-bearing McMurray Formation. Overburden materials from these layers are often used for road and dyke construction in the mine, depending on the soil properties and its mineral content. The oil-bearing rock type is the McMurray Formation, which is further classified into three rock types: the Upper McMurray (UKM), the Middle McMurray (MKM) and the Lower McMurray (LKM) (Masliyah et al., 2011). The McMurray Formation is comprised of coarse sand, fine sand, water, and bitumen. The main element of interest is bitumen, which exists in various grades across the formation. The formation rests with profound unconformity on Devonian carbonates and is unconformably overlain by the Clearwater Formation. The LKM is comprised of gravel, coarse sand, silt and clay, with siderite as cement. The UKM and MKM are comprised of micaceous, fine-to-medium-grained sand, silt, and clay, with rare siderite as cement and intra-clasts and pyrite nodules up to 10 cm in diameter (Hein and Cotterill, 2006). Lastly, the Devonian carbonates (DVN) is a rock layer that sits below the oil sands (McMurray Formation) and is comprised of numerous limestone outcrops. A sketch of the vertical profile of an oil sands formation is shown in Figure 2-1.

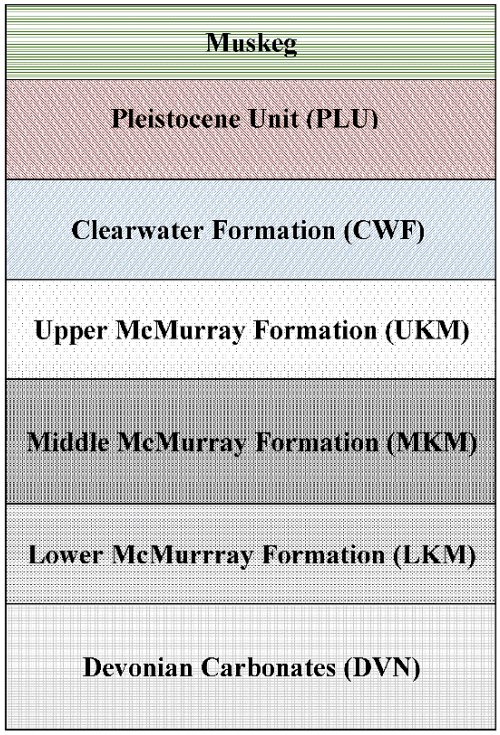

Figure 2-1: Sketch of the vertical profile of an oil sands formation.

2.3 Oil sands composition and properties

Oil sands are loose sand deposits that contain a very viscous form of petroleum known as bitumen. These unconsolidated sandstone deposits are comprised primarily of sand, clay, and water saturated with bitumen. Oil sands are sometimes referred to as tar sands or bituminous sand. Oil sands are composed of 55 – 80% solid contents (sand, silt, and clays), 0 – 16% bitumen and 0 – 7% water. The composition of oil sands is defined based on particle size. Any material that passes through a 44 µm (325 US mesh) sieve is referred to as fines, which includes silt, very fine quartz and clay minerals. In addition, heavy minerals have been found in the oil sands as part of the solid contents (Bichard, 1987). These heavy minerals are unique features of the solid contents in oil sands. The properties of the constituents of oil sands play a vital role in the processability of oil sands ores. Physical and chemical properties such as the nature of the bitumen, the coarse solid size distribution, the fine solids and clay content and the salt concentration and type in the formation water all contribute to the efficiency of bitumen floatation and recovery (Masliyah et al., 2011). Furthermore, the surface properties such as the surface tension, interfacial tension, and electric potentials of the sand grains and bitumen have an effect on the separation of bitumen from the sand grain with its ultimate floatation. These properties are affected by the temperature and water chemistry used in the bitumen extraction process (Long et al., 2008). Thus, it is important to understand the compositions and properties associated with oil sands and bitumen recovery. Each composition and property affecting bitumen recovery will be discussed further in subsequent sections.

2.3.1 Bitumen

Bitumen, an unconventional fossil fuel, is an extra heavy oil with high viscosity, density, and heavy metal concentrations and a low hydrogen to carbon ratio, which makes the oil extraction process more costly compared to conventional oil recovery (Masliyah et al., 2011). Bitumen from oil sands has a high molecular weight and low-volatility components, with typical compositions of 83% carbon, 10.6% hydrogen, 4.8% sulphur and, 0.4 % nitrogen (Basu et al., 1996). The density of bitumen is higher than that of heavy oil and is very close to water. This is an important factor in the bitumen flotation process since a density difference is required for oil to move to the top of the separation vessels. Therefore, air is also required as an additive to effectively reduce the density of the oil and cause the oil droplets to rise during frothing. Bitumen viscosity in-situ varies from several hundred to several tens of thousands of Pascal-seconds (Carrigy, 1967). The viscosity of

Athabasca bitumen is about 1,000,000 millipascal-seconds (mPa.s) at reservoir temperature, making the bitumen practically immobile; however, this characteristic gives enough material strength to the oil sands to be mineable (Mossop, 1980).

2.3.2 Organic-rich solids and ultra-fine clays

Fines are defined as all solid particles smaller than 44 micrometers (µm) in diameter, while clays are fines that are less than 2 µm in size, sometimes referred to as ultra-fines. Clays found in oil sands deposits are mostly comprised of kaolinite and illite. Most of the solids in the Athabasca oil sands are composed mainly of quartz sands grains, which are hydrophilic and 99% water wet. Sand particles with a diameter larger than 44 µm (microns are considered coarse, while those less than 44 µm are considered fine) (Bowman, 1971). Also, evidence has shown that a substantial amount of fines are present in the form of thin, discontinuous beds (Takamura, 1982). Some other trace minerals, such as mica, rutile, zircon, tourmaline, vanadium, and pyrite, are also found in the sand composition. Fines in the solid component mostly consist of silt and clay. Kaolinite (40 – 70%) and illite (28 – 45%) are the dominant clay minerals in Athabasca's oil sands (Chalaturnyk et al., 2002). The other common clay minerals are montmorillonite (1 – 5%), chlorite (~1%), smectite (~0.3 %), and mixed-layer clays (~1.7 %) such as kaolinite-smectite and illite-smectite. The amount of clays contained in an oil sands deposit is proportional to the number of fines. Therefore, all high fines ores have a high clay content and tend to have a lower bitumen content. These lower-grade ores are often described as poor processing since they disrupt the gravity separation process in extraction. Bitumen froth produced from high fines ores tend to be of a lower quality, typically containing less than 50 % bitumen and more than 40 % water (Masliyah et al., 2011).

Further studies have shown that certain mineral solid fractions are associated with significant amounts of organic-rich solids (ORS) (Kotlyar et al., 1988). These organic solids are composed of toluene insoluble organic matter that is physically or chemically adsorbed onto particle surfaces. ORS exist primarily as aggregates of clay, sand, and silt bound together by organic matter that is largely humic in origin. The solids interact strongly with bitumen. During bitumen separation, these solids may report to the bitumen froth, the primary tailings or remain with the middlings, depending on their size and density. As a result of their close association with bitumen, these solids are responsible for significant losses with certain ore types. In the tailings, the solids can contribute to flock structure formation through bonding between particles by free bitumen. In bitumen separation processes, the organic matter associated with various ORS fractions represents an

impediment to optimum bitumen separation and upgrading. In this sense, these solids are considered to be active relative to the inactive water wetted quartz particles comprising the bulk of the oil sands ore. Preliminary results indicate that the ORS content of an ore appears to be a better predictor for ore processability than the traditional use of bitumen or fines (~44 μm) contents. Two types of ORS have been recognized. The first is a coarser fraction, usually less than 44 μm but also occurring as particles greater than 100 μm in diameter. This coarser fraction of ORS typically occurs as aggregates of smaller particles bound together by humic matter and precipitated minerals. During the bitumen separation process, these heavy aggregates transport any associated bitumen into the aqueous tailings, thus reducing overall bitumen recovery. The second type of ORS is very thin, ultra-fine clay particles with a major dimension <0.3 μm in size. These ultra-fine clays, with a surface coating of organic matter, remain with bitumen during the separation process. In bitumen upgrading, these solids may be entrained with volatile overheads and cause problems in downstream operations (Kotlyar et al., 1988).

2.3.3 Heavy minerals in oil sands

The occurrence of heavy minerals in Alberta's oil sands has long been known and has been documented by many researchers (Kaminsky et al., 2008). The most relevant heavy minerals identified have been titanium-bearing, zirconium-bearing, and rare-earth-bearing minerals in a small fraction of mineral solids. These minerals are all important from the point of view of economic potential. In general, the heavy mineral concentration in most of the Athabasca McMurray Formation was found to be consistently low, averaging about 0.35% titanium dioxide (TiO_2) and 0.032% zirconium dioxide (ZrO_2) in feed grade oil sands. The TiO_2 content varied from 0.08 to 1.6%, and the ZrO_2 content ranged from 0.0012 to 0.13%. Other valuable elements identified in the oil sands included rare earth and trace amounts of palladium, platinum, and gold. However, none of these occurred in high enough grades to warrant economic extraction. The distribution of heavy minerals in a range of different sizes on the surface mineable area of the Athabasca oil sands was studied. It was observed that titanium and zirconium minerals were concentrated in fine size fractions and iron minerals in coarse size fractions. Silicon minerals seem to distribute evenly throughout the size range tested. In terms of size, heavy minerals are typically fine-grained. The 100-mesh (150 mm) size fraction is probably of most interest to potential users of the heavy minerals. It was reported that the 100-mesh fraction made up approximately 55 to 66% of the solid weight of the oil sands. Although bitumen is the primary target of commercial

extraction plants, there is a selective enrichment of the heavy minerals in bitumen extraction processes as well. Titanium and zirconium minerals seem to move with the bitumen. As the bitumen froths are treated to remove the mineral solids and water, these heavy minerals are concentrated in the froth treatment tailings. Thus, to make use of the heavy minerals contained in the oil sands tailings, the heavy minerals need to be separated and upgraded (Kaminsky et al., 2008).

2.3.4 Connate water in oil sands

The water present in oil sands contains dissolved ions such as sodium, calcium, magnesium, chloride, potassium, sulfate and bicarbonate. Bitumen is not in direct contact with the mineral phase because a thin film of water surrounds individual sand grains. The thickness of the water film is predicted to be about 10 nm (Hall et al., 1982). This water film is assumed to be stabilized by electrostatic forces coming from electrical double layers at the oil/water and water/sand grain interfaces (Hall et al., 1982). A thin layer of water separates bitumen from sand particles, and, as the water content of an ore increases, the bitumen content decreases (Takamura, 1982). For higher concentrations of ions in the water, the bitumen content will be lower; this means rich oil sands have a low salt content, and poor oil sands have a high salt content. The pH of most of the oil sands is between 8 and 9, with some acidic ores at higher depths (Masliyah et al., 2011). Also, the bitumen and water contents vary and depend on the ore variation and clay mineralogy (Kasperski, 2001). Takamura (1982) presented a description of the generic microscopic structure of Athabasca's oil sands. A microscopic structure of Alberta oil sands can be seen in Figure 2-2.

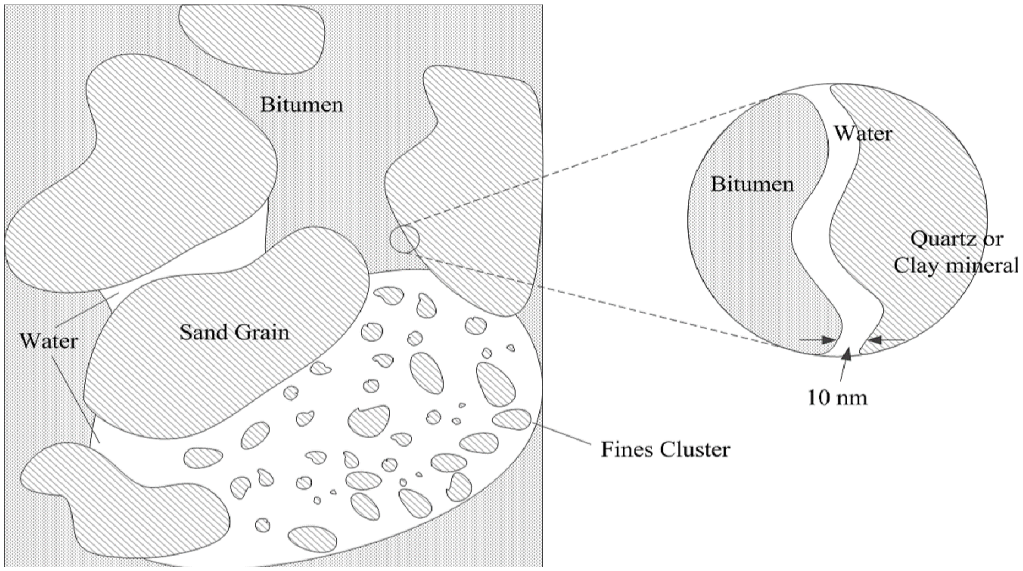

Figure 2-2: Microscopic structure of Alberta oil sands modified after Takamura (1982).

2.4 Oil sands material classification

There are several ore classification schemes for oil sands based on descriptors such as bitumen content, fines content and ore processability as indicated in Table 2-1 (Kaminsky et al., 2008). The most common method of classifying oil sands ore is based on its grade. A high-grade ore is considered to contain more than 10% bitumen, a mid-grade ore contains 8 – 10% bitumen, and a low-grade ore contains less than 8% bitumen. Further, a high-fines ore contains more than 18% fines, while a low-fines ore contains less than 6% fines. Processability indicates the quantity of bitumen recovered in the froth. When more than 80% of bitumen and 90% of total bitumen recovery are obtained from the froth, it is referred to as a good processing ore (Kasperski, 2001). Froth quality and settling behavior are also sometimes considered as factors in processability, but most processability curves report recovery only. A high-quality froth will contain about 66% bitumen, 25% water, and 9% solids or a 7:1 bitumen to solids ratio (Kasongo et al., 2010).

Table 2-1: Oil sands ore classification.

Methods classification	Content (%)	Ore classification
Bitumen	>10	High
	Between 8 and 10	Medium
	<8	Low
Fines Content	>18	High fines ore
	<6	Low fines ore
Processability	Bitumen >10	Good processing ore
	Fines < 6	Good processing ore
	Bitumen < 8	Poor processing ore
	Fines > 18	Poor processing ore

2.5 Effect of oil sands properties in bitumen recovery

According to the Alberta Energy Regulator (AER) (Regulator, 2016), to establish the volume of bitumen an operator is required to recover each year from its mining and processing operations, four criteria are used collectively:

- Cutoff grade: This is defined as the minimum bitumen content of the oil sands that would be classified as ore. The standard cutoff grade is 7 weight percent bitumen.
- Minimum mining thickness: This is the minimum thickness of ore that can be separated from waste or waste that can be separated from ore. It has been set at 3 meters.
- Total volume to bitumen in place ratio (TV:BIP): This the minimum value for TV/BIP that would be used to determine the pit crest limits. It is set at 12.
- Processing plant recovery: This is a variable factor based on the average bitumen content of the as-mined ore. If the average bitumen content of the as-mined ore is 11 weight percent

bitumen or greater, the recovery factor is 90 weight percent. If the average bitumen content of the as-mined ore is less than 11 weight percent bitumen, recovery is determined as shown in Equation (2.1), where x is the average weight percent bitumen content of the as-mined ore:

$$Recovery = -202.7 + 54.1(x) - 2.5(x^2) \quad (2.1)$$

The recovery of bitumen from oil sands ore during extraction depends on certain different process variables. The variables can be classified into three main groups: ore properties, water chemistry, and operating conditions. Extensive research works have been conducted to recognize and clarify the involved factors that could impact bitumen recovery during the bitumen extraction process from oil sands ores (Sanford, 1983; Dai and Chung, 1995; Basu et al., 1996; Masliyah et al., 2004; Long et al., 2008; Wallwork et al., 2008) as shown in Table 2-2. For this research, the focus is only on the ore properties since it involves the mine-to-mill concept. Thus, ore properties such as bitumen grade, fines contents, types of fines, and the mineralogy of sands and fines will be considered in this review.

Table 2-2: Factors affecting bitumen recovery and oil sands ore processability.

Ore properties	Water chemistry	Operating conditions
Bitumen grade	pH	Temperature
Fines content	Presence, valence and concentrations of ions	Mechanical mixing and residence time
Types of fines	Presence and concentrations of surfactants	Slurry density
Mineralogy of sands and fines	Presence and concentrations of carbonates	Aeration
Weathering of ores	Presence and concentrations of dispersants and polymers	Bubble size

2.5.1 Effect of organic-rich solids in bitumen recovery

Bitumen is usually separated from the inorganic matrix by a surface layer of interstitial water that allows bitumen to be liberated from its bulk mineral component. However, the humic content associated with ORS tends to adsorb and bind the bitumen to both internal and external aggregate surfaces, thus reducing the recovery of bitumen. The size and density of ORS are vital properties that need to be considered in bitumen recovery. Small particles of ORS report with the froth along with the liberated bitumen. If the particles are not discarded to the tailings during the froth cleaning process, they will remain with the bitumen froth and adversely affect the quality of the bitumen.

Coarser and higher-density aggregates will not float with the bitumen fraction; rather, they will remain with the middling or settle with the primary tailings along with the associated bitumen, thus reducing the recovery (Kotlyar et al., 1988).

2.5.2 Effect of ultra-fine clays in bitumen recovery

In the oil sands industry, it has been noted that coarse sands do not cause any problems throughout the extraction process. However, the fines (<44 μm) content plays an important role during this process. Bitumen recovery decreases with increasing fines content in warm water extraction (Chalaturnyk et al., 2002). It is illustrated that small particle size is a contributing factor affecting bitumen recovery, but comparison tests with ultra-fine silica particles (<0.3 μm) showed that mineralogy is a more important parameter (Wik et al., 2008). Nano- and micro-sized minerals, mainly clay minerals, are most detrimental (Hooshiar et al., 2012). These minerals may interact with the solvent, bitumen and connate water due to their small particle size, high specific surface area, swelling capacity, cation exchange capacity, layer charge and specific physicochemical properties. The presence of fine solids in oil sands has been found to interfere with interactions among bitumen droplets. Fine solids also minimize bitumen coalescence, thus producing small-sized bitumen droplets that are more difficult to contact with air bubbles. Coated bitumen droplets form steric barriers between bitumen and air bubbles, making it difficult for the bitumen to attach and engulf the bubbles. This results in an increase in slurry viscosity and possibly gelation, which slows the flotation of formed bitumen-bubble aggregates. The bitumen froth quality and its floatation kinetics are reduced due to the fine solids appearing in the froth with recovered bitumen and slurry water (Liu et al., 2008).

The mineralogy of clay minerals in the Alberta oil sands has been studied, and a summary of this mineralogy can be found in Mikula et al. (2008). Based on such studies, it has been reported that the mineralogy of clay minerals in the Alberta oil sands deposit differ across the deposit. Kaolinite, illite, smectite, and chlorite are found to be the major clay minerals in the Alberta oil sands deposits, with the first two being the most abundant. In one of the studies conducted by Zhou et al. (1999), to investigate the effects of fine solids with different surface properties on oil extraction, it was shown that the addition of fine solids has a negative impact on bitumen extraction. The bitumen recovery decreased with the solids addition, compared with no solids addition. Also, the smaller solids (e.g. <5 mm silica) reduced bitumen recovery more than the coarser solids (e.g. <40 mm silica). This could be attributed to the physical barriers created between bitumen and bubbles

by suspended fine solids, increasing the difficulty for direct bitumen-bubble contact. Since finer solids are more easily suspended in the slurry, they are more likely to block direct bitumen-bubble contact than coarser solids, thereby more severely retarding the flotation. In another study conducted by Liu et al. (2008) to investigate the role of fine clays in bitumen extraction from oil sands, it was observed that in the absence of calcium, montmorillonite clays were weakly attached on the bitumen surface. However, with the addition of 1 millimolar (mM) of calcium-montmorillonite, clay attached strongly to the bitumen surface. The reason for the strong attachment was due to a low electrostatic double-layer force and an increased adhesive force between bitumen and montmorillonite clays that acted as a barrier for bitumen-air attachment and bitumen-bitumen coagulation, thus contributing to poor bitumen recovery.

Even though numerous attempts have been made to demonstrate the impact of bulk properties of oil sands ore on bitumen recovery, it is also important to understand the clay characteristics and mineralogy of ORS and ultrafine clays in the oil sands. Both extraction recovery and product quality are significantly affected by interactions between organic materials and clay minerals. These interactions produce clay-organic matter complexes that make the bitumen extraction process more complicated. Therefore, it is crucial to understand the influence of water, bitumen, and solids on the extraction process (Hooshiar et al., 2012).

2.6 Open pit optimization and production scheduling

The operation and management of a large open pit mine is an enormous and complex task, especially those operations with a longer mine life. Optimization techniques can be successfully applied to resolve issues that arises in the production planning and management of a mine. These applications include ore-body modeling and ore reserve estimation, the design of optimum pits, the determination of optimal production schedules, the determination of optimal operating layouts, the determination of optimal blends, and the determination of equipment maintenance and replacement policies (Caccetta and Giannini, 1986).

One of the critical problems encountered in mine planning is determining the final pit limit of the mine. The final pit limit of a mine is defined as a contour which is an outcome of extracting the volume of material that provides the total maximum profit whilst satisfying the operational requirements of safe wall slopes. The final pit limit gives the shape of the mine at the end of its life. The contour is usually smoothed to produce the final pit outline. The design of the final pit outline plays a significant role in all stages of the life of mine: at the feasibility study stage, when

there is a need to produce a life-of-mine pit design; at the operating stage, when the pits need to be developed to respond to changes in metal prices, costs, ore reserves, and wall slopes; and towards the end of the mine's life, where the final pit design may allow the economic termination of the mining project. There is a need for constant monitoring of the optimum pit to facilitate the best long- term, medium-term and short-term mine planning and subsequent exploitation of the reserve at all stages of the mine life. The optimum pit and mine planning are dynamic concepts requiring constant review. In practice, it is necessary to construct a whole spectrum of pits, each corresponding to a specific set of parameters. The ultimate pit limit problem has been efficiently solved using the Lerchs-Grossmann and graph-theory algorithm, also known as Picard's network flow method. These methods are based on the block model of an orebody (Lerchs and Grossmann, 1965; Picard, 1976).

Open-pit production scheduling in surface mining is characterized by the sequence in which blocks of the orebody are extracted and transferred to maximize the net present value subject to mining, economic and processing constraints (Caccetta and Giannini, 1986). Usually, mining, milling and processing capacities are the constraints associated with the mining extraction sequence; mill feed and concentrate grades; stockpile-related restrictions; and a variety of logistics problems and different operational specifications, such as minimum pit bottom width and maximum vertical depth. The concept of block models provides a starting point for the optimization problem for production scheduling. Block models divide the orebody into smaller blocks that depend on pit slopes, orebody dip, grade distribution and the type of equipment used (Caccetta and Giannini, 1986). Using either kriging or interpolation techniques, each block is assigned some attributes such as density, grade and tonnages. The blocks are then divided into ore and waste using cut of grade optimization techniques. The total number of the blocks created can be in the order of millions, thus requiring movement over a long period of time. This movement of blocks demands adequate scheduling that considers the physical and operational constraints (Khan and Niemann-Delius, 2014).

The techniques used in mathematical programming for block extraction sequencing ensures consistent practical mining environment. Linear programing, integer programming and mixed integer linear programming use this type of technique for mining applications (Hickman, 2014). One of the major drawbacks of the conventional approach is that the optimal scenario is affected by uncertainties related to input parameters. These uncertainties are categorized as in-situ grade

uncertainty, economic uncertainties such as capital and operating costs and technical mining specification uncertainties such as extraction capacities and slope considerations (Dimitrakopoulos and Ramazan, 2008; Hickman, 2014). Meanwhile, the stochastic approach uses some form of randomness when searching for feasible solutions (Hickman, 2014). The input to the stochastic approach takes into account the variabilities of each input parameter. Their outputs do not give one good optimum solution but a range of solutions with some form of distribution to give the user a feel of variability for the solution. Stochastic techniques are more efficient when using evolutionary algorithms (Hickman, 2014).

Due to the complexity and size of the problem, all these approaches suffer from one or more of the following limitations: (i) they cannot provide to most of the constraints that arise, (ii) they yield only suboptimal solutions and in most cases without a quality measure, and (iii) they can only handle small-size problems. The most commonly applicable method is parameterization, which was initially introduced by Lerchs and Grossman (1965). In these techniques, a set of nested pits is generated, starting with the final pit contour, by varying the economic parameters. For each parameter value, the Lerchs-Grossman algorithm is applied to generate the optimum pit. The most widely used software packages that employ this method include Whittle's Four-D and Four-X and Earthworks NPV Scheduler (Ver. 3.2.5). The latter product has a restricted tree search procedure used to re-sequence the pushbacks in an effort to improve the NPV (Meagher et al., 2014).

Stochastic mine planning is a rather recent development aimed at addressing uncertainty in ore supply from an orebody, commodity prices and metal demand as well as other issues of uncertainty in mine planning. The application of stochastic optimization in mine scheduling problems has been reported to substantially increase the net present value to between 10% and 30% compared to other conventional approaches (Ramazan and Dimitrakopoulos, 2018; Kumar and Dimitrakopoulos, 2019; Paithankar et al., 2020). Production planning are collectively grouped into two main approaches: (i) the conventional approach and (ii) the uncertainty-based/stochastic approach. In the conventional approach, all inputs are assumed to have fixed known real values from which a sequence of possible solutions are generated.

2.7 Mathematical programming methods used in mine scheduling problems

Production scheduling requires efficient optimization techniques to realize the benefits. Optimization techniques such as linear programming (Johnson, 1969), mixed integer programming (Gershon, 1983), branch and cut (Caccetta and Hill, 2003), dynamic programming (Onur and

Dowd, 1993), fundamental tree algorithm (Ramazan, 2007), and Lagrangian parameterization methods (Dagdelen and Johnson, 1986) have been applied in mine scheduling problems. The key objectives of an open pit mine production scheduling problem is to specify the sequence in which blocks should be removed from the mine in order to maximize the total discounted profit from the mine subject to a variety of constraints. To achieve this goal, mathematical formulations that enforce these constraints are developed. These constraints include the mill throughput, the volume of material extracted per period, blending constraints, stockpile-related constraints, and logistics constraints (Gershon, 1983; Caccetta and Giannini, 1986).

2.7.1 Linear programming

The linear programming (LP) model was first proposed by Johnson as an optimization technique for mine scheduling problems (Johnson, 1969). The LP model considered the time value of money, different processing types and also the dynamic cutoff grade strategy. In order to solve the large multi-period production planning problem, the LP model was first broken down into a master problem and a set of sub-problems using Dantzig-Wolf decomposition principles (Johnson, 1969). Each sub-problem is then solved as a single-period problem that has the same characteristics as the ultimate pit limit problem. This can be achieved using a maximum network flow algorithm. Although Johnson's method generated optimum results for each period individually, it does not completely solve the mine scheduling problem in the long-term. The variables were linearly continuous, which was responsible for fractional block extraction. Also, the LP model provided situations in which some portion of a block was extracted, while all the overlying blocks have not been mined. This shortfall causes some percentage of overlying blocks to be suspended in air which is not representative of a typical mining extraction. Another drawback of this model was that there were too many constraints and there was a limitation on the number of blocks that could be handled by the model (Johnson, 1969).

2.7.2 Mixed integer programming

The problem of partial mining of blocks associated with Johnson's LP model was addressed by Gershon (1983), who discussed a mixed integer programming (MIP) model that would allow partial blocks to be mined if all precedent blocks have been completely removed. The key to this formulation was adding additional decision variables to Johnson's LP model. The advantages of the MIP model compared to the LP model were that it provided a more practical extraction sequence in mine scheduling. Partial block mining was allowed on the condition that all blocks

preceding the partially mined block have been mined. The net result of this model was that only one constraint per block was required. The main disadvantage of this model was its inability to handle large problems using commercial software because it contained too many binary variables. In addition, because of a larger model size, the dynamic cut-off grade concept could not be considered (Gershon, 1983).

2.7.3 Integer programming: Lagrangian relaxation approach

This is another research method used in solving mine scheduling problems. A binary linear programming formulation usually involves a large number of zero-one variables, which may exceed the capacity of current commercial packages. Several approaches have been proposed by researchers to solve such models. Dagdelen and Johnson (1986) used a Lagrangian relaxation approach to solve the open pit production planning problem. The objective function is to maximize the NPV, which is subject to block precedence and production capacity constraints (Dagdelen and Johnson, 1986). The authors relaxed the mining capacity constraints and added penalty in the objective function accordingly. A sub-gradient algorithm is used to update the Lagrangian multipliers for small-scale problems. The Lagrangian multipliers are adjusted until the optimum schedule is obtained. At each step, a problem similar to an ultimate pit limit problem should be solved. In cases where there are no multipliers that can result in a feasible solution for the constraints, this method may not converge to an optimum solution. This problem is called the gap problem. Caccetta et al. (1998) tested this method on a real orebody with 20,979 blocks and six time periods. The schedule obtained was within 5% of the theoretical optimum. Another drawback of this scheme is that it does not consider the dynamic cutoff grade concept during scheduling (Caccetta et al., 1998).

Akaike and Dagdelen (1999) extended the work by Dagdelen and Johnson (1986) using an iterative technique to update the values of the Lagrangian multipliers. Akaike and Dagdelen then transformed the integer programming model through use of the Lagrangian relaxation method so that the transformed problem has the same characteristics as the final pit design problem. This problem can be solved interactively by changing the Lagrangian multipliers by using a sub-gradient method until convergence to the optimal solution of the primal problem is reached, if at all. The authors also improved the efficiency of the sub-gradient method to reach the optimal solution much faster. The most important advantages of this algorithm are the use of the dynamic cutoff grade concept with the stockpile option with zero-one variables during the scheduling

process. This will improve the NPV of a mining project. The disadvantage of this method is the possibility of a gap problem occurring, which means that it may not lead to an optimum solution (Akaike and Dagdelen, 1999).

2.7.4 Integer programming: Clustering approach

The clustering approach was first implemented by Ramazan et al. (2005) to solve an integer programming (IP) model of production planning in an open pit mine. Clustering means classifying a large amount of data into relatively few classes of similar objects. This is the reason for the complexity reduction in the considered application, which allows for improved decisions based on the information gained. Ramazan et al. (2005) combined ore and waste blocks together to decrease the number of binary variables in the IP model. The authors introduced the fundamental tree as any combination of blocks within the pushbacks, such that the blocks can be profitability mined and obey the slope constraints. The clustering process is done using an LP formulation so that the information available for individual blocks is not lost (Ramazan et al., 2005). The authors created three fundamental trees that can be used for the LP model where the first tree is mined first followed by the second and third trees which are also feasible to mine. Their precedence relations are determined using the cone template after the fundamental trees are defined. Each fundamental tree is treated as a mining block that contains certain ore tonnage, metal content and quality parameters such as grade. Then, a binary variable is assigned to each fundamental tree for each production period except the last one. In order for the IP model that uses fundamental trees to be handled by commercial software such as CPLEX, the material within the final pit limit is divided into a smaller volume by defining three to five pushbacks. Finally, fundamental trees should be scheduled by an IP formulation that contains all the mining and milling operational constraints and tree sequence requirements. The advantages of using the clustering approach is that it reduces the binary variables and eliminates the gap problem. It also generates more NPV than other scheduling software. Its disadvantages are that it needs to generate pushbacks before scheduling. More than one iteration is needed to generate fundamental trees. Finally, its application is complicated, as the optimality of the solution depends on the optimality of the generated pushbacks (Ramazan et al., 2005).

2.7.5 Mixed integer linear programming

Most optimization problems that are formulated as mixed integer linear programming problems are solved using a branch-and-cut technique. This technique consists of a combination of cutting

plane methods and branch-and-bound algorithms that solves a sequence of linear programming relaxations of the IP problem. Cutting plane methods improve the relaxation of the problem to a more closely approximate IP problem. Branch-and-bound algorithms follow a state-of-the-art divide-and-conquer approach to solving optimization problems. It is usually not possible to solve a general IP problem efficiently using only a cutting plane approach; it is necessary to also use branching, which results in a branch-and-cut approach (Mitchell, 2002).

Caccetta and Hill (2003) proposed a branch-and-cut procedure for solving IP models of long-term production planning (LTPP) problems. However, they did not provide the complete information of their algorithm due to the commercialization of their software. They reported that a key advantage of their algorithm is its ability to incorporate explicitly all technical constraints such as maximum vertical depth, minimum pit bottom width and a stockpile option. Similarly, it could produce good solutions for medium term mine production planning problems. However, optimal solutions for large optimization problems were difficult to obtain. The authors applied the branch-and-cut technique on a large model containing about 209,600 blocks and ten scheduling periods. They could obtain a solution within 2.5% of the optimum within four hours. Another drawback of this scheme was that, it did not optimize the cutoff grade during the optimization process. It was reported that for large and/or difficult problems, branch-and-cut methods can be used in combination with heuristics or meta-heuristics to obtain a good (possibly optimal) solution and also to indicate how far it is from the optimal solution. MILP formulations do not consider the smoothness of the scheduled patterns, which relates to equipment movement in a period and equipment access. Also, geological uncertainty is ignored in the MILP optimization model. The usual approach is to first determine the final pit outline and then, through a series of refinements, mining schedules are generated. The final pit outline is determined by smoothing the contour produced by solving the ultimate pit limit problem. The ultimate pit limit is the maximum value pit resulting from the mining of ore and waste blocks under the assumption that all mining can be done in one period (Caccetta and Hill, 2003).

2.8 Stochastic integer programming (SIP) for mine production scheduling

The complexity of mine scheduling problems is increased by uncertainties due to the sparse variability of geological data. The uncertainty of the ore grade may cause discrepancies between planning expectations and actual production. Various authors proposed methodologies to account for grade uncertainty and demonstrate its impact. Godoy and Dimitrakopoulos (2004) presented a

new risk-inclusive long-term production planning (LTPP) approach based on simulated annealing. A multistage heuristic framework was proposed to generate a schedule that minimizes the risk of deviations from production targets. The authors reported a substantial improvement in NPV in the presence of uncertainty. These techniques can also be difficult to execute, and several criteria such as the size of the optimization problem and computation time may need to be considered in order to obtain acceptable results. A risk-based algorithm that focuses on surface mine planning was implemented by Osanloo et al.(2008). The author generated different schedules for a number of realizations of the ore grade. Parameters such as the selling price, mining cost, processing cost, ore grade and tonnage requirements were used as input for the mine scheduling problem. The proposed method led to multiple schedules reflecting the impact of grade uncertainty on the expected yearly production yield in the mine plan. Koushavand et al. (2014) used simulated orebodies to show the impact of grade uncertainty on production scheduling. They used simulated orebody realizations one at a time in traditional optimization approach, however, this sequential process does not optimize accounting for uncertainty. Ramazan and Dimitrakopoulos (2018) suggested a MILP model to maximize the NPV for each realization. Then, the probability of extraction of a block at each period is calculated. These probabilities are used in the second stage of optimization to arrive at one schedule; however, the uncertainty is not used directly in the optimization process. Ramazan and Dimitrakopoulos (2018) also presented a linear integer programming (LIP) model to generate optimal production schedules. Multiple realizations of the block model were considered. This model has a penalty function; the cost of deviations from the target production is calculated based on the geological risk discount rate (GDR), which is the discounted unit cost of deviation from target production. It is not clear how to define the GDR parameter (Ramazan and Dimitrakopoulos, 2018).

Stochastic integer programming (SIP) provides a framework for optimizing mine production scheduling while considering uncertainty. A specific SIP formulation is briefly shown to generate optimal production schedules, using equally probable simulated orebody models as input, without averaging the related grades (Dimitrakopoulos and Ramazan, 2008). The optimal production schedule generated is the schedule that produces a maximum achievable discounted total value from the project, given the orebody uncertainty described through a set of simulated orebody realizations. The proposed SIP model allows the consideration of geological risk in terms of not meeting planned targets during actual operation which is different from traditional scheduling

methods that use a single orebody model, where risk is randomly distributed between production periods, and there is no control over the extent of risks on the schedule. The objective function of the SIP model is constructed as the 'maximization of a profit function'. The profit function is defined as the difference between the total expected NPV and the cost of deviations from planned production targets. A single orebody model combined with a smooth image of the deposit, is used conventionally to maximizing the NPV. However, when the expected deviations from the planned amount of ore tonnage with a planned grade and quality in a schedule are high in actual mining operations, the conventional model is unlikely to achieve the resultant NPV of the planned schedule. Therefore, the SIP model is developed to consider the minimum of the deviations together with the maximization of the NPV to generate a feasible NPV (Ramazan and Dimitrakopoulos, 2018).

A constant value is initially assigned for each of the cost parameters representing the cost at time 0 (base cost) during the formulation of the objective function. The risk discounting parameter is introduced to determine the cost at different time periods by discounting the base cost using the discount parameter (Dimitrakopoulos and Ramazan, 2003). If the parameter is set to 0, the deviations in production targets can be expected to result in more or less the same level between different production periods because the cost of a unit deviation will be the same in all periods. However, the distribution of deviations will depend on the distribution of the grade and ore tonnage variations over the deposit and the relative magnitude of the costs for the deviations used in the SIP model compared to the economic values of the blocks. The geological risk discount rate (GDR) allows the management of risk associated with grade uncertainty to be distributed between periods. The application of a high GDR parameter will ensure that the least risky areas in terms of meeting production targets is extracted earlier, and the riskiest parts will be left for later periods. If a very small GDR or a GDR of zero is used, the risk will be distributed at a more balanced rate among production periods, depending on the distribution of uncertainty within the mineralized deposit. The variables in the objective function are used to define a risk profile for the production, and the NPV produced is the optimum for the defined risk profile. It is considered that if the expected deviations from the planned amount of ore tonnage with a planned grade and quality in a schedule are high in actual mining operations, it is unlikely to achieve the resultant NPV of the planned schedule. Therefore, the stochastic integer programming model contains the minimization of the

deviations together with the NPV maximization to generate practical and feasible schedules with achievable cash flows (Dimitrakopoulos and Ramazan, 2003).

2.9 Geostatistics approaches in reserve estimation

Estimating recoverable reserves is one of the most essential procedures in the mining industry during production stages for grade control as well as mine planning. The primary goal of grade control in mine production is to differentiate between material that is above the cut-off grade and below the cut-off grade through recoverable reserve estimation. Recoverable reserve estimation focuses on the quality (grade) and quantity (tonnage) over an area wherein drilling has not been performed, since drilling does not cover the whole area to be mined. From the estimates, a decision can be made as to whether to mine the material as ore (above cut-off) or waste (below cut-off). According to Sinclair and Blackwell (2002), the term local/recoverable reserve estimation is not strongly stated however, it is used in the overall context of point estimation or small block estimation of a selective mining unit. The most important goal is to provide estimates that are accurate and reliable. In mining, geostatistical techniques are preferred for recoverable reserve estimation compared to other techniques. Since not all areas where mining is done are drilled and sampled, geostatistical techniques were developed to aid in estimating the grade and tonnage of an area based on nearby sample values. Geostatistics has been described as the application of the theory of regionalized variables to the estimation of mineral deposits (Matheron, 1971). The idea of using geostatistics for local reserve estimation in the mining industry was introduced by Krige (1951).

Apart from reserve estimation, geostatistical methods are often used for mine planning as well as for testing the value of using various types of machinery on the output of the mine (Journel and Huijbregts, 1978; Clark, 1979) Currently, this technique is commonly used in other disciplines such as hydrogeology, meteorology and contour mapping (Royle et al., 1981). In mining, geostatistics stresses the geological context of the data and the spatial relationship between data. Compared to other methods of resource estimation, geostatistics is a better method for estimating the grade and tonnage of mineral deposits. In contrast to conventional approaches that analyze the statistical distribution of the sample data, geostatistics integrates both the statistical distribution of the sample data and the spatial connection between the sample data, thereby addressing more concerns in relation to the geology. Geostatistical techniques use variograms that depend on the spatial distribution and internal structure of the data and not just the actual values. If a variogram

is good, it offers estimates that are a good representation of the spatial distribution of the input data (Samal et al., 2008). The technique is based on the principle of regionalized variables, which notes that the interpolation from space points should not be based on a smooth continuous object (Matheron, 1971). Unlike classical statistics, which considers grade to be uniformly distributed, geostatistical techniques consider the shifts of mineralization in relation to the orebody's trend direction. It often recognizes the field of influence and continuity or lack of continuity of mineralization within the orebody. While other techniques such as inverse distance weighting and polygonal estimation do not provide a measure of the accuracy of estimates, geostatistical techniques provide not only the estimates but also measurements of the accuracy of the estimates. Apart from the estimated mean grade, they also reflect the estimated variance or grade distribution. The calculated variance in conditional simulation is often used in risk analysis of the reserve estimate (Samal et al., 2008).

2.9.1 Geostatistical kriging

Ordinary kriging is one of the estimation techniques commonly used in mining for grade control for mineral resource estimation. It was first introduced by Daniel Krige, who developed the technique while performing ore reserve estimation work in a South African gold mine (Krige, 1951). Ordinary kriging is considered linear interpolator since it is based on the linear weighted average and assumes that the mean of the data is constant and it is unknown (Clark, 1979). The distance from the estimation point and spatial correlation of the samples greatly impact the sample weight values. In Ordinary Kriging, the spatial correlation structure of the data is influenced the weighting of neighboring data points rather than by the power of their inverted distance like the inverse distance weighting method. Upon close evaluation, the kriging approach is similar to the inverse distance weighting method. However, the major difference is that Ordinary Kriging weights are assigned based on the variogram model, thereby taking the spatial relationship into consideration. Using a variogram in computing weights helps minimize the expected error in the least square way. For this reason, Ordinary Kriging is sometimes said to produce the best linear unbiased estimate, and some authors have associated Ordinary Kriging with the acronym BLUE - Best Linear Unbiased Estimator (Isaaks and Srivastava, 1989; Sinclair and Blackwell, 2002). It is considered BLUE since the weighted linear combination of data make up the estimate. The unbiased nature of the results is due to the absence of residual and estimation errors. Ordinary Kriging is considered to be best choice since the variance of errors is mostly minimized through a

linear combination of surrounding samples that are used to make predictions by assigning weights to the surrounding samples. Some estimation techniques like inverse distance weighting are also considered unbiased as well as linear like the Ordinary Kriging technique; however, Ordinary Kriging aims to reduce the error variance, distinguishing it from these techniques. However, kriged estimates tend to be smoother than the input data.

2.9.2 Conditional simulation

In the context of mining, simulation refers to the imitation of conditions (Sinclair and Blackwell, 2002). A simulation is conditional if the resulting realizations honor the actual/original data at their locations. This method is mainly used for continuous variables like grade, height and age, and its basic principle is to obtain an appropriate simulation of a point, and its value must be drawn from its conditional distribution given the values at some of the nearest points. In the mining industry, it is used in the study of grade continuity, recoverable reserve estimation, optimizing sampling plans for advanced exploration, evaluation of resource estimation and also in mine planning (Sinclair and Blackwell, 2002), mill optimization (Journel and Isaaks, 1984) and financial risk analysis (Ravenscroft, 1992). In mining, it can also be used to assess the variability of the spatial distribution of the mineralization, risk sensitivity analysis in the mine planning process and effect of block size on ore variability. During mine production, conditional simulation can be used for reconciliation by comparing predicted grades in the resource model with the actual grades sampled at the process plant. In geostatistical simulation, a system of model realizations is generated that presents a range of possibilities (Vann et al., 2002).

Conditional simulation generates maps of the grades of mineralization at a point honoring the sample grades' histogram, the variogram or spatial continuity of the sample grades as well as the grades at sample locations (Deutsch and Journel, 1992). The calculated histogram and variogram of simulated values and sample grades should be identical. Typically, small-scale ore variation is what causes ore misclassification in mining, and it is critical that these small-scale variations are repeated such as in conditional simulation because they affect the ore-waste selection process. Conditional simulation has been implemented in mineral resource estimation to correct the smoothing effect of other techniques like Ordinary Kriging (Matheron, 1971). As an estimation technique, it creates a pattern of values with statistical and spatial characteristics similar to true grades which minimizes the smoothing of the data. In contrast to other estimation techniques, the uncertainty attached to each estimate can be known when conditional simulation has been used. In

the simulation of production, a spatial model can be used for simulation purposes rather than estimation when assessing the uncertainty attached to the prediction of a variable or when realistic scenarios are required for post-processing algorithms (Rambert, 2005). Simulation generates a series of realistic outcomes that have equal probabilities, with each outcome honoring the input data, the spatial model and also the distribution model as compared to kriging. Simulation replicates the statistical and geostatistical characteristics of variability such as the histogram and variogram of the data (Vizi, 2008). Consequently, geostatistical simulation generates a set of values, forming one of infinite possible realizations that have simulated values with the same model of variogram as the experimental one and simulated values with the same distribution as the experimental ones. It is not uncommon for estimates to have some error or uncertainty since predictions can be inaccurate. Some of the errors can be due to the use of widely spaced data, geological variability, approximations made in the estimation process and the limitations of the models used. Apart from resource estimation, conditional simulation is considered the best option in predicting uncertainty for estimates since the uncertainty can be predicted at different scales by simply averaging up the simulated values (Rossi and Deutsch, 2013).

In addition, a set of simulated realizations obtained by conditional simulation can be used to provide a model of uncertainty at each location. The model can be analyzed by predicting uncertainty at locations where there is data from drill holes or previous production data. Rossi and Deutsch (2013) further stated that probability intervals can then be created by counting the number of times the true values fall within those intervals, thus determining if the predicted percentage is verified. The uncertainty model depends on the random function model used and can be used to characterize risk. In grade control, risk analysis is used in making economic decisions through evaluating the consequence of grade uncertainty, and the best choice is considered based on the maximum profit or minimum loss choice. Furthermore, the uncertainty model as described in the findings provides all the information needed to optimize decision-making under uncertainty (Rossi and Deutsch, 2013). Techniques such as ordinary kriging does not allow the confidence interval to be calculated using its variance. However, simulations can be used to build confidence intervals empirically since many variable simulations are calculated, and access to a complete empirical distribution is provided. This allows an assessment of the probability that a given variable will take a value that belongs to a given interval (Vann et al., 2002).

The main aim is to reproduce the variance of the input data, both in a univariate sense and spatially through the histogram and the variogram or another covariance model, respectively. Conditional simulation offers an effective platform to study any variability problem in situations where other techniques such as ordinary kriging cannot be applicable (Vann et al., 2002). Sinclair and Blackwell (2002) have summed up the technique by generating a number of realizations belonging to the same spatial location. These individual simulations at each point give a probability distribution of the grade at that specific point. These distributions are then used in several cases where one of its application is to estimate the probability of the grade that falls above a certain cut-off grade at any particular point. Some of the simulation techniques used in the mining industry are turning bands, sequential Gaussian simulation, and sequential indicator simulation.

2.9.3 Sequential Gaussian simulation

There are several simulation algorithms, but one of the simplest and most commonly used is Sequential Gaussian Simulation (SGS). Kriging provides an approximation of a variable's mean and standard deviation at a point, so the variable can be interpreted as a random variable following a normal distribution at each point. SGS utilizes a random variance from the normal distribution chosen as an approximation according to a uniform random number representing the degree of probability rather than the mean (Bohling, 2005). SGS is an algorithm that sequentially simulates nodes after each other and then uses simulated values as conditioning data. It is necessary to note that standard Gaussian values must be used in the SGS method; thus, the data is converted into Gaussian space. SGS produces standard random variables and performs a Gaussian transformation of the data. The simulated value at the visited point is randomly drawn from the conditional cumulative distribution function, defined by the kriging mean and variance, based on neighborhood values. At a new randomly visited point, the simulated value depends on the original data and previously simulated values. Finally, the simulated normal values are back transformed to the simulated values for the original variables. The process is repeated until all points are simulated for each realization throughout the grid (Deutsch and Journel, 1992). For this research, the SGS technique was carried out using the steps outlined by Deutsch and Journel (1992) to generate a successful geostatistical conditional realization. An illustrated summary of the basic steps in SGS is shown in Figure 2-3.

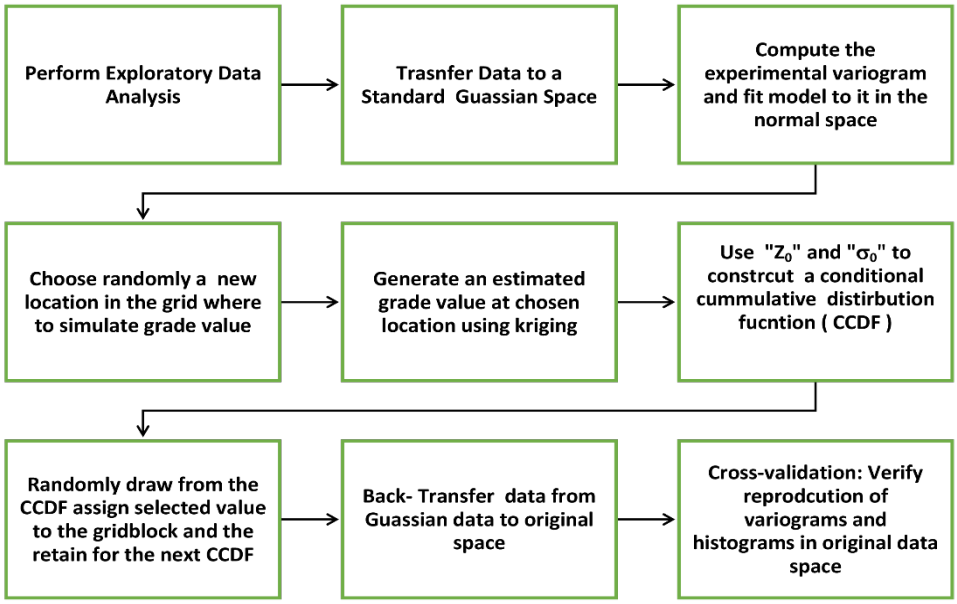

Figure 2-3: Basic steps in the SGS algorithm modified after Deutsch and Journel (1992)

The exploratory data analysis, which is conducted in the first step, relates to conventional first-order statistics including the central tendency such as the declustered mean, mode, median and ore grade distribution histograms. The study of spatial data analysis is denoted by the measurement of the variograms that are used at a later stage along with histograms of data as an element for cross-validation of the generated simulations.

2.10 Comparison between conditional simulation and ordinary kriging

Conditional simulation and Ordinary Kriging are geostatistical techniques that are applicable in the mining industry for resource assessment purposes. Both techniques use variograms in determining the spatial relationship of data. Estimation errors due to kriging variance and block variance are reported along with the kriged estimates. The kriging variance forms the basis for conditional simulation. While both techniques are used to provide estimates, their primary purposes are distinct. Ordinary kriging seeks to provide specific estimates that minimize the local estimation variance, while the purpose of conditional simulation is to provide a set of maps showing the grade, which honors the known reality and is likely to represent unknown reality at any location (Schofield and Rolley, 1997). Its ability to represent the spatial continuity of high grades and local variability of grades makes conditional simulation a superior technique compared to Ordinary Kriging. Whilst these techniques are commonly used for resource estimation, conditional simulation, unlike Ordinary Kriging, also provides a direct way of quantifying

uncertainty in knowing the true grade of a mining unit attached to an estimate in the form of a histogram. Since conditional simulation produces multiple simulations of the deposit grade, there is direct access to a histogram of the possible grade for any particular point in the deposit.

In addition, each value generated is independent of other values generated in other simulations and is more likely to be the true grade value at that specific location. Conditional simulation is thus a superior technique compared with Ordinary Kriging in defining the local variability of grade mineralization by direct access to the grade distribution through the average of simulated values. While kriging variance can be used to provide a definition of uncertainty, it requires assumptions about the shape of the potential grade histogram which are not practically realistic. Another drawback of kriging variance is that, it suggests that the estimation variance is independent of the magnitude of the grade being estimated, as compared to other mineral deposits having a variability of grades directly related to the magnitude of the grade (Schofield and Rolley, 1997). This becomes a limitation of using kriging variance to describe uncertainty in knowing the true grade of a mining unit. It does not permit the evaluation of the risk associated with the resource. Simulations aims to sample at an unknown location using constraints such as statistical moments of data; thus, the criteria for stationarity are stricter than those for kriging. Although Ordinary Kriging is considered the best linear unbiased estimator, the smoothing effect it has on estimates is one of the key drawbacks of the technique. Local variability is not retained due to smoothing and does not replicate well the histograms of the original data, as opposed to conditional simulation (Asghari et al., 2009). Conditional simulation honors the original data, has less of a smoothening impact on estimates and retains local variability. However, with the sampling density, such as that of blast-hole data, maps from both Ordinary Kriging and Conditional Simulation gradually become closer to the true map, since they both honor the sample values at sample locations (Deutsch and Journel, 1992; Asghari et al., 2009).

2.11 Effect of grade uncertainty in mine production planning and optimization

One of the most important economic criteria, as stated by the AER, used to separate ore from waste material is the cut-off grade (COG) (Regulator, 2016). The COG controls the movement of quantities of materials among mining, processing and refining stages of an open pit mining operation. The flow of the materials in the mining operation relies on the grade-tonnage curve. If the COG is determined to be too low, it will result in increasing the life of the operation with no economic justification. On the other hand, if the COG is set too high, it will result in the waste of

some valuable materials (Bascetin and Nieto, 2007). The variation in the grade tonnage curve is unavoidable, and it is therefore crucial that the methods that define the COG policy must abide the consideration of grade uncertainty. The fundamentals of COG were introduced by Lane (1988). Two theoretical COG thresholds were introduced: (i) Marginal COG and (ii) Breakeven COG. The marginal COG was considered as the critical grade that is high enough to pay for the cost of processing the mined material, even if it does not pay for the cost of mining that material completely. Meanwhile the breakeven COG is considered a critical grade that is high enough to pay for both the cost of processing and mining of that material. The results of a simple break-even computation can generate a constant COG schedule for the life of mine within a pre-defined pit limit (Taylor, 1972; Lane, 1988). However, it has been reported that a break-even calculation cannot maximize the NPV of the mining project since it ignores the variability in the geology of the mineral deposit and the operational constraints (Taylor, 1972; Hall et al., 1982).

During prefeasibility studies of ore resource estimation, the unavailability of sufficient data creates uncertainty, and, if ignored, this can impact the mining operation. Uncertainty can be quantified using geostatistical conditional realizations. The grade uncertainty can impact the mine production plan in two different ways: (1) it could cause variations in the tonnages of ore that is sent to the mill for processing and (2) the expected EBV from all realizations could be different than that calculated from the average grade (Koushavand et al., 2014). For example, in Figure 2-4 the estimated copper grade value for *block i* denoted by *g* for a single estimated block and seven conditionally simulated estimated blocks of synthetic data are shown, where *B* is the block tonnage in tonnes, *r* is the metal recovery in percent, *pr* is the copper price, and *mc* and *pc* are mining and processing costs, respectively. Assuming the economic parameters of the synthetic data are B = 18650 tonnes, r = 90%, pr = \$4030/tonne, mc = \$2/tonne and pc = \$6/tonne and the marginal COG is 0.23%Cu, if the economic block value of *block i* is computed using only the single estimated block grade, the EBV becomes -\$69,381 if that block is extracted as ore and -\$37,300 if that block is extracted as waste. Since the block estimate lies below the marginal COG, the mine optimizer will economically consider the block as waste and will mine it as waste with an EBV of -\$37,300. In the simulated model, some of the block grade estimates in the realizations seem to be above the marginal COG and could be considered as ore block in such scenarios. If the EBV for *block i* in each realization is computed, the EBV will become \$26,673, -\$37,300, -\$37,300, \$87,552, \$35,467, -\$37,300, and -\$37,300 for Realizations 1, 2, 3, 4, 5, 6, and 7, respectively. Thus, the

expected EBV for *block i* will become $70.29. Under a stochastic approach, a stochastic optimizer will choose to mine this block with an expected value of $70.29 as an ore block. However, sending this particular block to feed a mill up to its capacity might be risky and would require stochastic optimization to minimize this risk in addition to its potential value.

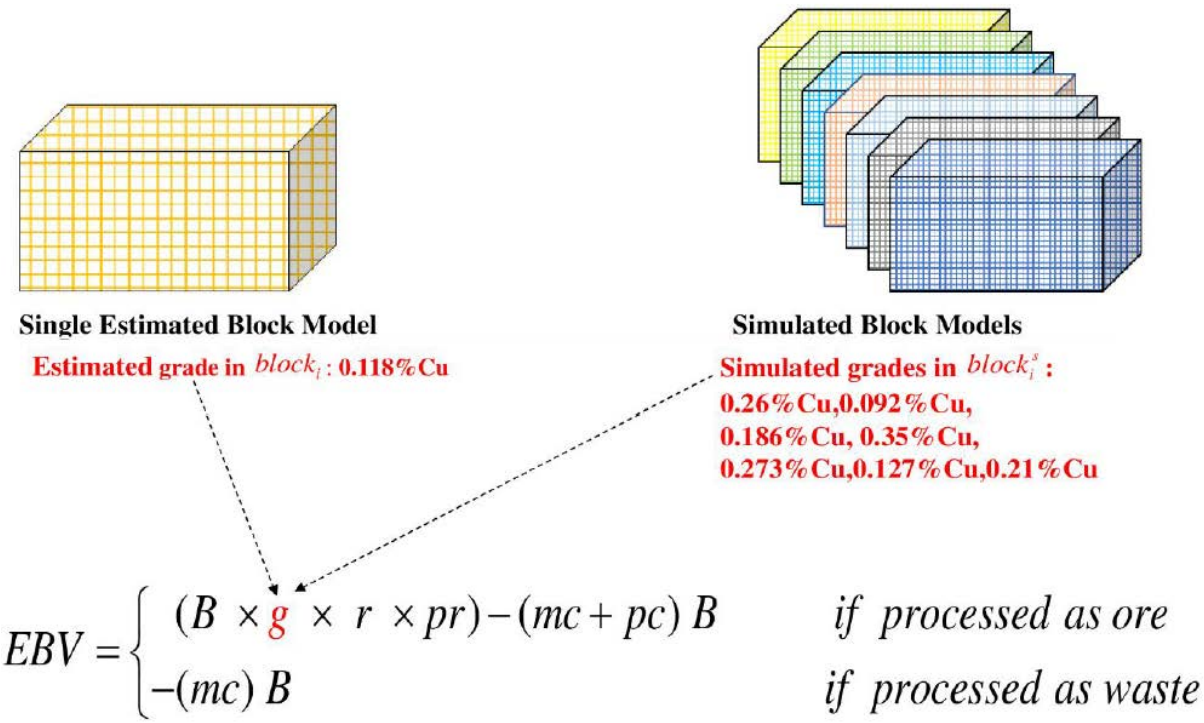

Figure 2-4: EBV computations using a single estimated block model versus simulated block models of a synthetic data

The average EBV of the simulated model is higher than the single estimated model. This means that even for a block with an average grade above the marginal COG, the average EBV from simulations could be less than zero, and so not economic to be processed. This is due to the nonlinearity of the EBV calculation. Therefore, with the methods that maximize NPV based on a single block model, this block is assumed to be waste and will not be processed. Grade uncertainty has two main effects on long-term production planning: (1) it may cause deviations from production targets. This can be calculated from the simulated realizations by taking into account the grade variation and the input COG; and (2) grade uncertainty has a direct effect on the EBV calculation and consequently on the NPV of the project. Thus, in this research, the model that is developed is designed to account for the risks of not meeting the production targets as well as maximizing the NPV.

2.12 Summary and conclusions

Integrated strategic long-term mine plans define the complex strategy of the displacement of ore, waste, overburden, and tailings over the mine life. The objective of long-term mine plans is to minimize the environmental footprint and maximize the cash flow. Limitations of space because of lease conditions, the scale of operations and the construction of external and in-pit dyke impoundments add to the complexity of planning in oil sands mining. Oil sands long-term mine plans are driven by the quantity and quality of tailings produced downstream. Oil sands mine planning and waste management face limitations such as the risks and uncertainties associated with the tailings process and relying on conventional models.

Stochastic integer programming models offer a framework to address uncertainty in key inputs of mine production schedules in areas such as grade and rock type uncertainties. Mathematical formulations based on stochastic integer programming have been shown to generate the optimal production schedule while considering grade uncertainty (Ramazan and Dimitrakopoulos, 2018). Hence, its application in mine planning will allow the management of uncertainties associated with ore grades and waste disposal.

CHAPTER 3

THEORETICAL FRAMEWORK

3.1 Background

This chapter focuses on the theoretical framework for optimizing an integrated long-term production schedule for open pit oil sands mining. The goal of this research is to develop an integrated mine planning optimization framework using stochastic mixed integer linear programming (SMILP) model that will incorporate bitumen grade uncertainty including decisions on material extraction and waste management. The challenge of bitumen grade uncertainty is met by the execution of a sequence of steps beginning with the collection of drillhole samples aimed at providing the largest possible amount of information regarding the orebody's composition, concentration and geometry. However due to the typically large dimensions of viable deposits and the monetary and time expenditures associated with exploration work, actual sampling of the orebody can never be exhaustive. Thus, given the relatively small number of sample data collected at the exploration stage, geologists are then forced to cope with limited information in building geological models of the mineral deposit. Building of such models necessarily entail estimating the grades at locations previously not sampled which results in significant degree of uncertainty. In order to fully understand the concept of ore grade uncertainty, it is necessary to recognize the theoretical concepts that are associated and relevant to geostatistical techniques which are applicable to ore resource estimation. Geostatistical techniques provide a framework that allows the geologist to estimate geological attributes such as ore grades, rock type and porosity.

In this chapter, a methodology is presented on how the sets of multiple equally probable orebody realizations are generated. The procedural methodology ensures the accurate modelling of the ore grades that are under consideration to be able to quantify and verify the uncertainty surrounding the ore grades estimates for decision making process. The formulation and implementation of a stochastic mixed integer linear programming model for production scheduling optimization while considering grade uncertainty is then elaborated in detail. The objective function of the stochastic model is to maximize NPV while minimizing dyke construction cost and fluctuations in the processed ore grades and tonnages.

3.2 Procedural steps of the proposed methodology

Grade uncertainty in a long-term production plan has been reported to affect the NPV of a mining project due to the variability between the actual and planned production targets especially in the early years of mine life (Osanloo et al., 2008). Hence, to address the issue of grade uncertainty, instead of using a single estimated block model for production scheduling, twenty simulated realizations, that are representative of grade variability were used as input to the SMILP model. A conventional model which was based on Ordinary Kriging estimation was considered as the base case model. Both models including the E-type model were then compared to demonstrate the impact of grade uncertainty on the integrated mine production scheduling and waste management plan. Figure 3-1 shows a summary of the procedural steps used in the proposed methodology. The workflow used in this research to generate an oil sands production schedule in the presence of grade uncertainty from the SMILP framework are as follows;

1. Create an oil sands geologic and economic block models from the given oil sands drill hole data sets using Geovia GEMS software (Geovia Dassault Systems, 2018). This first block model is estimated using Ordinary Kriging and considered as the base case model of this research.
2. Implement geostatistical modelling to generate multiple realizations of the oil sands block model using Sequential Gaussian Simulation (SGS) algorithm to map out ore bitumen grade uncertainty in the blocks. In this step, Geostatistical Software Library also known as GSLIB will be used (Deutsch and Journel, 1998).
3. Determine the final economic pit limits of the OK block model (base case block model), the E-type block model, and the stochastic block models using 3D LG algorithm in Geovia Whittle software (Geovia Dassault Systems, 2018). The pit limits generated for each block model are verified and validated by ensuring that the total tonnages (including ore and waste) are consistent for all block models. These pit limits can be used to investigate the uncertainty in the final pit limit tonnages but that is not the focus of this research.
4. Export the blocks contained in the final economic pit limits of the OK block model into Geovia GEMS software and design the final pit outline guided by the OK pitshell from Geovia Whittle software. This becomes our reference final pit design
5. Select the sets of blocks within the reference designed final pit for the OK, E-type and stochastic block models. Save the selected blocks in ASCI file format.

6. For the case study, define the input scheduling parameters to be used by the SMILP model. For each of the OK and E-type scenarios, the number of input block model is one while for the stochastic scenario, the number of input block models are the twenty SGS realizations.
7. Import the files obtained in Step 5 and implement the developed mathematical formulations in MATLAB. IBM/CPLEX is used as the solver for the optimization problem (Mathworks, 2018; ILOG, 2019).
8. Perform comparative and sensitivity analysis based on the schedule results from the optimization run.

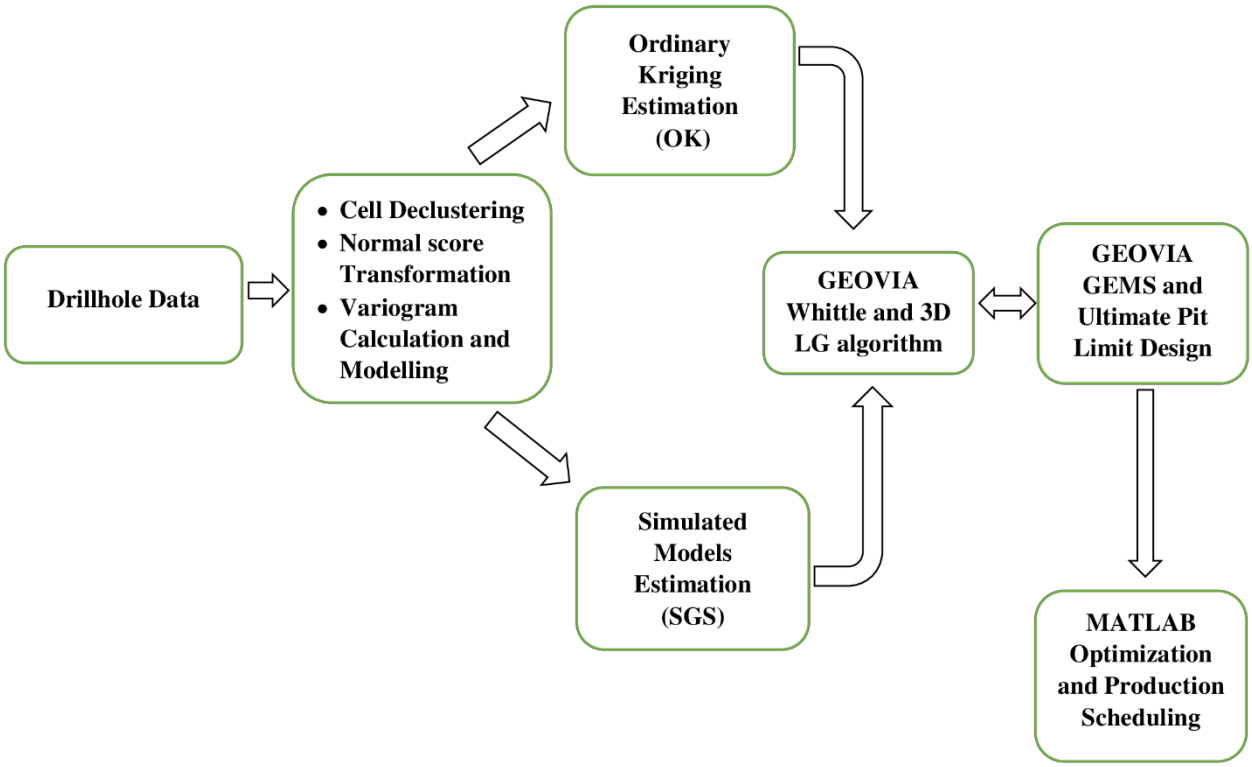

Figure 3-1: Summary of procedural steps in the proposed methodology

3.3 Conceptual oil sands mining and waste management system under grade uncertainty

The primary purpose of oil sands mine planning and waste management is to provide processable ore blends from the mine pit to the processing plant and contain the tailings in-pit through maximization of the net present value and minimization of dyke construction cost (Boratynec, 2003; Azam and Scott, 2005; Ben-Awuah and Askari-Nasab, 2013). A schematic view of a conceptual oil sands mining and waste management system can be seen in Figure 3-2

A conceptual oil sands mining model consists of an oil sands deposit which is to be extracted coupled with an in-pit waste management strategy. The first stage in any typical oil sands mining operation is to remove approximately 30 m of overburden material before the oil sand ore can be mined. Each oil sands mining block is made up of ore, overburden and interburden dyke material also known as OI dyke material, and waste. Overburden material and the oil sands ore are mined using large shovels. The completely mined out areas can be concurrently used as in-pit tailings storage areas as directional mining progresses and the in-pit tailings dyke footprints are released. Mined waste which includes overburden and interburden material is hauled to waste dump. Suitable waste however is used to construct facilities such as tailings dykes. Some of the mined waste are unsuitable for construction because they are too wet or contain undesirable materials such as clays.

The effect of grade uncertainty on mining operations contributes to unsustainable waste management planning. This could result in environmental challenges leading to major financial liabilities and mine closure by regulatory agencies. If grade uncertainty is not considered in oil sands mine planning, there may be excess waste produced than the waste management plan can handle or an over-design of the waste management system to handle less waste than planned. This will result in lost opportunities in terms of waste management cost. Also, the misinterpretation of materials due to grade uncertainty and variability associated with the amount of waste produced can impact both the stakeholders and profitability of the mining complex. Therefore, in the proposed SMILP framework, waste management will be incorporated in the optimization framework by first simulating equally probable stochastic orebody models to identify the underlying uncertainty and variability of the different material types. Secondly, each component in the conceptual mining system is optimized simultaneously to investigate the impact of grade uncertainty on the mine plan. The grade variability in the mined materials should be taken into consideration during production scheduling so as to avoid under- or over-representing the proportions of materials classified as either ore or waste. Thus, the material classified as waste can be represented accurately and the environmental impacts and waste management cost minimized globally.

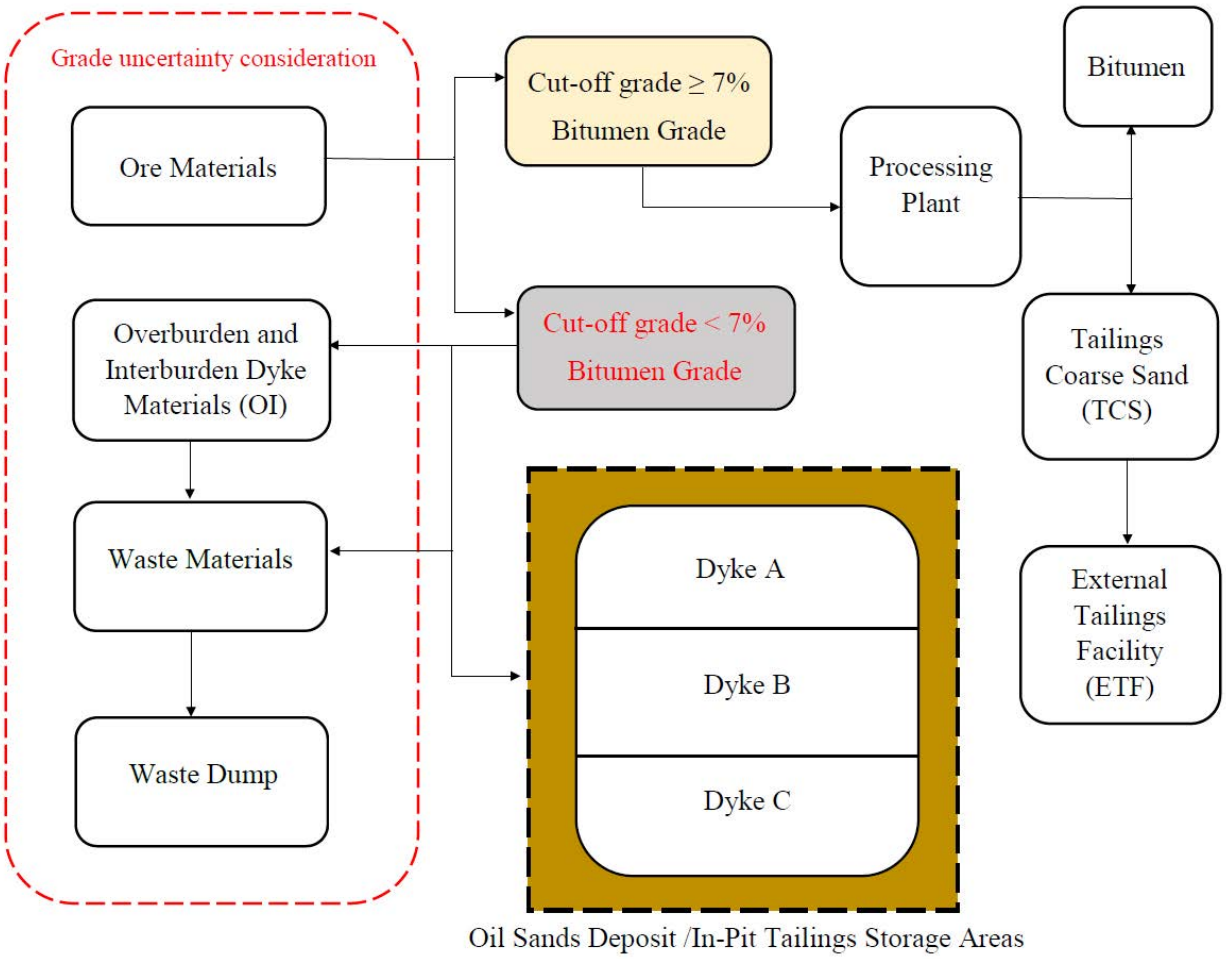

Figure 3-2: Conceptual mining and waste management system

3.4 Geostatistical modelling

Geostatistical modelling techniques are widely used to model uncertainty in any given geological data by creating conditional realizations. All the realizations are equally probable and can be considered as plausible representations of the geological complexity. Geostatistical Software Library also known as GSLIB (Deutsch and Journel, 1998) was used to implement Sequential Gaussian Simulation (SGS) for the given data. This software provides a comprehensive collection of geostatistical estimation and simulation algorithms. The rocktype modelling was done using Geovia GEMS software where polygons were created to distinguish between rocktype that were classified as ore or waste. In this research, rocktype MMF was classified as ore because it is the oil-bearing rock type containing bitumen grades while the OB rocktype was considered as waste because it is the topmost layer of a typical oil sands formation and materials from these layers are often used for road and dyke construction in the mine, depending on the soil properties and its mineral content (Masliyah et al., 2011). Figure 3-3 illustrates summary of steps for the

geostatistical modelling of the given datasets. Koushavand et al. (2014) presents detailed procedural steps on how to use GSLIB to implement SGS for a given data set.

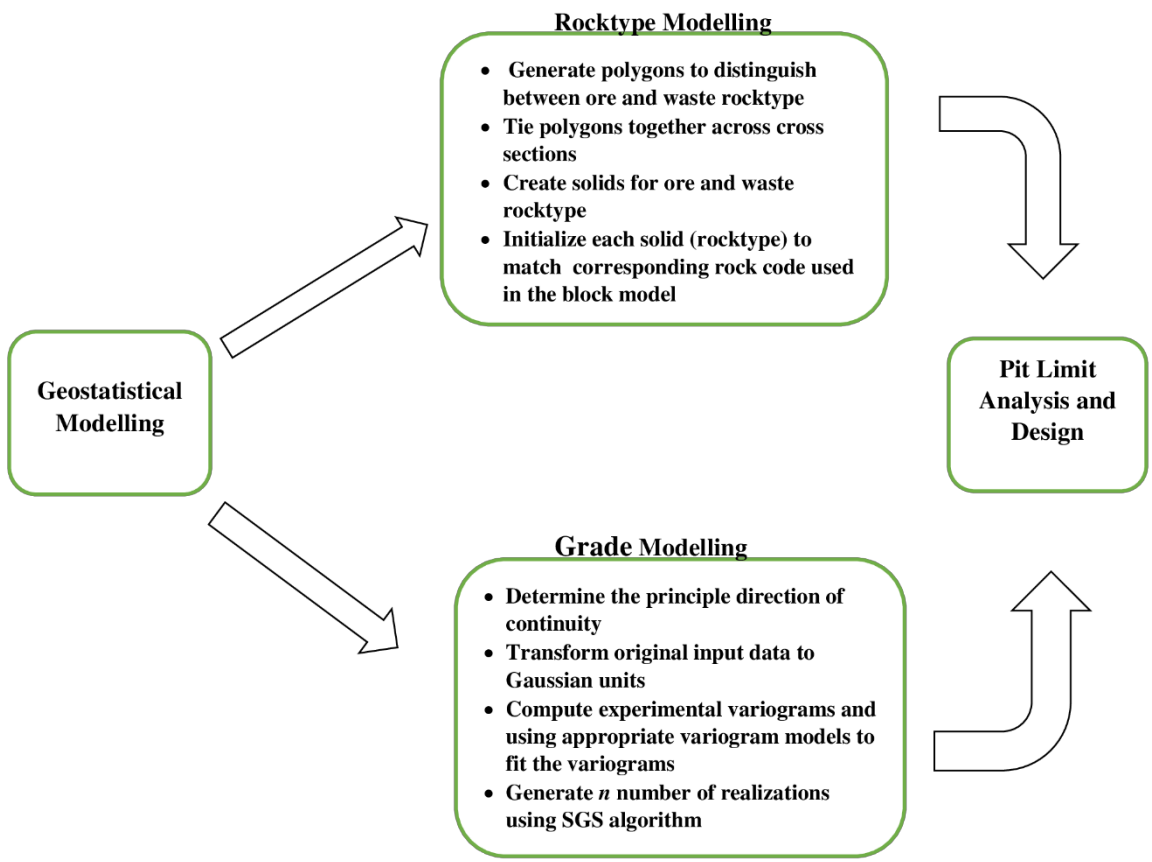

Figure 3-3: Summary of steps in geostatiscal modelling

3.5 Stochastic mixed integer linear programming (SMILP) formulation

The SMILP optimization framework for generating long-term production schedule in the presence of grade uncertainty was modelled using multiple realizations from Sequential Gaussian Simulation (SGS). In the optimization model, the objective function will proceed in three parts. The first part aims at maximizing the net present value. The second part aims at minimizing dyke construction cost for waste management. The third part, referred to as the cost of uncertainty, aims at minimizing the deviations/risks associated with ore tonnage and ore grade uncertainty. Thus, the costs associated with variations in ore tonnages and ore grades from the equally probable orebody realizations are included in the model to reduce the risk of not meeting planned production targets. The general form of the proposed SMILP formulation for this research is written as:

Maximize: Net Present Value – Dyke Construction Costs – Cost of Uncertainty

Subject to:

- *Mining Capacity Constraints*
- *Processing Capacity Constraints*
- *OI and TCS Dyke Material Capacity Constraints*
- *Ore Grade Blending Constraints*
- *Dyke Material Grade Blending Constraints*
- *Sand-to-fines Blending Constraints*
- *Variable Relations Constraints*
- *Mining Precedence Constraints*
- *Non-negativity Constraints*

The SMILP production schedule is subject to variety of technical, physical and economic constraints that will enforce the mining and dyke construction capacities, and blending requirements. The notations used in the formulation of the SMILP model have been classified as indices, sets, decision variables and parameters.

3.5.1 Indices

$a \in \{1,....,A\}$	index for possible mining locations (pits)
$e \in \{1,....,E\}$	index for the element of interest in each block
$j \in \{1,....,J\}$	index for phases (pushbacks)
$n \in \{1,....,N\}$	index for blocks
$s \in \{1,....,S\}$	index for realizations
$t \in \{1,....,T\}$	index for scheduling periods
$u \in \{1,....,U\}$	index for possible destinations for materials

3.5.2 Sets

$A = \{1,....,A\}$	set of all index for possible mining locations (pits) in the model
$J = \{1,....,J\}$	set of all the phases in the model
$N = \{1,....,N\}$	set of all the blocks in the model

$S = \{1,....,S\}$	set of all equally probable orebody realizations
$U = \{1,....,U\}$	set of all the possible destinations in the model
$D_n(J)$	for each block, $D_n(J)$ includes all the blocks that must be extracted prior to mining block n to ensure that block n is exposed for mining with safe pit slopes, where J is the total number of blocks in $D_n(J)$
$C_n(L)$	for each block, $C_n(L) \subset D_n(J)$ defines the immediate predecessor blocks that must be extracted prior to extraction of the block *n*, where *L* is the total number of blocks in $C_n(L)$

3.5.3 Decision variables

$a_n^t \in \{0,1\}$	binary integer variable controlling the precedence of extraction of blocks a_n^t which is equal to one if extraction of block *n* has started by or in period *t* and is equal to zero, otherwise
$x_n^{u,t} \in [0,1]$	continuous variable representing the ore portion of block *n*, that is to be extracted and processed at destination *u* in period *t*
$y_n^{a,t} \in [0,1]$	continuous variable representing the portion of block *n* to be mined in period *t* from location *a*, which includes both ore, overburden, interburden and waste from the associated blocks
$c_n^{u,t} \in [0,1]$	continuous variable representing the interburden dyke material portion of block *n* to be extracted and used for dyke construction at destination *u* in period *t*
$gdev_{s,-}^t$	continuous variable representing the shortage of the grade upper bound in period *t* for realization *s*
$gdev_{s,+}^t$	continuous variable representing the surplus of the grade lower bound in period *t* for realization *s*
$odev_{s,-}^t$	continuous variable representing the shortage of the ore tonnage upper bound in period *t* for realization *s*
$odev_{s,+}^t$	continuous variable representing the surplus of the ore tonnage lower bound in period *t* for realization *s*

$k_n^{u,t} \in [0,1]$	continuous variable representing the tailing coarse sand dyke material portion of block n to be extracted and used for dyke construction at destination u in period t
$l_n^{u,t} \in [0,1]$	continuous variable representing the overburden dyke material portion of block n to be extracted and used for dyke construction at destination u in period t

3.5.4 Parameters

$cb^{u,t}$	cost in present value terms per tonne of interburden dyke material for dyke construction at destination u
$ck^{u,t}$	cost in present value terms per tonne of overburden dyke material for dyke construction at destination u
$ct^{u,t}$	cost in present value terms per tonne of tailings coarse sand dyke material for dyke construction at destination u
$cm^{a,t}$	cost in present value terms of mining a tonne of waste in period t from location a
$cp^{u,e,t}$	extra cost in present value terms per tonne of ore for mining and processing at destination u
$cs^{e,t}$	selling cost of element e in present value terms per unit of product
$d_{n,s}^{u,t}$	discounted economic block value obtained by extracting block n and sending it to destination u in period t for realization s
$d_{n,s}$	overburden dyke material tonnage in block n of realization s
$dgeo$	geological discount rate
$f_{n,s}^{e}$	average percent of fines of element e in ore portion of block n for realization s
$\underline{f}^{u,t,e}$	lower bound on the required average fines percent of element e in period t at processing destination u
$\overline{f}^{u,t,e}$	upper bound on the required average fines percent of element e in period t at processing destination u
$f_{n,s}^{d}$	average percent of fines in interburden dyke material portion of block n of realization s

$\underline{f}^{u,t,d}$	lower bound on the required average fines percent of interburden dyke material in period *t* at dyke construction destination *u*
$\overline{f}^{u,t,d}$	upper bound on the required average fines percent of interburden dyke material in period *t* at dyke construction destination *u*
$g_{n,s}^{e}$	average grade of element *e* in ore portion of block *n* for realization *s*
$\underline{g}^{u,t,e}$	lower bound on the required average head grade of element *e* in period *t* at processing destination *u*
$\overline{g}^{u,t,e}$	upper bound on the required average head grade of element *e* in period *t* at processing destination *u*
$h_{n,s}^{u,t}$	discounted cost of mining tailings course sand dyke material in block *n* of realization *s* and in period *t* for construction at destination *u*
$i_{n,s}$	interburden dyke material tonnage in block *n* of realization *s*
$j_{n,s}$	tailings coarse sand dyke material tonnage in block *n* of realization *s*
$m_{n,s}^{u,t}$	extra discounted cost of mining all the material in block *n* of realization *s* and in period *t* as interburden dyke material for construction at destination *u*
$o_{n,s}$	ore tonnage in block *n* of realization *s*
$p_{n,s}^{u,t}$	extra discounted cost of mining all the material in block *n* of realization *s* and in period *t* as overburden dyke material for construction at destination *u*
$P^{e,t}$	price of element *e* in present value terms per unit of product
pc_{g-}^{t}	penalty cost for lower grade target deviation in period *t*
pc_{g+}^{t}	penalty cost for upper grade target deviation in period *t*
pc_{o-}^{t}	penalty cost for lower ore tonnage target deviation in period *t*
pc_{o+}^{t}	penalty cost for upper ore tonnage target deviation in period *t*
$q_{n,s}^{a,t}$	discounted cost of mining all the materials in block *n* in period *t* as waste from location *a* for realization *s*
r	interest rate

$r^{u,e}$	proportion of element *e* recovered if it is processed at destination *u*
$\underline{T_m}^{a,t}$	lower bound on mining capacity in period *t* at location *a* (tonnes)
$\overline{T_m}^{a,t}$	upper bound on mining capacity in period *t* at location *a* (tonnes)
$\underline{T_{pr}}^{u,t}$	lower bound on processing capacity in period *t* at destination *u* (tonnes)
$\overline{T_{pr}}^{u,t}$	upper bound on processing capacity in period *t* at destination *u* (tonnes)
$\underline{T_{od}}^{u,t}$	lower bound on overburden dyke material in period *t* at destination *u* (tonnes)
$\overline{T_{od}}^{u,t}$	upper bound on overburden dyke material in period *t* at destination *u* (tonnes)
$\underline{T_{id}}^{u,t}$	lower bound on interburden dyke material in period *t* at destination *u* (tonnes)
$\overline{T_{id}}^{u,t}$	upper bound on interburden dyke material in period *t* at destination *u* (tonnes)
$\underline{T_{td}}^{u,t}$	lower bound on tailing coarse sand dyke material in period *t* at destination *u* (tonnes)
$\overline{T_{td}}^{u,t}$	upper bound on tailing coarse sand dyke material in period *t* at destination *u* (tonnes)
$sf^{e}_{n,s}$	average ratio of sand-to-fines of element *e* in ore portion of block *n* for realization *s*
$\underline{sf}^{u,t,e}$	lower bound on the required average sand-to-fines ratio of element *e* in period *t* at processing destination *u*
$\overline{sf}^{u,t,e}$	upper bound on the required average sand-to-fines ratio of element *e* in period *t* at processing destination *u*
$w_{n,s}$	waste tonnage in block *n* of realization *s*
$v^{u,t}_{n,s}$	discounted revenue obtained by selling the final product within block *n* of realization *s* in period *t* if it is sent to destination *u*, minus extra discounted cost of mining all the material in block *n* as ore from location *a* and processing it destination *u*

3.5.5 Numerical modelling of economic block value

A parameter known as the Economic Block Value (EBV) is calculated for each block in each realization. The EBV of a block is the revenue generated by selling the final product less all the costs involved in extracting and processing the block. The mining cost of a block is a function of the distance between its location and its final destination. Since the long-term production plan is a multi-period optimization problem and blocks are extracted in different periods, a discount rate is applied to calculate the present value of the EBV, the revenue and the costs. Therefore, the Discounted Economic Block Value (DEBV) for the stochastic model is calculated using Equation (3.1).

$$d_{n,s}^{u,t} = v_{n,s}^{u,t} - q_{n,s}^{a,t} - p_{n,s}^{u,t} - m_{n,s}^{u,t} - h_{n,s}^{u,t} \tag{3.1}$$

The parameters stated in Equation (3.2) to Equation (3.6) are used to calculate the Discounted Economic Block Value (DEBV).

$$v_{n,s}^{u,t} = \sum_{e=1}^{E} o_{n,s} \times g_{n,s}^{e} \times r^{u,e} \times (P^{e,t} - cs^{e,t}) - \sum_{e=1}^{E} o_{n,s} \times cp^{u,e,t} \tag{3.2}$$

$$q_{n,s}^{a,t} = (o_{n,s} + d_{n,s} + i_{n,s} + w_{n,s}) \times cm^{a,t} \tag{3.3}$$

$$p_{n,s}^{u,t} = d_{n,s} \times ck^{u,t} \tag{3.4}$$

$$m_{n,s}^{u,t} = i_{n,s} \times cb^{u,t} \tag{3.5}$$

$$h_{n,s}^{u,t} = j_{n,s} \times ct^{u,t} \tag{3.6}$$

The discounted revenue shown in Equation (3.2) is the present value of the ore minus the cost of processing the ore. The discounted cost of mining all the materials such as ore, overburden, interburden and waste is shown is represented in Equation (3.3). The extra discounted cost of mining materials such as overburden, interburden, and tailing coarse sands for the purpose of dyke construction for waste management are represented by Equation (3.4) to Equation (3.6) respectively.

3.5.6 SMILP model objective function

The objective function for the SMILP model for integrated long-term production planning and waste management is formulated in three main parts: 1) maximizing the net present value of the mining operation (Equation (3.7)), 2) minimizing the dyke construction cost for the waste management plan (Equation (3.8)), and 3) minimizing the uncertainty cost associated with

deviating from the operating targets, including ore grade and ore tonnage deviations (Equations (3.9) and (3.10)). These control variabilities from grade targets and ore tonnage targets.

$$Max \frac{1}{S}\sum_{s=1}^{S}\sum_{t=1}^{T}\sum_{a=1}^{A}\sum_{u=1}^{U}\sum_{n=1}^{N}\left(\frac{v_{n,s}^{u,t}\times x_{n}^{u,t}-q_{n,s}\times y_{n}^{a,t}}{(1+r)^{t}}\right) \qquad \textit{Part 1} \tag{3.7}$$

$$Min \frac{1}{S}\sum_{s=1}^{S}\sum_{t=1}^{T}\sum_{a=1}^{A}\sum_{u=1}^{U}\sum_{n=1}^{N}\left(\frac{p_{n,s}^{u,t}\times l_{n}^{u,t}+m_{n,s}^{u,t}\times c_{n}^{u,t}+h_{n,s}^{u,t}\times k_{n}^{u,t}}{(1+r)^{t}}\right) \qquad \textit{Part 2} \tag{3.8}$$

$$Min \frac{1}{S}\sum_{s=1}^{S}\sum_{t=1}^{T}\left(\frac{pc_{o+}^{t}\times odev_{s,+}^{t}+pc_{o-}^{t}\times odev_{s,-}^{t}}{(1+dgeo)^{t}}\right) \tag{3.9}$$

$$Min \frac{1}{S}\sum_{s=1}^{S}\sum_{t=1}^{T}\left(\frac{pc_{g+}^{t}\times gdev_{s,+}^{t}+pc_{g-}^{t}\times gdev_{s,-}^{t}}{(1+dgeo)^{t}}\right) \tag{3.10}$$

Part 3

Equation (3.7) to Equation (3.10) can be combined as a single objective function as shown in Equation (3.11)

$$Max \frac{1}{S}\sum_{s=1}^{S}\sum_{t=1}^{T}\sum_{a=1}^{A}\sum_{u=1}^{U}\sum_{n=1}^{N}\left(\begin{array}{l}\left(\frac{v_{n,s}^{u,t}\times x_{n}^{u,t}-q_{n,s}\times y_{n}^{a,t}}{(1+r)^{t}}\right)- \\ \left(\frac{p_{n,s}^{u,t}\times l_{n}^{u,t}+m_{n,s}^{u,t}\times c_{n}^{u,t}+h_{n,s}^{u,t}\times k_{n}^{u,t}}{(1+r)^{t}}\right)- \\ \left(\frac{pc_{o+}^{t}\times odev_{s,+}^{t}+pc_{o-}^{t}\times odev_{s,-}^{t}}{(1+dgeo)^{t}}\right)- \\ \left(\frac{pc_{g+}^{t}\times gdev_{s,+}^{t}+pc_{g-}^{t}\times gdev_{s,-}^{t}}{(1+dgeo)^{t}}\right)\end{array}\right) \tag{3.11}$$

In the objective function, the SMILP model consists of continuous and binary decision variables. Other than the *gdev* and *odev* variables, the remaining continuous decision variables can take any value between 0 and 1, while the binary variable value can take either 0 or 1. The binary variable a_n^t controls the precedence of block extraction while the continuous variables such as $x_n^{u,t}$, $y_n^{a,t}$, $l_n^{u,t}$, $c_n^{u,t}$, $k_n^{u,t}$ control the portion of blocks that is to be extracted. Other continuous decision variables like *gdev* and *odev* provide range of acceptable deviation from ore grade and ore tonnage targets to minimize the financial risk of not meeting the production targets.

In Part 1 of the SMILP model objective function, there are two decision variables for each block *n*. These decision variables are $x_n^{u,t}$ and $y_n^{a,t}$. The first decision variable $x_n^{u,t}$ represents the portion of block *n* that is to be processed (if it is ore) in period *t* while the second decision variable $y_n^{a,t}$ represents the portion of block *n* that is to be extracted in period *t*. By using two different decision variables for extraction and processing of each block, the optimizer decides whether a block should be processed or sent to the waste dump. Therefore, cut-off grade is implicitly implemented in the optimization process. By using two decision variables, it is possible to generate a schedule that may send low quality ore blocks located on upper benches to waste dump (or a low-grade stockpile), in order to gain access to high-quality ore blocks on the lower levels. This generates more revenue in early periods of the mine life increasing the total profit from the project. In the objective function, the profit from each block is always a function of the grade of the block. In earlier versions of long-term optimization models, cut-off grade was predetermined and fixed prior to optimization. Therefore, the destinations of the blocks were decided by a static cut-off grade, and in the optimization process only the period and portion of extraction was determined.

In Part 2 of the SMILP objective function, the aim is to minimize the cost of dyke materials extraction for dyke construction in line with waste management practices in oil sands mining. Continuous decision variables $l_n^{u,t}$, $c_n^{u,t}$, $k_n^{u,t}$ are used to respectively control the overburden, interburden and tailing coarse sand dyke material portions of a block extracted for dyke construction. To integrate grade uncertainty modelled with SGS realizations, continuous deviation variables $gdev_{s-}^t$, $gdev_{s+}^t$, $odev_{s-}^t$ and $odev_{s+}^t$ are introduced in Part 3 of the SMILP objective function, with their respective penalties pc_{g-}^t, pc_{g+}^t, pc_{o-}^t and pc_{o+}^t for managing deviations from ore grade and ore tonnage production targets. Also, a geological discount rate *(dgeo)* is applied to the cost of deviation to defer the risk of not meeting production targets to later periods. The application of the geological discount rate *(dgeo)* causes the objective function to be penalized to the extent that the deviations from the set targets in the early years is smaller compared to that in later years. Therefore, earlier periods have higher penalty compared to later periods. The higher penalty in the earlier periods forces the optimizer to reduce deviations from the set target early in the mine life and delay the extraction of uncertain areas to later periods when more geological knowledge of the deposit becomes available.

3.5.7 SMILP model constraints

3.5.7.1 Mining capacity constraints

The tonnage of material mined is equal to the sum of ore tonnage, waste tonnage and overburden-interburden dyke material tonnages. The total material mined may not be extracted all together in a specific period, however the fractional extraction of material has been made possible using decision variable $y_n^{a,t}$. The mining capacity constraints are represented by Equation (3.12) and Equation (3.13).

$$\sum_{n=1}^{N}\left(o_{n,s}+d_{n,s}+i_{n,s}+w_{n,s}\right)\times y_n^{a,t}\leq \overline{T_m}^{a,t} \tag{3.12}$$

$$\sum_{n=1}^{N}\left(o_{n,s}+d_{n,s}+i_{n,s}+w_{n,s}\right)\times y_n^{a,t}\geq \underline{T_m}^{a,t} \tag{3.13}$$

Equation (3.12) ensures that the total amount of material mined in each period does not exceed the targeted maximum capacity of equipment. Equation (3.13) ensures that the minimum amount of material that needs to be mined is achieved. The constraints are controlled by the continuous decision variable $y_n^{a,t}$.

3.5.7.2 Processing capacity constraints

Equation (3.14) and Equation (3.15) control the mill feed or processing capacity. These constraints are used to achieve an overall mine-to-mill integration during the mine production scheduling by providing a uniform feed throughout the mine life. Equation (3.14) and Equation (3.15) are at block level, which means decisions are made based upon the tonnage of ore above the cut-off grade for individual blocks. In practice, the processing capacity constraints must be set within tight upper and lower bounds to provide a uniform feed to the mill. Based on the shape of the orebody and distribution of ore grades, these constraints might not be satisfied under some circumstances, which will lead to an infeasible problem. Pre-stripping could be achieved by setting the upper and lower bounds of processing capacity constraints equal to zero for the desired periods. This approach will enforce the optimizer to mine only waste blocks in the early periods. The decision variables $odev_{s-}^{t}$ and $odev_{s+}^{t}$ in Equation (3.14) and Equation (3.15) are used as buffers to allow for deviations but are penalized in the objective function.

$$\sum_{n=1}^{N}\left(o_{n,s}\times x_n^{u,t}\right)-odev_{s,+}^{t}\leq \overline{T_{pr}}^{u,t} \tag{3.14}$$

$$\sum_{n=1}^{N}\left(o_{n,s}\times x_n^{u,t}\right)+odev_{s,-}^{t}\geq \underline{T_{pr}}^{u,t} \tag{3.15}$$

3.5.7.3 Dyke material capacity constraints

In the proposed model, the overburden-interburden dyke materials are used for dyke construction. Equation (3.16) to Equation (3.21) represent the dyke material constraints used to control dyke construction scheduling. These constraints are used to schedule the required dyke materials for all dyke construction destinations. The TCS dyke material generated from the processing plant is directly dependent on the mill feed. Thus, the schedules generated from the SMILP model give the mine planner good control over dyke material and provides an effective dyke construction and tailings storage management plan. Movement of dyke material for dyke construction can be well integrated into the mining fleet management plan and timely tailings containment areas can be created for the storage of fluid fine tailings. In oil sands mining, being able to efficiently plan the waste management strategy results in a more profitable and sustainable operation.

$$\frac{1}{S}\sum_{s=1}^{S}\sum_{n=1}^{N}d_{n,s}\times l_n^{u,t}\leq \overline{T_{od}}^{u,t} \tag{3.16}$$

$$\frac{1}{S}\sum_{s=1}^{S}\sum_{n=1}^{N}d_{n,s}\times l_n^{u,t}\geq \underline{T_{od}}^{u,t} \tag{3.17}$$

$$\frac{1}{S}\sum_{s=1}^{S}\sum_{n=1}^{N}i_{n,s}\times c_n^{u,t}\leq \overline{T_{id}}^{u,t} \tag{3.18}$$

$$\frac{1}{S}\sum_{s=1}^{S}\sum_{n=1}^{N}i_{n,s}\times c_n^{u,t}\geq \underline{T_{id}}^{u,t} \tag{3.19}$$

$$\frac{1}{S}\sum_{s=1}^{S}\sum_{n=1}^{N}j_{n,s}\times k_n^{u,t}\leq \overline{T_{td}}^{u,t} \tag{3.20}$$

$$\frac{1}{S}\sum_{s=1}^{S}\sum_{n=1}^{N}j_{n,s}\times k_n^{u,t}\geq \underline{T_{td}}^{u,t} \tag{3.21}$$

3.5.7.4 Ore grade blending constraints

The ore grade blending constraints control the grade of ore bitumen and ore fines in the mined material for the processing plant. The ore feed must follow pre-determined criteria to be acceptable for processing. The most important criteria in bitumen processing plants are the head grade of bitumen and fines in the ore feed. Ore blending constraints control the average grades and

guarantee that all the material entering the bitumen extraction process has a minimum acceptable grade of bitumen and a maximum acceptable grade of fines. The blending constraints for bitumen and fines are shown in Equation (3.22) to Equation (3.25) respectively. The shortage variable $gdev_{s-}^{t}$ and excess variable $gdev_{s+}^{t}$ of the average grade of bitumen are penalized in each period in the objective function.

$$\sum_{n=1}^{N} g_{n,s}^{e}\left(o_{n,s}\times x_{n}^{u,t}\right)-\sum_{n=1}^{N}\overline{g}^{u,t,e}\left(o_{n,s}\times x_{n}^{u,t}\right)-gdev_{s,+}^{t}\leq 0 \tag{3.22}$$

$$\sum_{n=1}^{N} g_{n,s}^{e}\left(o_{n,s}\times x_{n}^{u,t}\right)-\sum_{n=1}^{N}\underline{g}^{u,t,e}\left(o_{n,s}\times x_{n}^{u,t}\right)+gdev_{s,-}^{t}\geq 0 \tag{3.23}$$

$$\frac{1}{S}\sum_{s=1}^{S}\left(\sum_{n=1}^{N} f_{n,s}^{e}(o_{n,s}\times x_{n}^{u,t})-\sum_{n=1}^{N}\overline{f}^{u,t,e}(o_{n,s}\times x_{n}^{u,t})\right)\leq 0 \tag{3.24}$$

$$\frac{1}{S}\sum_{s=1}^{S}\left(\sum_{n=1}^{N} f_{n,s}^{e}(o_{n,s}\times x_{n}^{u,t})-\sum_{n=1}^{N}\underline{f}^{u,t,e}(o_{n,s}\times x_{n}^{u,t})\right)\geq 0 \tag{3.25}$$

3.5.7.5 Dyke material grade blending constraints

Equation (3.26) and Equation (3.27) control the interburden dyke material quality such as the average grade of fines in the interburden dyke material. Thus, ensuring that the fines in the interburden dyke material does not violate the limits of the maximum and minimum acceptable ranges of fines. Dyke material fines content plays a vital role in the stability of the dyke walls constructed.

$$\frac{1}{S}\sum_{s=1}^{S}\left(\sum_{n=1}^{N} f_{n,s}^{d}(i_{n,s}\times c_{n}^{u,t})-\sum_{n=1}^{N}\overline{f}^{u,t,d}(i_{n,s}\times c_{n}^{u,t})\right)\leq 0 \tag{3.26}$$

$$\frac{1}{S}\sum_{s=1}^{S}\left(\sum_{n=1}^{N} f_{n,s}^{d}(i_{n,s}\times c_{n}^{u,t})-\sum_{n=1}^{N}\underline{f}^{u,t,d}(i_{n,s}\times c_{n}^{u,t})\right)\geq 0 \tag{3.27}$$

3.5.7.6 Sand-to-fines blending constraints

The sand-to-fines ratio constraints are introduced to control the proportion of ore material sand and fines sent to the processing plant. By implementing this constraint, it enables faster reclamation of the tailings after deposition. The sand-to-fines ratio constraints are represented by Equation (3.28) and Equation (3.29)

$$\frac{1}{S}\sum_{s=1}^{S}\left(\sum_{n=1}^{N} sf_{n,s}^{e}\left(o_{n,s}\times x_{n}^{u,t}\right)-\sum_{n=1}^{N}\overline{sf}^{u,t,e}\left(o_{n,s}\times x_{n}^{u,t}\right)\right)\leq 0 \tag{3.28}$$

$$\frac{1}{S}\sum_{s=1}^{S}\left(\sum_{n=1}^{N} sf_{n,s}^{e}\left(o_{n,s}\times x_{n}^{u,t}\right)-\sum_{n=1}^{N}\underline{sf}^{u,t,e}\left(o_{n,s}\times x_{n}^{u,t}\right)\right)\geq 0 \tag{3.29}$$

3.5.7.7 Ore, dyke materials and mining variables relation constraints

The ore, dyke materials and mining variables relation constraints in the SMILP model consist of inequalities that ensure that the total material mined in any given scheduling period from any mining location exceeds or is equal to the sum of the ore, and overburden and interburden dyke materials mined for all realizations. It is also assumed that blocks scheduled for each period are extracted uniformly. Equation (3.30) represents the ore, dyke materials and mining variables relation constraints.

$$\sum_{u=1}^{U}\sum_{n=1}^{N}\left(o_{n,s}\times x_{n}^{u,t}+d_{n,s}\times l_{n}^{u,t}+i_{n,s}\times c_{n}^{u,t}\right)\leq\sum_{a=1}^{A}\sum_{n=1}^{N}\left(y_{n}^{a,t}\left(o_{n,s}+d_{n,s}+i_{n,s}+w_{n,s}\right)\right) \tag{3.30}$$

3.5.7.8 Ore and tailings coarse sand variables relation constraints

The ore and tailings coarse sand variables relation constraints shown in Equation (3.31) is defined to control the relation between the ore decision variable and the TCS decision variable. The constraint states that the fraction of TCS dyke material generated in each period should not exceed the portion of ore mined for all destinations. The TCS dyke material is only generated when ore is processed for bitumen extraction.

$$\sum_{u=1}^{U} k_{n}^{u,t}\leq\sum_{u=1}^{U} x_{n}^{u,t} \tag{3.31}$$

3.5.7.9 Ore and mining variables control constraints

The continuous decision variables used to control ore extraction and mining over the mine life are represented by Equation (3.32) and Equation (3.33) respectively. These inequalities ensure that the sum of partial extractions is at most one for every block that is processed or mined over all time periods. These inequalities also monitor all the different portions of blocks that are scheduled for all destinations.

$$\sum_{u=1}^{U}\sum_{t=1}^{T} x_{n}^{u,t}\leq 1 \tag{3.32}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} y_{n}^{a,t}\leq 1 \tag{3.33}$$

3.5.7.10 Dyke materials variables control constraints

Equation (3.34) to Equation (3.36) are the respective variables' control constraints for overburden, interburden and TCS dyke materials. These continuous decision variables are used to monitor the partial extraction of overburden and interburden dyke materials, and TCS dyke material respectively. These inequalities ensure that the sum of the partial extractions add up to or is less than one for every block that is mined for dyke construction over all time periods. These relations also monitor all the different portions of blocks that are scheduled for all destinations.

$$\sum_{u=1}^{U}\sum_{t=1}^{T} l_n^{u,t} \leq 1 \tag{3.34}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} c_n^{u,t} \leq 1 \tag{3.35}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} k_n^{u,t} \leq 1 \tag{3.36}$$

3.5.7.11 Mining precedence and block relations constraints

The extraction of mining blocks must follow the precedence order, based on the spatial location of the mining blocks. The precedence for blocks extraction is defined in two sets such as vertical precedence and horizontal precedence. For vertical precedence, the algorithm is defined such that before the extraction of a specific mining block, all the blocks on top must already have been extracted so that the block is accessible. In the case of horizontal precedence, it is defined such that the extraction operations follow a pre-determined mining direction. For instance, consider a set of mining blocks that is set to be scheduled. For simplicity, let's assume that the model consists of four mining blocks that is scheduled to be extracted over four periods (T = 4) as shown in Figure 3-4. The immediate predecessor blocks are labelled with directed-arcs graphs pointing from parent to child node. A directed graph constructs the precedence relationships between mining blocks. The directed graphs tag the mining blocks that must be extracted prior to extracting each mining Block *n*. Based on the figure, the immediate predecessors' set is C(L) = (2,3,4) which would represent the mining blocks that must be extracted before the extraction of Block 1.

The required equations for the mining precedence are defined by Equation (3.37) to Equation (3.40). The set $C_n(L)$, shown in Equation (3.37) represents the set of immediate mining blocks that are on top of mining block *n.* The set $D_n(J)$ shown in Equation (3.38) represents the sets of

immediate mining blocks in a specified horizontal mining direction that must be extracted prior to the extraction of mining block *n*. Equation (3.37) and Equation (3.38) ensure that the vertical and horizontal precedence requirements are met.

Equation (3.39) relates the binary integer decision variable a_n^t (used to control the vertical and horizontal precedence) with the continuous decision variable $y_n^{a,i}$. The binary integer decision variable a_n^t is equal to one if the extraction of the mining block *n* has started by or in period *t*, otherwise it is zero. Equation (3.40) states that when the extraction of a block starts in a specific period, then that block is accessible and available for extraction in all other periods after.

$$a_n^t - \sum_{a=1}^{a}\sum_{i=1}^{t} y_s^{a,i} \le 0 \qquad s \in C_n(L) \tag{3.37}$$

$$a_n^t - \sum_{a=1}^{a}\sum_{i=1}^{t} y_r^{a,i} \le 0 \qquad r \in D_n(J) \tag{3.38}$$

$$\sum_{a=1}^{a}\sum_{i=1}^{t} y_n^{a,i} - a_n^t \le 0 \tag{3.39}$$

$$a_n^t - a_n^{t+1} \le 0 \tag{3.40}$$

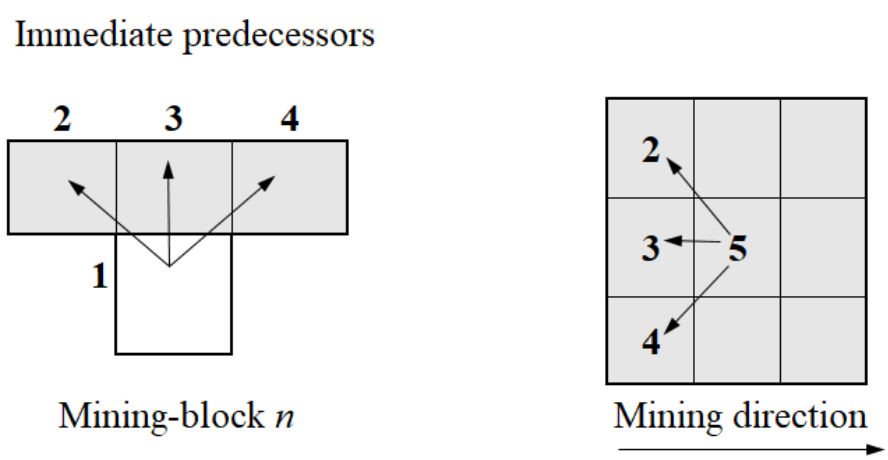

Figure 3-4: Mining block extraction precedence in the SMILP model modified after Ben-Awuah and Askari-Nasab (2011): a) vertical precedence and b) horizontal precedence

3.5.7.12 Non-negativity constraints

Equation (3.41) defines the non-negativity constraints of the decision variables for mining, processing, OB dyke material, IB dyke material and TCS dyke material. Equation (3.42) defines

the non-negativity constraints of the deviational variables that control the risk of not meeting the planned production targets.

$$x_n^{u,t}, y_n^{a,t}, l_n^{u,t}, c_n^{u,t}, k_n^{u,t} \geq 0 \tag{3.41}$$

$$gdev_{s,-}^t, gdev_{s,+}^t, odev_{s,-}^t, odev_{s,+}^t \geq 0 \tag{3.42}$$

3.6 Implementation of the SMILP model formulation

The SMILP formulation was implemented in a MATLAB programming environment (Mathworks, 2018). IBM/CPLEX (ILOG, 2019) was used as the optimization solver. The CPLEX solver approaches the optimization problem by first solving for the relaxed LP solution. During this process, all integer variables are relaxed by being changed to continuous variables. The relaxed LP solution then serves as the upper bound to the IP problem. CPLEX then uses the branch-and-cut optimization algorithm to reach a feasible integer solution. Branch-and-cut is a culmination of branch-and-bound and cutting plane methods (Horst and Tuy, 1997). CPLEX terminates the optimization once it reaches an absolute tolerance of the gap between the best integer objective and the objective of the best node remaining in the branch-and-bound algorithm. The termination of the optimization process indicates that a feasible solution has been found within the set tolerance limit which was specified by the user prior to the optimization run.

IBM/CPLEX uses a general form as an input for all optimization problems. The general structure for a MILP problem is stated by Equation (3.43) to Equation (3.46).

$$\min_x f(z) = f^{'}.x \tag{3.43}$$

Subject to:

$$Aineq.x \leq bineq \tag{3.44}$$

$$Aeq.x = beq \tag{3.45}$$

$$lb \leq x \leq ub \tag{3.46}$$

Where;

- f is a coefficient vector of the decision variables in the objective function. This is a $n \times 1$ vector; n is the total number of decision variables.
- x is the vector of the decision variables which includes binary and continuous variables; x is a vector of $n \times 1$.

- $Aineq$ is the coefficient matrix of the decision variables in the inequality constraints. The coefficient matrix of $Aineq$ is a $m \times n$ matrix; m is the total number of linear constraints.
- $bineq$ is a vector in the inequality constraints.
- Aeq is the coefficient matrix of the decision variables in the equality constraints.
- beq is a vector in the equality constraints.
- lb and ub define the lower and upper boundary constraints of the decision variables; which are vectors of $m \times 1$.

Detailed formulation and programming techniques in implementing the SMILP model for integrated stochastic oil sands long-term production planning and waste management using MATLAB and IBM/CPLEX can be found in the publications of Ben-Awuah et al. (2013) and Koushavand et al. (2014).

3.8 Summary and conclusions

In mining, geostatistics emphasizes the geological context of data and the spatial relationship between data. This is very useful to predict the quality and quantity over an area where drilling was not done, from a limited amount of data. The procedural steps of Sequential Gaussian Simulation algorithm were highlighted in this chapter. The implementation of the uncertainty-based SMILP model that integrates oil sands production planning and waste disposal management in the presence of grade uncertainty was also documented in this chapter. The interactions and relations of the decision variables in an oil sands conceptual mining system was fully discussed with the objective of providing an optimal production schedule that aims at (i) maximizing the NPV of the oil sands mining operation; (ii) minimizing the dyke construction cost for the waste disposal management plan; (iii) minimizing the risks associated with deviations from the grade blending and ore processing targets. The objective function of the SMILP model is subject to practical constraints as discussed in the chapter. The numerical model of the SMILP formulation was implemented in a MATLAB environment (Mathworks, 2018) and the optimization problem solved with IBM/CPLEX (ILOG, 2019).

CHAPTER 4

APPLICATION OF METHODOLOGY AND DISCUSSION OF RESULTS

4.1 Background

This chapter focuses on the implementation and verification of the workflow and SMILP model that was formulated in Chapter 3. For this research, a case study consisting of different data sets from oil sands deposits were considered and the methodologies were implemented for these data sets.

An exploratory data analysis was conducted first for the case study where univariate and bivariate descriptions of the assays were calculated. Secondly, measurement of spatial continuity was then performed to verify that data samples which are closer in space will possess similar properties than samples that are further from each other in space. Thirdly, variogram modelling was employed so as to display the variability between the data points as a function of distance and to highlight any anisotropy. Ordinary Kriging (OK) estimation was then performed for the base case model followed by conditional Sequential Gaussian Simulation (SGS) for the simulated model. Also, an average block model from the simulated realizations, also known as the E-type block model, was created. Blocks within the ultimate pit limit were used as the input data for the SMILP model for subsequent integrated stochastic production scheduling and waste disposal planning.

MATLAB (Mathworks, 2018) environment was used as the programming platform for the SMILP model and IBM/CPLEX (ILOG, 2019) solver which uses a branch and cut optimization algorithm was used to solve the resulting production scheduling optimization problem. The SMILP model was implemented on a Lenovo ThinkStation with CPU E5-1650 @ 3.60 GHz and 64 GB of RAM. The verification of the SMILP model formulation was investigated by comparing the optimized uncertainty-based production schedule results from the simulated block models to the optimized deterministic production schedule results from the OK block model. Further investigation was done by also comparing the uncertainty-based production schedule results to the deterministic production schedule results from the E-type block model. The experiments designed to validate the integrated stochastic production schedule and waste

disposal plan, incorporates simulated ore bodies to generate a practical schedule with an optimal net present value compared to using a single estimated orebody.

4.2 Verification of the SMILP model

The purpose of model verification is to determine whether a system has been built adequately to the set specifications. Thus, to verify the implementation of the SMILP model, the response to the question whether the developed application meets the set standard is sought for. As indicated in Chapter 3, the main objectives of the SMILP model is to (i) maximize the net present value of the mining operation, (ii) minimize the construction costs of dykes for waste management (iii) minimize the cost of uncertainty associated with targeted ore grade and tonnage. These objectives are subject to technical and practical constraints. The verification of the uncertainty-based production schedule was done by comparing it to the deterministic production schedules from the OK block model and the E-type block model. Also, the results from the SMILP model are verified by applying the scheduled results to three randomly selected realizations, and then repeating the same process with the deterministic results to compare them accordingly.

4.3 Experimental design framework for the SMILP model

The procedure used in handling the integrated oil sands mining and waste management problem in the SMILP framework comprises of a solution strategy that is based on the branch and cut optimization algorithm which is implemented in IBM/CPLEX (Holmström et al., 2009; ILOG, 2019). In order to obtain a consistent experimental result, the solution scheme used in solving the optimization problem should be able to highlight the complete definition of the integrated oil sands production scheduling and waste disposal problem which includes a conceptual mining framework, a dyke construction strategy, and an optimized production schedule in the presence of grade uncertainty. The modelling assumptions are primarily based on the knowledge of practical mining environments and the framework for the application of operations research methods in mining.

The SMILP model framework is validated using dataset for an oil sands case study. The case study consists of three scenarios based on block models from Ordinary Kriging and Sequential Gaussian Simulation. In the first scenario, OK estimation technique is used to construct the geologic block model and subsequently the economic block model for the oil sands deposit. In the second scenario, a single average block model of all the generated SGS realizations is used to construct the geologic block model and the economic block model for the oil sands deposit.

Blocks in the ultimate pit limit serves as input to the SMILP model framework to generate an integrated stochastic production schedule and waste disposal plan. Mining, processing and dyke material scheduling are implemented with blocks as the scheduling units. It must be noted that in the first and second scenarios, a single estimated orebody model is considered in the scheduling process. Also, grade uncertainty is not considered in both scenarios for the case study.

Scenario 3 focuses on the deployment of the SMILP model framework for a stochastic block model from SGS realizations based on grade uncertainty. The inclusion of grade uncertainty in the mine production plan will minimize financial risk by providing reliable tonnage and grade estimates in the life of mine plan. The robustness of the schedule results is investigated by applying each schedule to randomly selected realizations. The resulting schedules for the selected realizations are compared based on ore tonnage, waste tonnage, average grade and NPVs. Lastly, sensitivity analysis was conducted based on the risk parameter, geological discount rate (GDR), to assess it impact on the overall net present value of the mining project. Table 4-1 shows the experimental design layout for the case study and the associated scenarios.

Table 4-1: Summary of experimental design layout for the case study

	Scenarios	Objective function	Design considerations
Case Study	**Scenario 1: OK block model (Conventional case)**	$Revenue - Mining\ cost - Dyke\ construction\ costs$	• No financial risk assessment • No ore grade and ore tonnage uncertainty cost
	Scenario 2: E-type block model	$Revenue - Mining\ cost - Dyke\ construction$	• No financial risk assessment • No ore grade and ore tonnage uncertainty cost
	Scenario 3: Stochastic block model (SGS realizations)	$Revenue - Mining\ cost - Dyke\ construction\ costs - Cost\ of\ uncertainty$	• Introduction of GDR parameter • Financial risk assessment • Ore tonnage and ore grade uncertainty costs implemeted in the objective function • Sensitivity analysis on GDR and penalty

4.4 Case study

4.4.1 Exploratory data analysis: Summary statistics

The input data sets that was used for the case study is for an oil sands deposit which consisted of 104 vertical exploration drill holes with azimuth and dip of 0° and -90° respectively. The drill holes data contain samples with assays for bitumen, fines, organic rich solids, and water. Summary statistics of the assays can be seen in Table 4-2. The summary statistics were obtained by analyzing the histogram and cumulative distribution function of the assay grades as shown in Figure 4-1 and Figure 4-2 respectively.

Table 4-2: Summary statistics of samples assays for Case study

Parameter (Units)	Bitumen	Fines	Organic Rich Solids	Water
Number of samples	2964	2964	2964	2964
Minimum value (%m)	0.00	0.00	0.710	2.30
Maximum value (%m)	17.90	98.20	3.81	19.49
Mean (%m)	9.385	14.913	1.733	4.910
Median (%m)	10.105	1.155	1.475	2.505
Geometric Mean (%m)	-	-	1.578	3.875
Variance $(\%m)^2$	20.623	492.403	0.607	15.081
Standard Deviation (%m)	4.541	22.190	0.779	3.883
Coefficient of Variation	0.483	1.487	0.449	0.790
Skewness	-0.331	1.588	0.863	1.588
Kurtosis	1.932	4.635	2.655	4.634
10.0 Percentile (%m)	2.395	0	0.940	2.30
50.0 Percentile (median) (%m)	10.105	1.155	1.475	2.505
90.0 Percentile (%m)	14.875	51.610	3.045	11.335

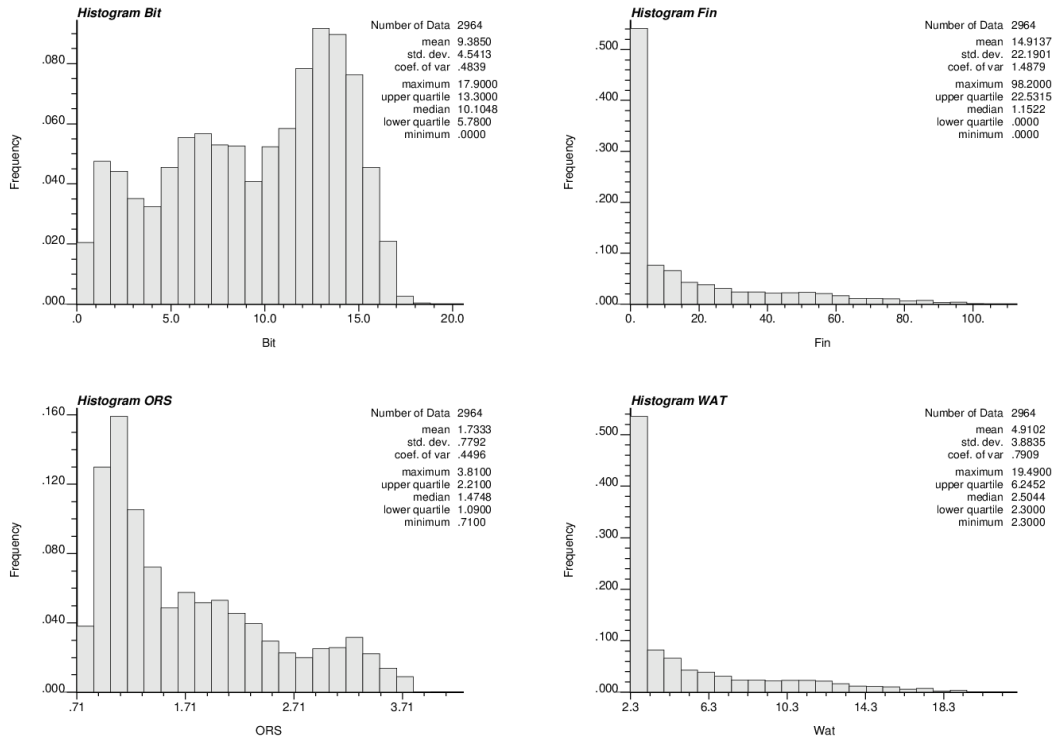

Figure 4-1: Histogram of assay samples

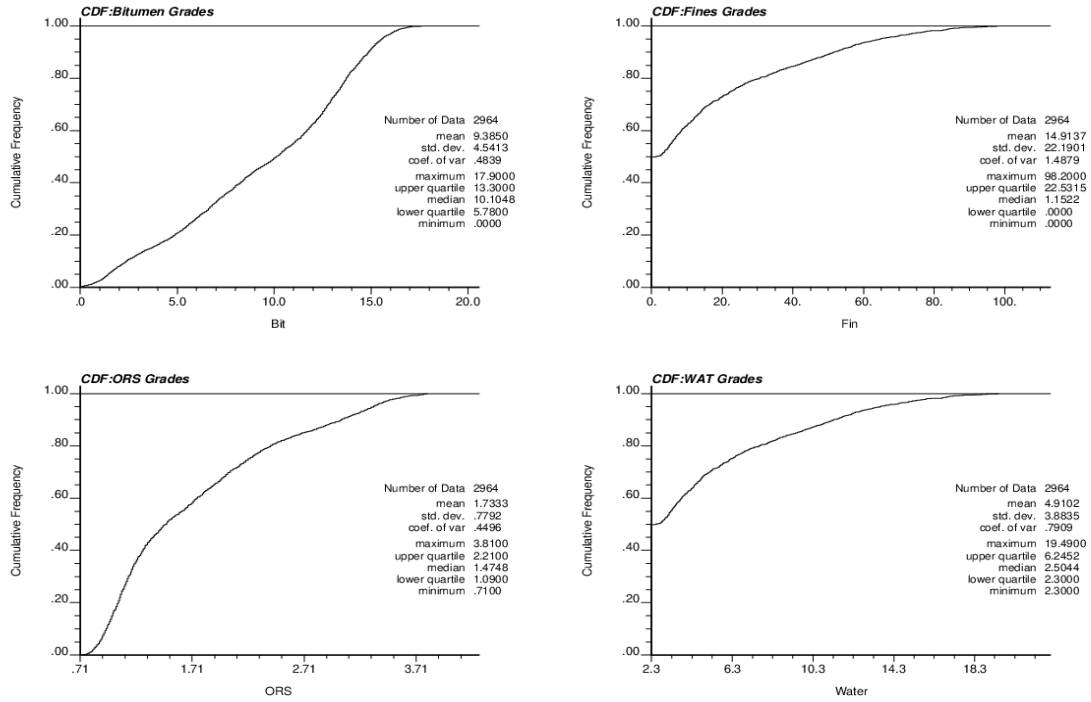

Figure 4-2: Cummulative distribution function of the samples assays

From the histogram shown in Figure 4-1, it was observed that the assay samples are uni-modal and non-symmetrical with skewness. The histogram for bitumen grade data follow a negatively skewed distribution with most of its values concentrating on the right tail of the distribution graph while the left tail of the distribution is longer. The histogram for fines, organic rich solids and water grades follow a positively skewed distribution with most of its values concentrating on the left tail of the distribution graph while the right tail of the distribution is longer. Having a skewed data impacts negatively on the efficiency and reliability of the statistical data analysis, therefore in order to resolve this problem, the sample data is log transformed in the subsequent exploratory data analysis so as to make the skewed distribution more symmetric for geostatistical modelling. The application of log transformation on a skewed data also pulls together very large data values which have been thinly spread out at the high end of the sample data.

For this research, the element of interest is bitumen grades since its variability in estimation creates uncertainty which could potentially impact the overall net present value of the mining project (Dimitrakopoulos and Ramazan, 2008; Osanloo et al., 2008; Hickman, 2014; Koushavand et al., 2014; Ramazan and Dimitrakopoulos, 2018).

4.4.2 Cell declustering of sample data

When obtaining sample data for mining, it is rarely collected in a regular pattern as it makes more sense to obtain sample data in areas that have higher grade zones. As a result of this, the collected sample data used for resource estimation are always clustered and if the sample data is not declustered before applying grade estimation, it can lead to biased results in resource estimation (Journel and Huijbregts, 1978; Isaaks and Srivastava, 1989; Sinclair and Blackwell, 2002; Rossi and Deutsch, 2013).

Declustering was applied on the data set so as to reduce the effect of clustered samples. By applying declustering, the histogram and summary statistics are adjusted to be representative of the entire volume of interest. Cell declustering technique was performed on the data set to adjust the global mean of the data. Figure 4-3 shows a 2D location map of the drill holes with bitumen grade distribution while Figure 4-4 shows the influence of declustering on the given data set of oil sands drill composites. By applying declustering, the lowest weights were assigned to locations with less dense samples and the highest weights were assigned to locations with sparse samples. A plot of the declustered mean against the cell sizes to highlight the impact of cell declustering technique is shown in Figure 4-5. It was observed that the naïve mean or the average

of the bitumen grade without any declustering was about 9.38 %m; however, after applying declustering, the mean of the bitumen grade was adjusted to 9.21 %m. This indicates that the undeclustered mean, may overestimate the global bitumen reserve by up to 1.81%. The cell size that minimizes the declustered mean was about 550 m (estimated from Figure 4-5). This makes sense given our previous observation of the drill hole data spacing.

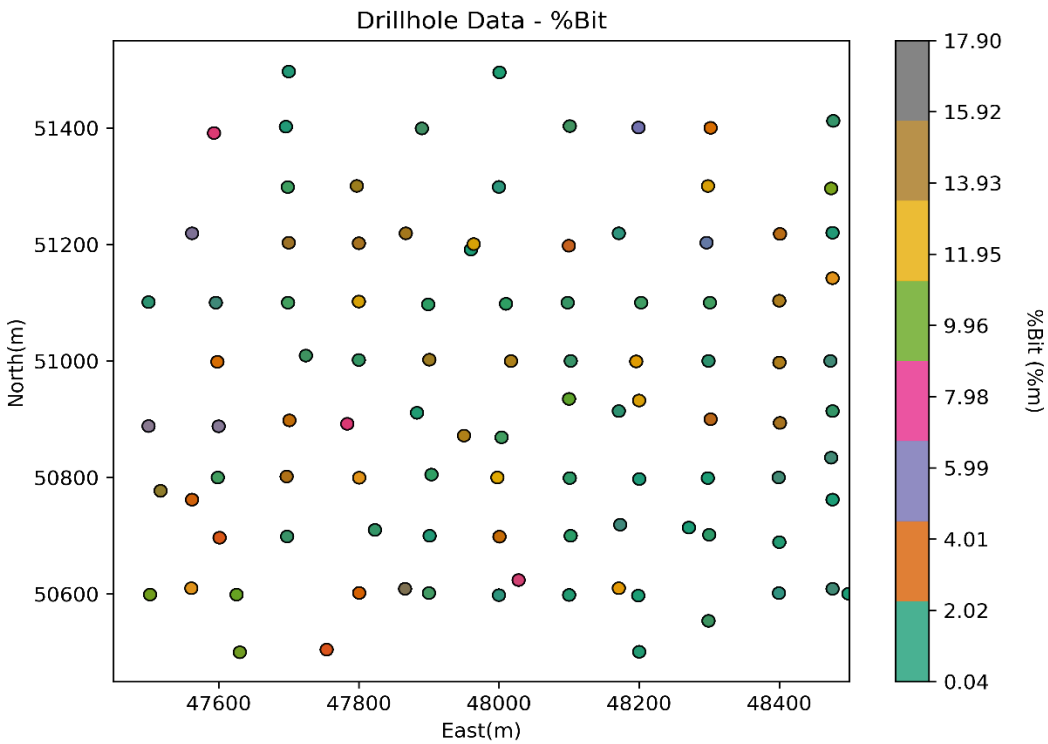

Figure 4-3: 2D location map of the drill holes with bitumen grade distribution

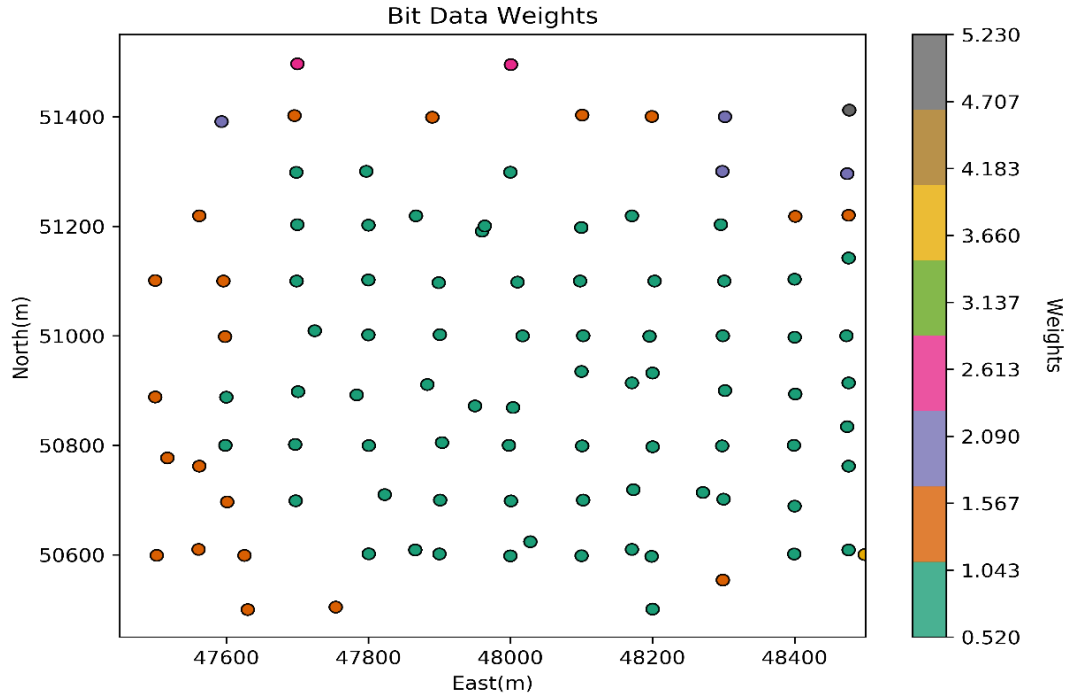

Figure 4-4: Location map of the declustered weights for bitumen grades

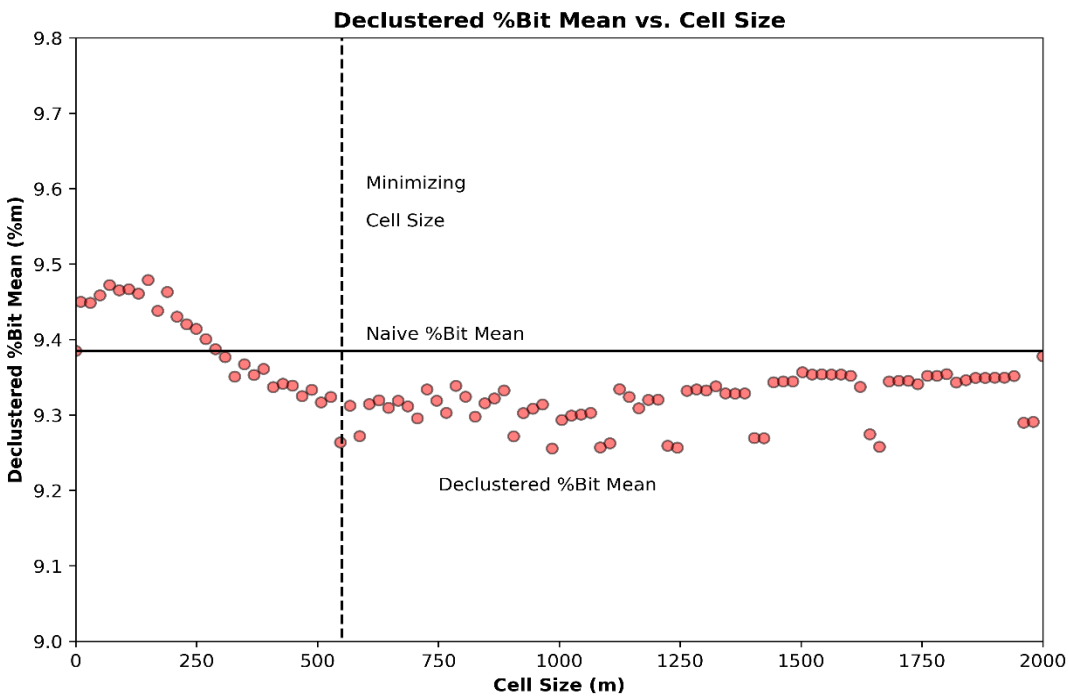

Figure 4-5: Plot of declustered bitumen mean vs cell size

4.4.3 Spatial description of sample data

The measurement of spatial continuity was employed so as to understand the correlation between the observations of the univariate samples at different locations. This is very useful to detect the presence of general trends in the data. Geostatistical techniques as explained by Issaak and Srivastava (1989) were used to analyze spatial variability and distribution of the given sample data for estimating parameters at unsampled locations. According to Issaak and Srivastava (1989), application of geostatistics has three primary steps; (1) assumption of stationarity, (2) spatial modelling of sample data, and (3) estimation of variable value at unsampled location. There are many parameters to analyze the spatial characteristics of sample data like covariance, correlation coefficient and variogram. The measurement of spatial continuity was achieved using semi-variogram and variogram modelling. Ninety percent of geostatistical reservoir characterization studies use variogram-based geostatistical modelling methods (Gringarten and Deutsch, 1999).

The original data set containing bitumen grades was transformed to a Gaussian space using the standard normal score transformation which is applicable in geostatistical analysis (Manchuk et al., 2005). Transformation of data to normal score distribution can satisfy the assumption of stationarity of data. Normal score transformation has some benefits like, dampening of differences between extreme values and mapping the theoretical sill to one. The transformed normal score data also serves as input data in the stochastic Gaussian simulation process. Figure 4-6 shows the histogram of declustered bitumen grade and normal score bitumen grade.

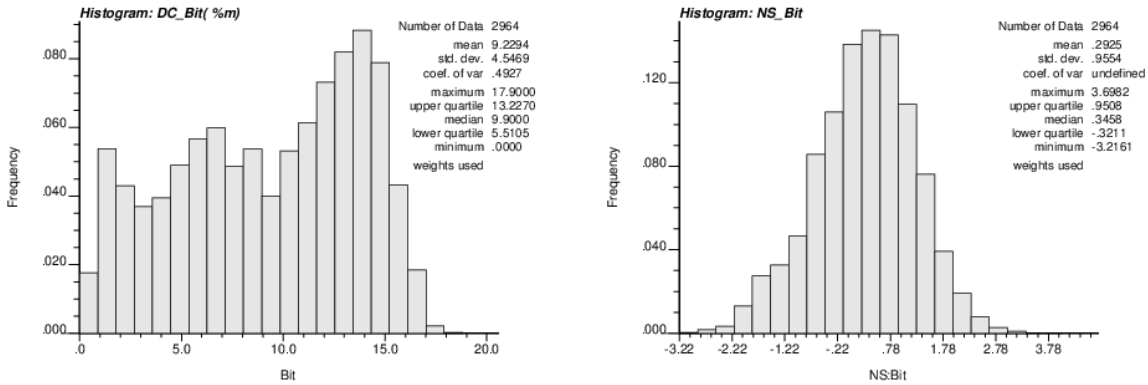

Figure 4-6: Histogram of declustered bitumen grade (left) and normal score bitumen grade (right)

The measurement of spatial continuity was assessed by plotting the location maps of the normal score transforms for bitumen grades and by observation, the plotted data was inspected to detect any presence of geologic trend which will assist in the parameter selection needed for variogram

modelling. Also, the variogram map containing the number of pairs was plotted so as to observe the direction of continuity.

Spatial data directionality was calculated using multiple experimental variograms. Omnidirectional variogram for bitumen grades was first computed to identify the sill while vertical variograms were used to identify the nugget effect. Primary variogram maps were calculated to determine the orientation of the major axis in the presence of anisotropy. Directional experimental variograms were calculated and a theoretical variogram model was fitted to the experimental variogram points. The azimuth of the directional experimental variograms that were considered were 0°, 22.5°, 45°, 67.5°, 90°, 112.5°, 135° and 157.5° for 10 lags with a lag separation distance of 80 m and a lag tolerance of about 45 m. For the directional experimental variogram, an azimuth tolerance of 45°, and a dip of 0° with no dip tolerance and a vertical and horizontal bandwidth of 20° were used as parameters for the directional experimental variograms. For the omnidirectional variogram, an azimuth of 0° and azimuth tolerance of 90° were used. These parameters were chosen after considering the area geology of the ore deposit, the size of the domain and the spacing between the drillhole data. Table 4-3 shows the observations that were obtained from the experimental variogram. Figure 4-7 shows the results of the experimental variogram calculation for the respective azimuths.

Table 4-3: Directional experimental variogram observations

Azimuth (°)	Nugget	Range (m)	Geological Trend
0°	0.62176	311	Cyclicity
22.5° $(minor)$	0.61415	438	Cyclicity
45°	0.61415	552	Cyclicity
67.5°$(major)$	0.80785	578	Cyclicity
90°	0.82638	-	Geometric anisotropy
112.5°	0.85257	560	Cyclicity
135°	0.85257	423	Cyclicity
157.5°	0.62176	-	Geometric anisotropy
Omnidirectional	0.62176	311	Cyclicity

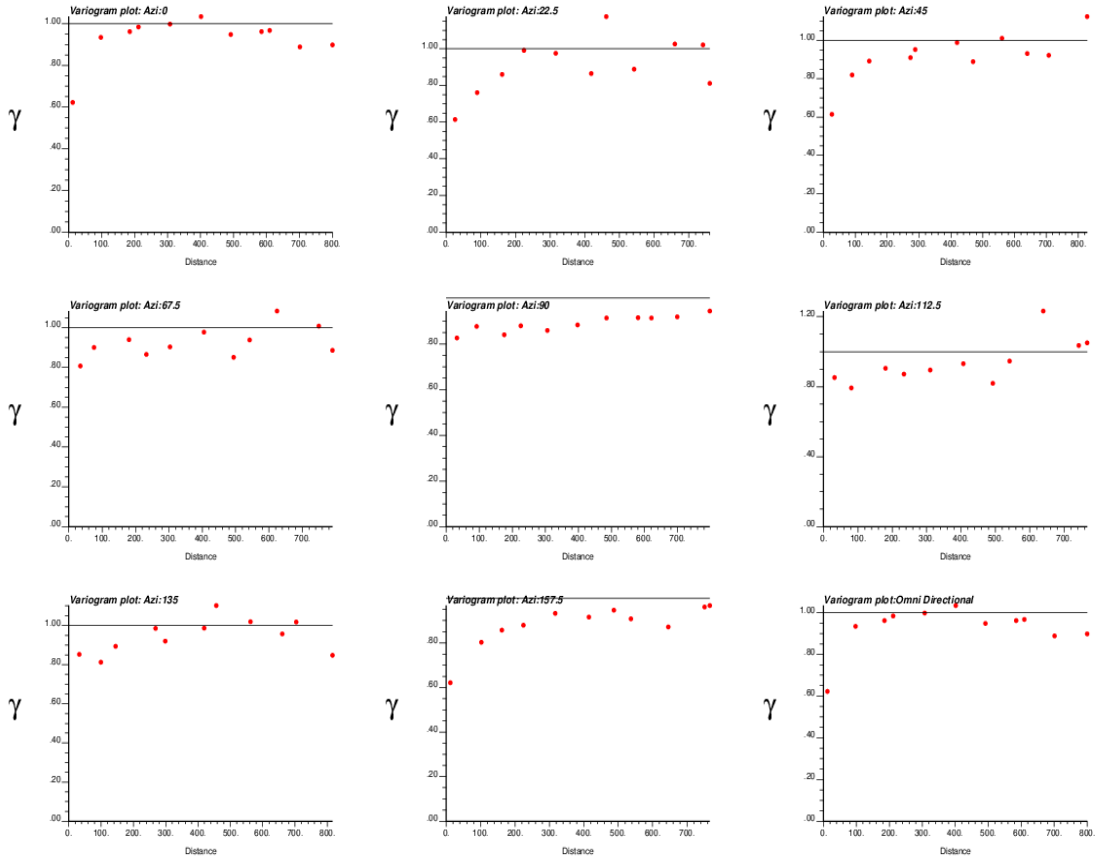

Figure 4-7: Directional experimental variogram calculation plots

The azimuths of the minor and major directions were found to be 22.5° and 67.5°. This selection was done by observing the ranges in the semi-variogram plots. It could be seen that the azimuth with the longest range was 67.5° with a distance of about 578 m while the azimuth with the shortest range was 22.5° with a distance of about 438 m. Also, from Figure 4-7, it could be seen that there was a number of trends found in the data. Cyclicity was observed in most of the directions used to compute the experimental variograms except for azimuth 90° and 157.5°, which indicated geometric anisotropy. The presence of cyclicity could be linked to the underlying geologic periodicity and also due to limited data availability. A variogram model was fitted to the experimental variogram points since a variogram is needed for all distances and in all directions (Sinclair and Blackwell, 2002).

A single nested variogram model based on exponential variogram for the major, minor and vertical directions were fitted. Figure 4-8 shows the experimental and the fitted variogram models in the major and minor directions. The equation for the exponential model is shown in

Equation (4.1), where $h = 3a$. It means when $h = 3a$, $\gamma(h) \approx C_0 + C$ and the effective range becomes $3a$. The parameters used to model the experimental variogram can be seen in Table 4-4.

$$\gamma(h) = \begin{cases} 0, & h = 0 \\ C_0 + C\left(1 - e^{-\left(\frac{h}{a}\right)}\right), & h < 0 \end{cases} \tag{4.1}$$

Table 4-4: Parameters used for the variogram modelling of bitumen grades

Direction	Azimuth	Nested Variogram model	Sill Contribution	hmin (m)	Hmax (m)	Nugget
Vertical	N0.0S	Exponential	0.43	180	220	0.57
Minor	N22.5E	Exponential	0.58	320	800	0.42
Major	N67.5E	Exponential	0.18	610	800	0.81
		Exponential	0.01	1200	1600	0.19

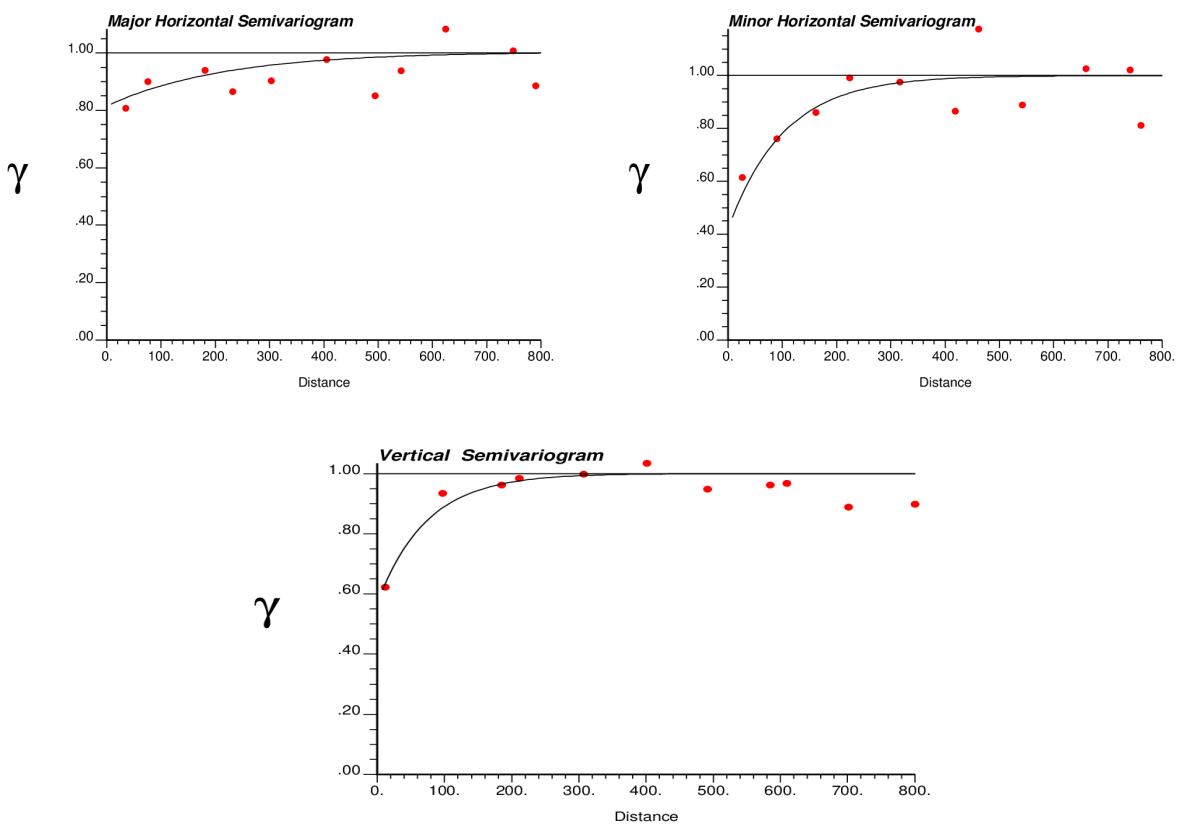

Figure 4-8: Experimental directional variograms (dots) and fitted variogram models (solid lines) for bitumen grades of ore blocks, distance units in meters

The next step was to define a regular grid that will be used to perform OK estimation and SGS. The parameters used to define the grid includes: (i) the grid dimensions which consist of the number of blocks in each X, Y and Z directions; (ii) the block size which consists of the dimensions of a single block size; and (iii) origin coordinates. The selection of a suitable block size was very critical and carefully considered. The use of small block sizes increases the reliability of the model and the overall number of blocks in the model. Similarly, having too many blocks increase the computational time for modeling and the optimization process. Also, for a relatively homogeneous oil sands deposit, a larger block size can be used. On the other hand, extremely large block sizes will smoothen out the grade fluctuations and therefore the effect of grade uncertainty will be diminished. Considering all these factors, the size of the blocks is chosen to be 50 by 50 by 15 meters. The height of each level or bench is considered as the height of one block which is 15 meters. To estimate (or simulate) at the block level, the scale difference of the input point data and that of the blocks should be taken into account. Therefore, the correct way to create a geostatistical model at a block scale is to build a high-resolution grid for estimation and simulation, and then re-block to the appropriate block size. For this case study, each block was discretized into 5 points in length, 5 points in width and 3 points in height. That means inside each block there are 75-point scale simulation or estimation values. The definition of the high-resolution grid for point scale modeling for the case study is summarized in Table 4-5. There are 6587600 nodes in this grid. The block scale grid definition is also given in Table 4-6. There are 94,864 blocks in this grid.

Table 4-5: High resolution grid definition for point scale estimation

Parameter (Unit)	**Direction (Easting)**	**Direction (Northing)**	**Direction (Elevation)**
Number of Nodes (#)	383	430	40
Origin Coordinates (m)	45878	49128	340
Grid Spacing (m)	10	10	5

Table 4-6: Block scale grid definition

Parameter (Unit)	**Direction (Easting)**	**Direction (Northing)**	**Direction (Elevation)**
Number of Nodes (#)	77	86	14
Origin Coordinates (m)	45878	49128	340
Grid Spacing (m)	50	50	15

4.4.4 Ordinary kriging estimation results

Search ellipse profiles and semi-variogram profiles were updated from the semi-variogram models and used for interpolation within the ore rock types for bitumen and fines grades. The paramaters from variogram modelling were used as input for ordinary kriging and simulation. Ordinary kriging estimation is done with the sill, the nugget effect and the range to calculate the ordinary kriging bitumen grade and variance for each mining block in the block model through block kriging (Gringarten and Deutsch, 1999). Based on the ordinary kriging estimates, the bitumen grade values were assigned to each block in original units (without normal score transform). Figure 4-9 shows a 2D plot of OK estimates and the estimation variance that was calculated for this case study. As anticipated, due to the smoothing effect of kriging, the standard deviation is decreased as compared to the sample data. The average bitumen grade of the OK estimates is 9.22 %m.

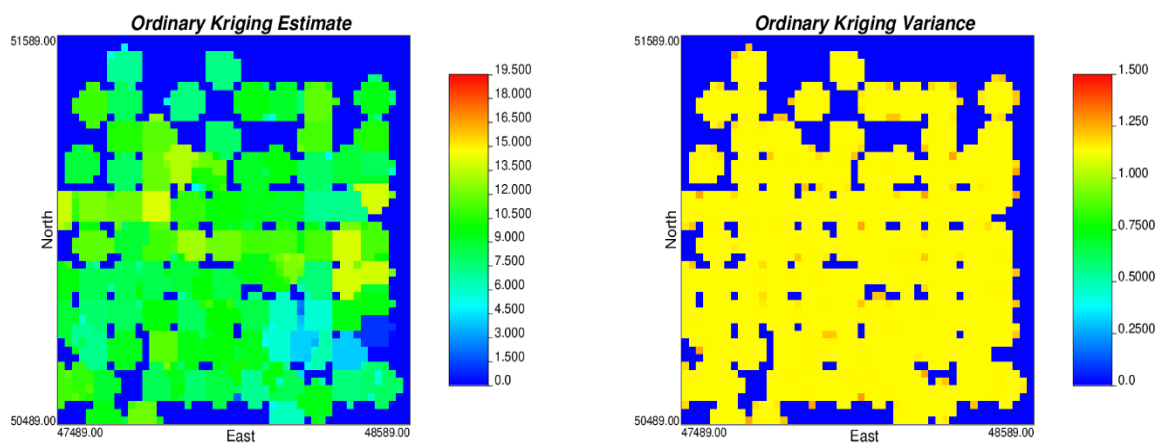

Figure 4-9: 2D plot of ordinary kriging estimates (left) and kriging variance (right)

In order to compare and validate the estimated kriged results, a Q-Q plot was created by plotting two sets of quantiles against one another. In this case, the estimated kriged bitumen grades were compared to the true bitumen grade values. The Q-Q plot analysis showed acceptable linear trend (located on the 45-degree line) between sample data and the estimated kriged results. Figure 4-10 shows a histogram plot of the kriged bitumen grades and Q-Q plot of the kriged bitumen grades and the true values of the bitumen grades. Based on the Q-Q plot, it could be observed that the OK estimation method has a minor deviation from the mentioned trend line. The standard deviation of the OK estimates was 2.34 %m. while that of the true bitumen grade values was 4.07 %m.. Also, comparing the estimated kriged values to the true sample values, it

was observed that the standard deviation from the kriged estimates was less than that from the true sample data. This was as a result of the smoothening effect associated with ordinary kriging estimation.

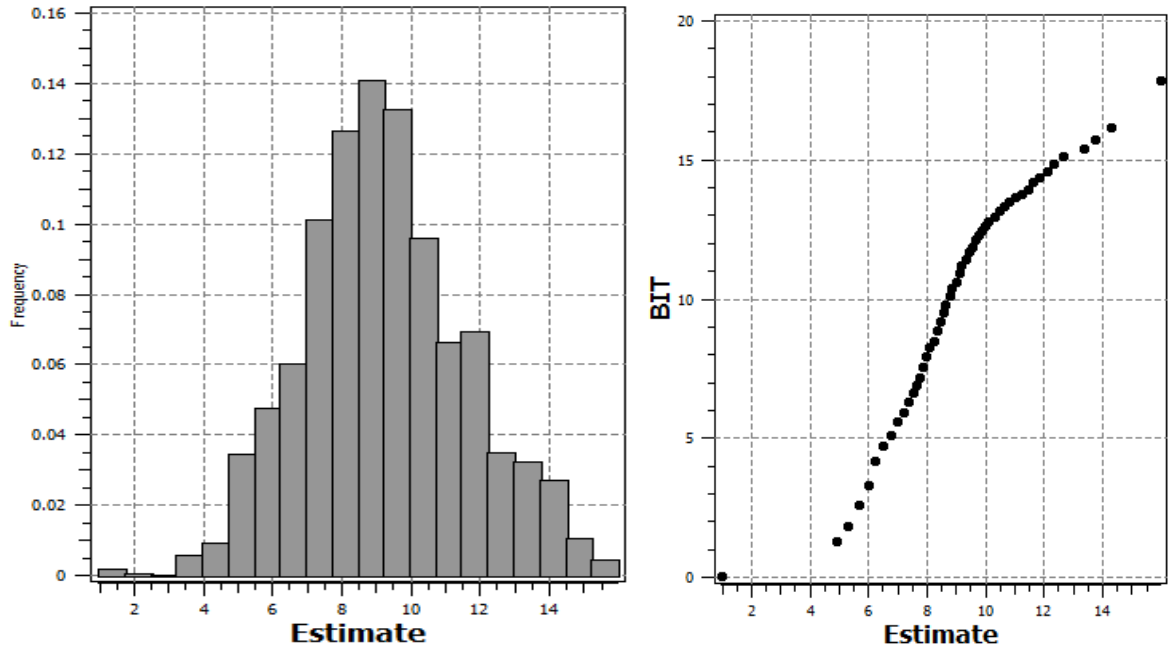

Figure 4-10: Histogram plot of kriged bitumen grades (left) and Q-Q plot of kriged bitumen grades versus true bitumen grades (right)

4.4.5 Sequential Gaussian simulation results

The results of 20 orebody realizations for bitumen grades were obtained after running the sequential Gaussian simulation (SGS) algorithm as shown in Figure 4-11. The SGS algorithm is capable of providing a set of maps showing the grade, which honors the known variability and is likely to represent unknown reality at any location (Schofield and Rolley, 1997). The SGS algorithm simulates nodes after each other sequentially, subsequently using simulated values as a conditioning data. It is necessary to use standard Gaussian values in SGS method; therefore, the input data was transformed into Gaussian space. SGS creates realizations of normal random variables and performs a Gaussian transformation of the data. A simulated value at a visited point is randomly drawn from the conditional cumulative distribution function, defined by the kriging mean and variance, based on neighborhood values. At a new randomly visited point, the simulated value is conditional to the original data and previously simulated values. Finally, the simulated normal values are transformed back into the simulated values for the original variable. The process is repeated until all points are simulated for each realization throughout the grid (Deutsch and Journel, 1992).

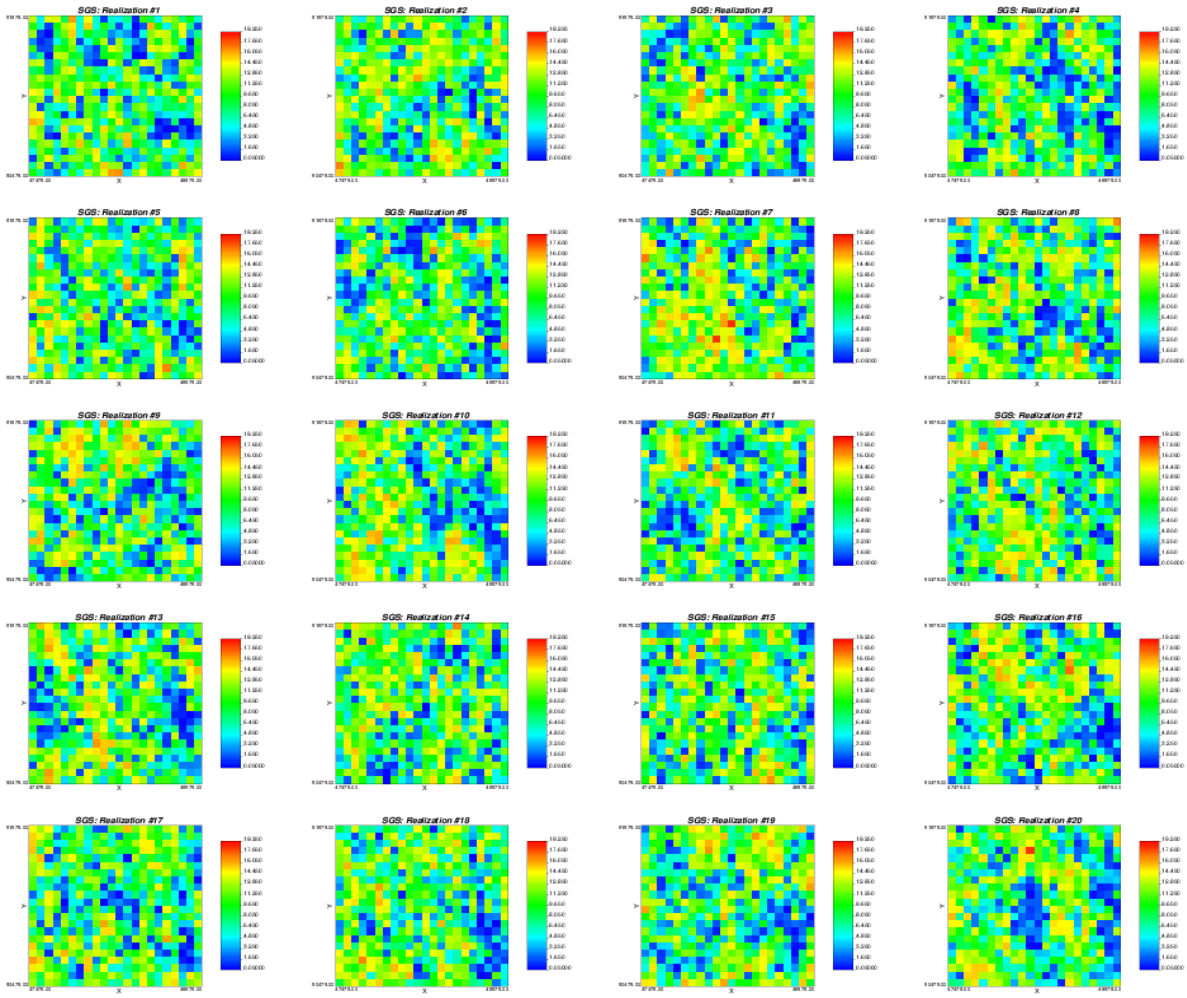

Figure 4-11: Realizations generated from SGS algorithm for the case study

Post-processing of simulated results was performed to obtain the average simulated (E-type) bitumen grade values from all the realizations at block scale and its conditional variance. In other words, the E-type mean is simply the average of the simulated values in each block. These post-processed results can be seen in Figure 4-12. The main difference between simulation and estimation is that, the estimated model does not reproduce the input histogram and variogram while the simulation honors the original input data and produces less smoothing effect. Figure 4-13 shows the variogram reproduction in the major, minor and vertical directions obtained from the realizations. Since the variograms are reproduced quite reasonably well, the generated realizations are considered representative of the grade uncertainty.

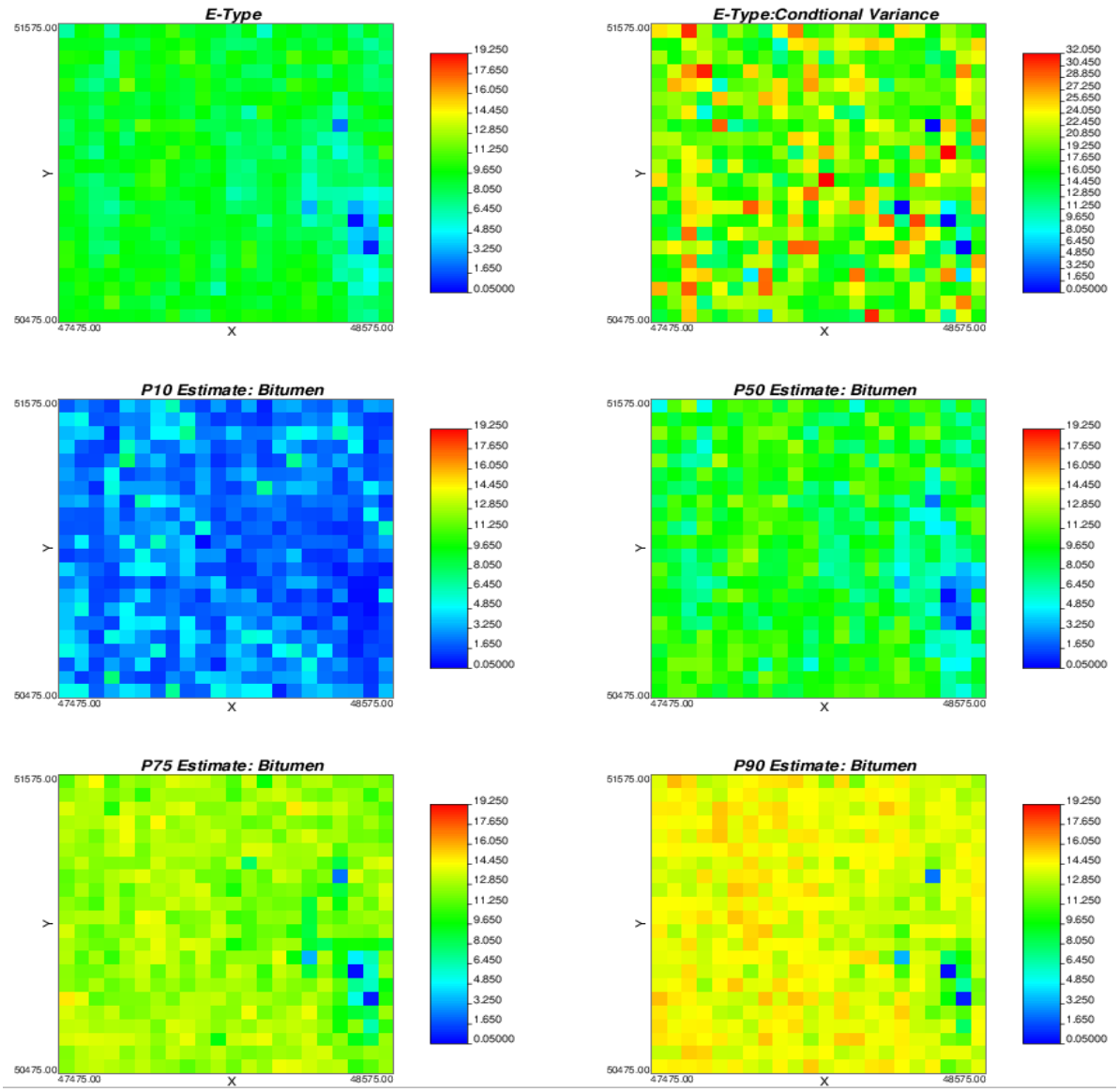

Figure 4-12: Post-process results generated from SGS realizations for case study

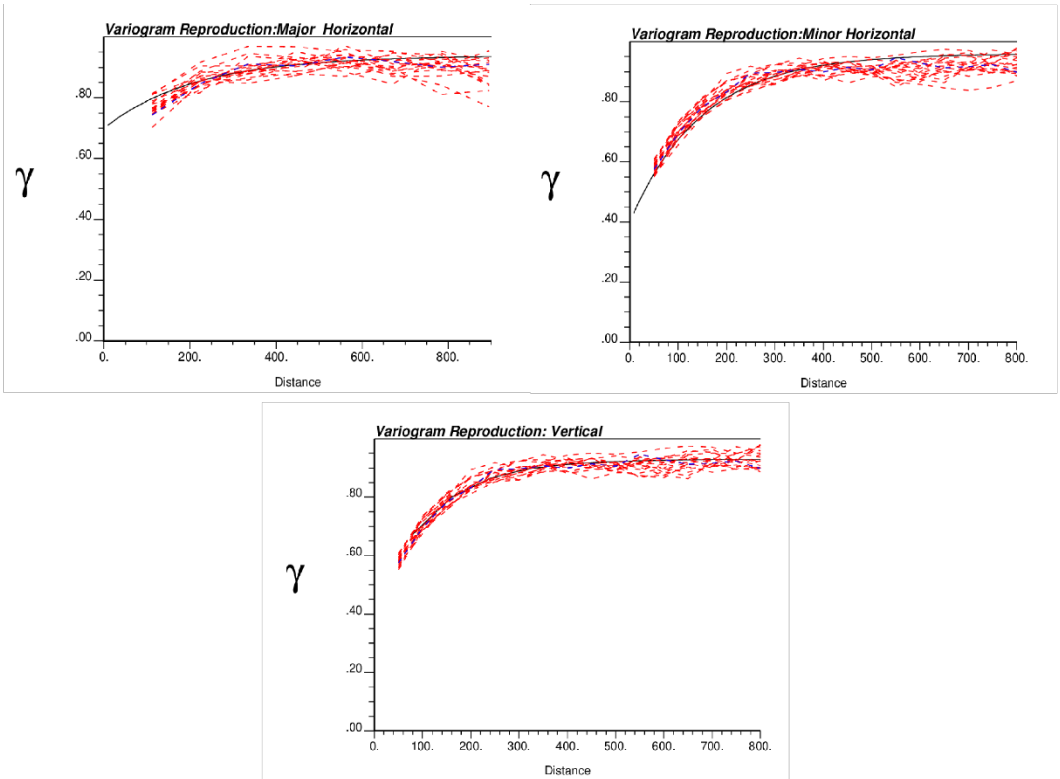

Figure 4-13: Variogram reproduction at Gaussian units for conditional simulation realizations (major, minor and vertical directions)

In order to validate the results obtained from the SGS, the results of each realization for bitumen grades were compared to the true bitumen grade values. Q-Q plots were used to compare the back-transformed simulated results to the original input sample data. The Q-Q plot analysis procedure in this work gave acceptable linear trends (located on the 45-degree line) between the sample data and the simulated results. Figure 4-14, Figure 4-15, Figure 4-16, Figure 4-17, show the histogram on the left and Q-Q plots on the right for the fifth, eight, and eighteenth realizations, and the E-type results of the simulated bitumen grades respectively. Based on the Q-Q plot on the right side of each figure, it can be observed that the distributions of the individual realizations are very similar to the original input sample data. The fifth, eight, and eighteenth realizations have mean of 10.02 %m, 9.85 %m, and 10.00 %m respectively as compared to the original sample data that has a mean of 10.12 %m. On the other hand, the E-type mean is about 9.45 %m and it is slightly different from the ordinary kriging mean which is 9.22 %m. This is because in OK estimation, kriging is applied in original units; theoretically they will be the same in Gaussian units.

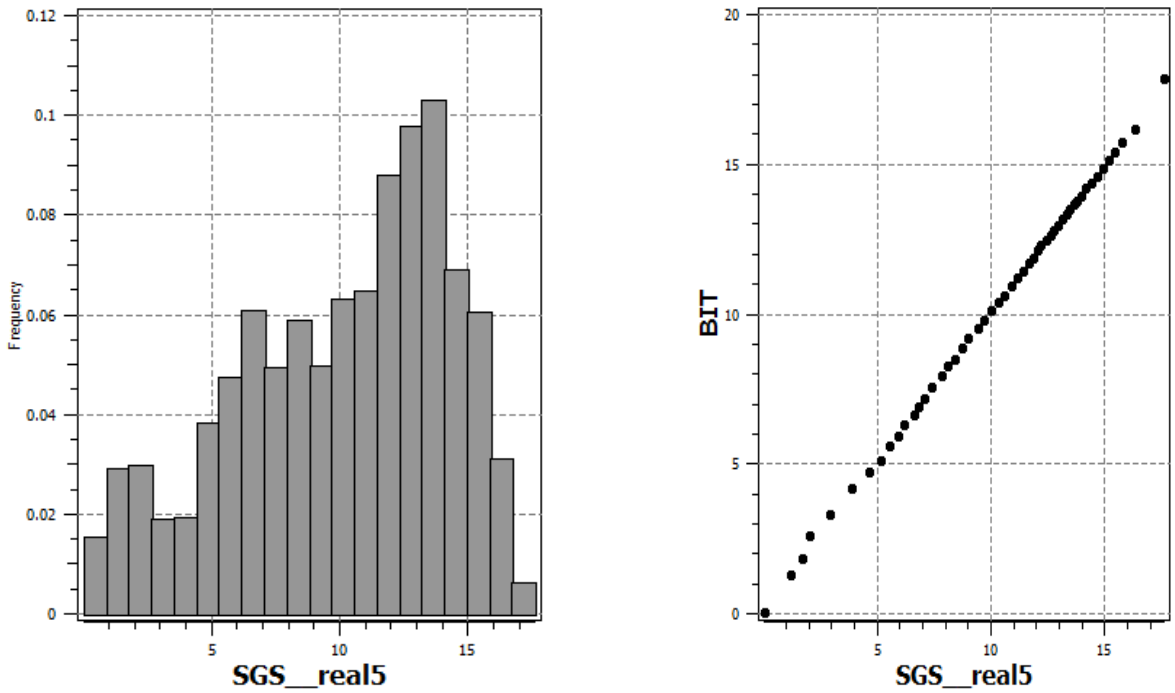

Figure 4-14: Histogram plot of Realization 5 (left) and Q-Q plot of Realization 5 with original input bitumen grades (right)

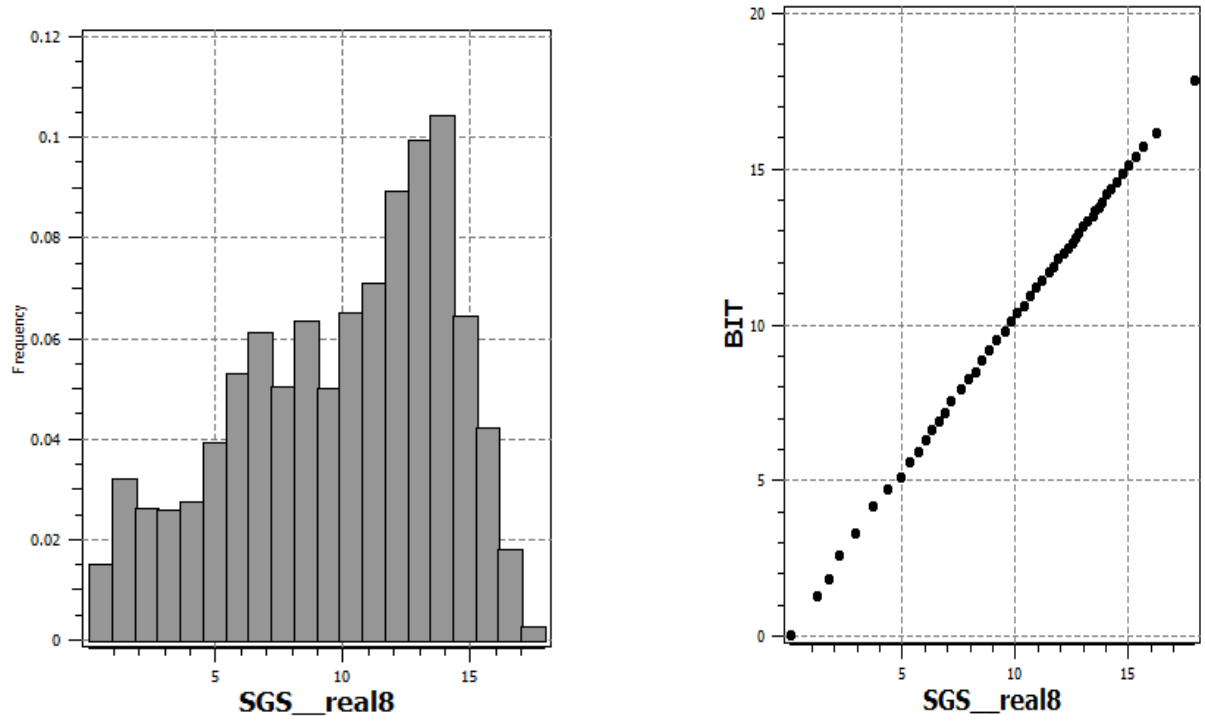

Figure 4-15: Histogram plot of Realization 8 (left) and Q-Q plot of Realization 8 with original input bitumen grades (right)

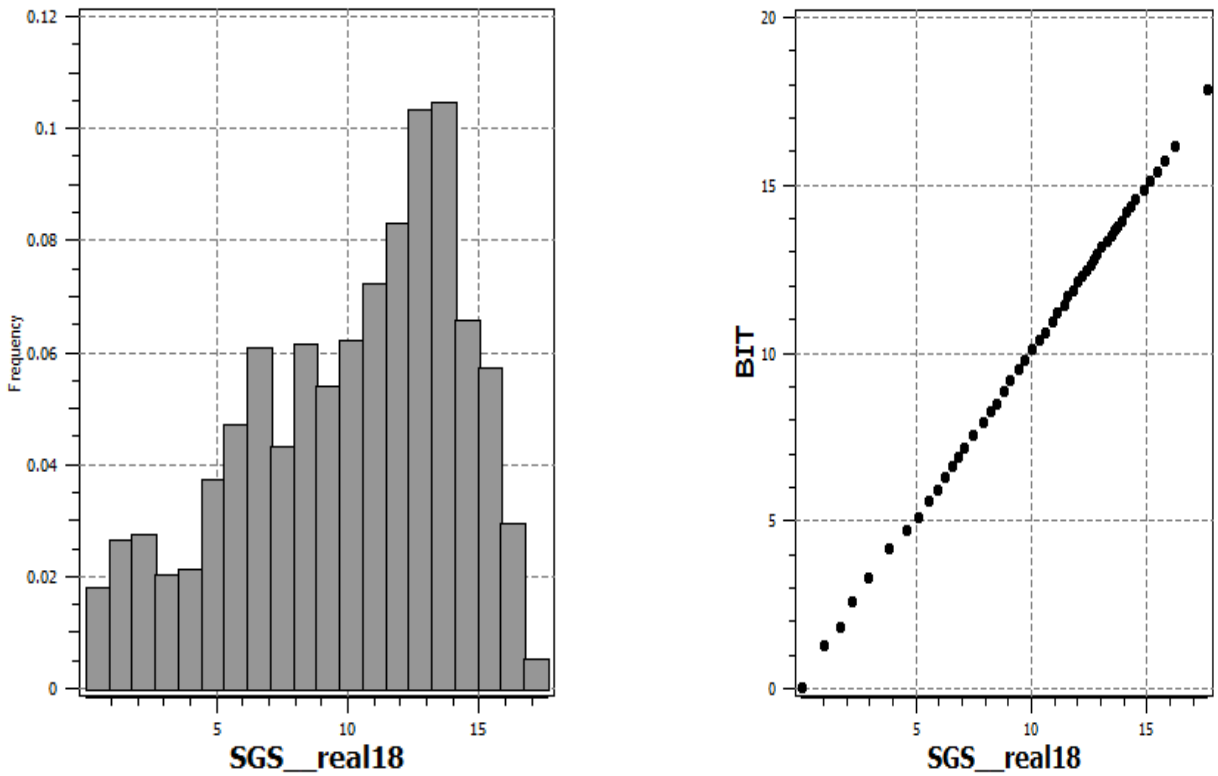

Figure 4-16: Histogram plot of Realization 18 (left) and Q-Q plot of Realization 18 with original input bitumen grades (right)

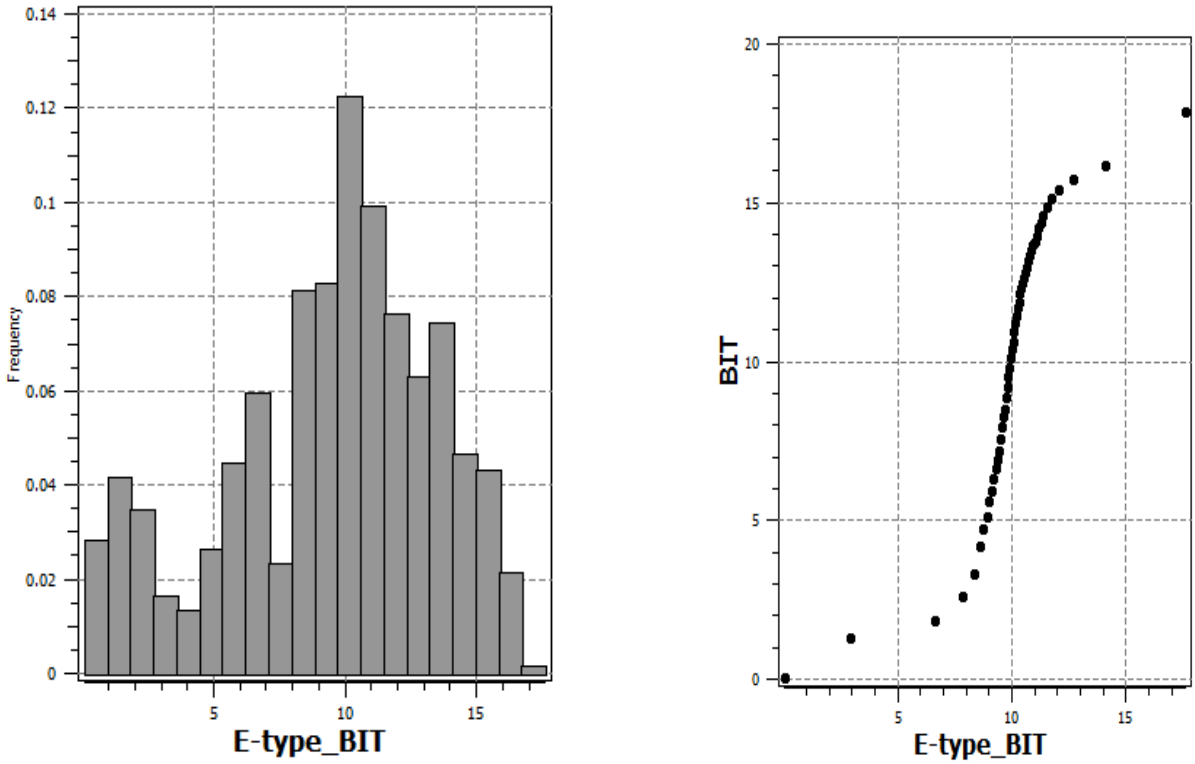

Figure 4-17: Histogram plot of E-type (left) and Q-Q plot of E-type with original input bitumen grades (right)

4.4.6 Ultimate pit limit determination and final pit limit design

The ultimate pits for the OK, E-type and SGS block models were generated using Geovia Whittle (Geovia Dassault Systems, 2018). The ultimate pit limits for these block models were compared while considering the number of pit shells that was generated in each instance. In comparison, the ultimate pit shells generated for each block model were similar with a total number of 16 pit shells using revenue factors ranging from 0.36 to 2.0 and an overall pit slope of 14 degrees. Due to this consistency, the ultimate pit limit for the OK block model was considered as the reference pit limit and subsequently used for pit design in Geovia GEMS (Geovia Dassault Systems, 2018).

Table 4-7 shows the errors of the pit slope for the ultimate pit limit which are within the acceptable range in the industry. Usually, the average slope error should not exceed 1 degree (Geovia Dassault Systems, 2018). Table 4-8 shows the 16 pit shells generated for OK block model with different revenue factors. By increasing the revenue factor, the selling price increases and therefore the size of the pit and the strip ratio increases. Revenue factors less than one create smaller pit shells aimed to extract more ore blocks with higher grades. Lower revenue factors create more traditional pit limits where the price of the mineral is predicted to be lower than the current price. The base case pit shell is generated with a revenue factor equal to one which can be seen as pit shell number 16. In the final pit, there are 191.6 million tonnes of ore and 129.8 million tonnes of waste. Figure 4-18 shows the ultimate pit outline generated using 3D LG algorithm.

Table 4-7: Errors of generated pit slope by 3D LG algorithm

Parameter (Units)	Value
Minimum slope error (°)	0.00
Average slope error (°)	0.20
Maximum slope error (°)	0.50

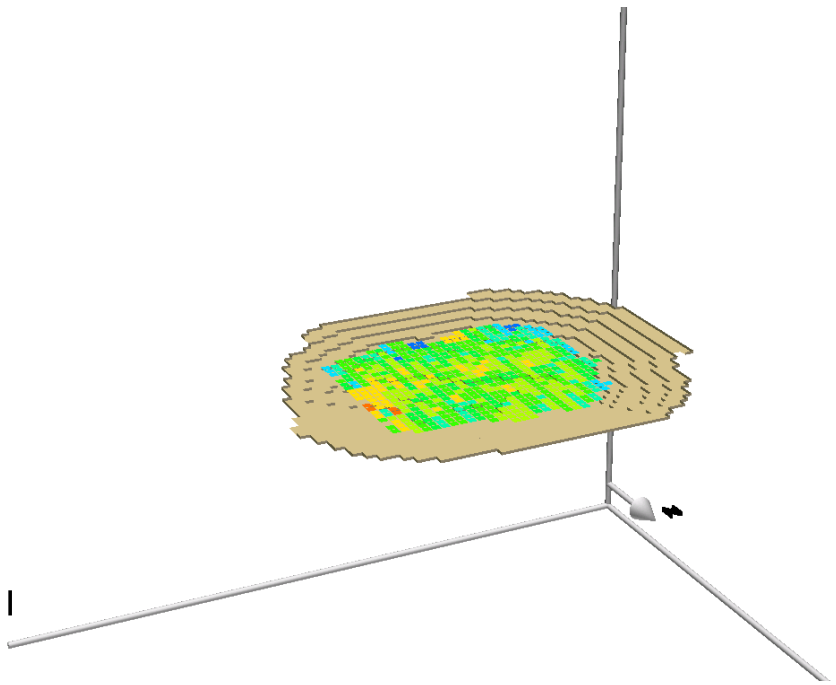

Figure 4-18: Ultimate pit outline generated using 3D LG algorithm

Table 4-8: Generated pit shells using different revenue factors for OK block model

Pit	Revenue Factor	Rock (Mtonnes)	Ore (Mtonnes)	Stripping Ratio	Max Bench	Min Bench	Bitumen Quantity (Mtonnes)	Average Bitumen Grade (%m)
1	0.36	5.47	1.58	2.47	14	12	0.10	6.18
2	0.38	79.77	19.14	3.17	14	9	1.69	8.85
3	0.40	93.31	26.30	2.55	14	9	2.34	8.88
4	0.42	121.53	41.42	1.93	14	9	3.78	9.12
5	0.44	149.49	57.96	1.58	14	8	5.41	9.33
6	0.46	168.21	72.76	1.31	14	8	6.89	9.48
7	0.48	173.61	79.22	1.19	14	8	7.54	9.52
8	0.50	206.41	96.15	1.15	14	8	9.18	9.55
9	0.52	222.76	111.27	1	14	8	10.55	9.48
10	0.54	232.87	119.38	0.95	14	8	11.28	9.45
11	0.56	256.06	135.84	0.88	14	8	12.75	9.39
12	0.58	277.97	152.14	0.83	14	8	14.33	9.42
13	0.60	295.91	167.19	0.77	14	8	15.84	9.48
14	0.62	302.36	174.41	0.73	14	8	16.57	9.50
15	0.64	316.85	187.17	0.69	14	8	17.88	9.55
16	0.66 – 2.0	321.43	191.55	0.68	14	8	18.32	9.56

Table 4-9: Design parameters for final pit

Parameter (Units)	Value
Berm width (m)	12.00
Bench height (m)	14.00
Pit slope angle (°)	14.00
Batter angle (°)	17.30
Tolerance angle (°)	3.00
Tolerance distance (m)	1.00

The next step was to export the ultimate pit shell from Geovia Whittle to Geovia GEMS for the final pit design. Table 4-9 shows the design parameters used for the final pit. During pit design, it is important to ensure the pit slopes follow the ultimate pit shell. To validate the pit design, volumetric analysis was performed by comparing the total amount of material in the ultimate pit shell with that in the final pit design. The total amount of materials in the final pit design was 314.23 million tonnes. The difference in material tonnage between the ultimate pit shell and final pit design was 2.24%, which is below the 10% tolerance allowed in the industry. The final pit design was considered as acceptable based on this criteria.

4.5 Implementation of the SMILP framework: Case study

This section provides a detailed documentation of the first case study experimental design and application of the SMILP model framework to an oil sands deposit. In this case study, there were three scenarios to be optimized. A single orebody model which was based on Ordinary Kriging estimation was considered as the first scenario and the base or conventional case study of this research. The second scenario was based on the E-type block model which is the average simulated block model generated by post-processing of the SGS realizations. The E-type estimates were slightly different than the Ordinary Kriging model. However, theoretically the E-type model is identical with the kriging results at Gaussian space (Journel and Huijbregts, 1978; Journel and Isaaks, 1984). Lastly, the third scenario was based on stochastic block models represented by 20 SGS realizations. The SGS realizations provide a set of equiprobable block models that are used to capture and assess uncertainty in the final pit outline and production schedule. Generally, recent studies based on stochastic orebody modelling and stochastic long-term production scheduling has shown that the scheduled results are not sensitive to the use of more realizations after ten, and twenty to four hundred will not provide different results in terms

of schedules and forecasts (Albor and Dimitrakopoulos, 2009; Koushavand et al., 2014; Garcia and Dimitrakopoulos, 2018).

Figure 4-19 shows the implementation scenarios that were investigated for this case study. A summarized information of the oil sands materials which include ore, dyke and waste materials contained in the ultimate pit design are presented in Table 4-10. The minimum and maximum limits of the material quantity and quality requirements for ore bitumen grades, ore fines grades, IB dyke material fines grade, and sand-to-fines blend ratio for the case study can be seen in Table 4-11. The operational capacity constraints and mine life were determined based on Taylor's mine life rule which is used to estimate the production rate and life of mine during the ultimate pit design stage (Taylor, 1986). The quality requirements were determined based on the recommendations of the Alberta Energy Regulator (AER) (Regulator, 2016) on waste management performance and operating criteria for oil sands mining schemes (McFadyen, 2009; Regulator, 2016). The economic parameters used in the case study for all scenarios are shown in Table 4-12. The economic parameters used for the case study were obtained from Ben-Awuah and Askari-Nasab, (2011). The risk parameters shown in Table 4-13 were primarily for Scenario 3 to minimize the risk associated with ore bitumen grade and ore tonnage deviations from the set targets during production scheduling. These risk parameter data were considered as the starting point of the optimization run for Scenario 3 and the parameters were subjected to sensitivity analysis to assess its performance and impact in the SMILP model. In Scenarios 1 and 2, there was no consideration of uncertainty and the risk parameters were set to zeros. Comparative analysis based on the three scenarios evaluated demonstrate the benefit of incorporating grade uncertainty and waste management in oil sands production scheduling to minimize financial losses and the opportunity cost of an over- or under-design of a suitable waste management plan. In addition, sensitivity analysis of the SMILP production schedule based on the GDR and penalties parameters were examined.

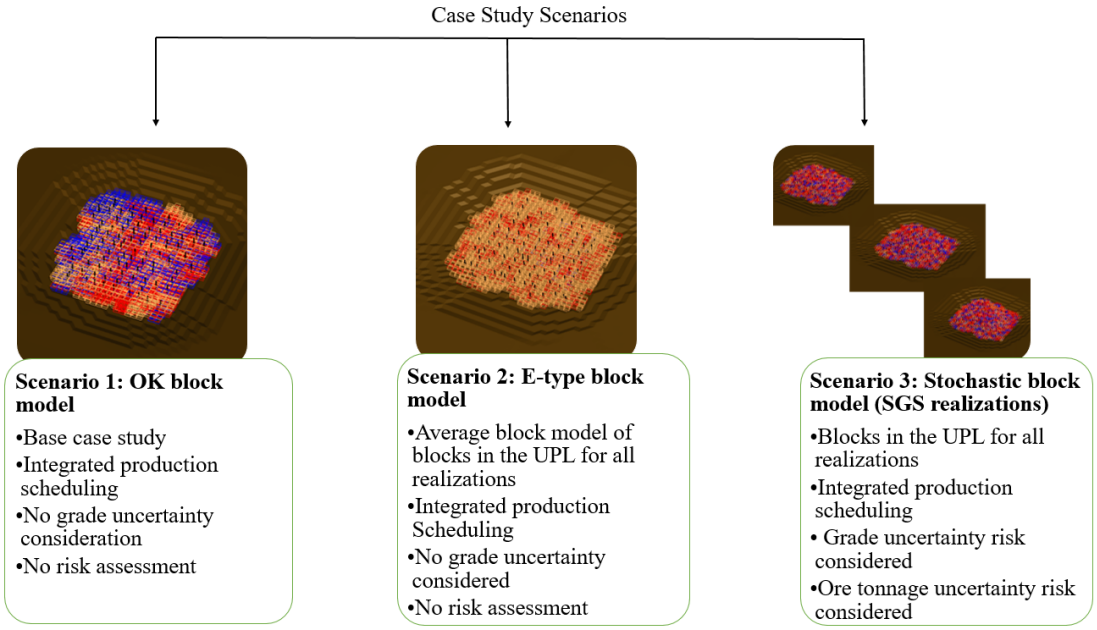

Figure 4-19: Implementation of the case study

Table 4-10: Block model data of oil sands materials for each scenario of the case study

Parameter (Unit)	Scenario 1 (OK block model)	Scenario 2 (E-type block model)	Scenario 3 (Stochastic block model - 20 SGS realizations)	
			Min	Max
Total block tonnage (Mt)	314.23			
Total mineralized tonnage (Mt)	270.04			
Total ore tonnage (Mt)	126.17	163.88	121.44	128.48
Total IB dyke material tonnage (Mt)	143.87	106.15	148.59	141.56
Total TCS dyke material tonnage (Mt)	87.21	110.05	78.51	84.00
Total OB dyke material tonnage (Mt)	38.10			
Block dimensions (m)	50 x 50 x 15			
Number of blocks (#)	4476			
Number of benches	6			
Mine life (Years)	10			

Table 4-11: Operational capacities and quality requirements for all case study scenarios

Parameter (unit)	Lower bound	Upper bound
Mining capacity (Mt/Year)	31.40	32.00
Processing capacity (Mt/Year)	10.00	14.00
OB dyke material capacity (Mt/Year)	1.00	3.80
IB dyke material capacity (Mt/Year)	1.00	4.00
TCS dyke material capacity (Mt/Year)	1.00	8.00
Ore bitumen grade (%m)	7.00	16.00
Ore fines grade (%m)	0.00	30.00
IB dyke material fines grade (%m)	0.00	50.00
Sand-to-fines ratio	0.00	6.00

Table 4-12: Economic parameters for all case study scenarios

Parameter (unit)	Value	Parameter (unit)	Value
Mining cost ($/tonne)	4.60	OB dyke material cost ($/tonne)	1.38
Processing cost ($/tonne)	5.03	IB dyke material cost ($/tonne)	1.38
Selling price ($/bitumen %mass)	4.50	TCS dyke material cost ($/tonne)	0.92
Economic discount rate (%)	10.00		

Table 4-13: Risk parameters for all case study scenarios

Parameter (unit)	Scenario 1 (OK)	Scenario 2 (E-type)	Scenario 3 (SGS)
Number of realizations (#)	1	1	20
Geological risk discount rate (%)	0	0	20
Cost of shortage in ore production ($/tonne)	0	0	5
Cost of excess in ore production ($/tonne)	0	0	10
Cost of shortage in ore bitumen grade ($/%m)	0	0	2.5
Cost of excess in ore bitumen grade ($/%m)	0	0	1.5

4.5.1 Discussion of Scenario 1 results: Case study with OK block model (conventional case)

This scenario features the conventional approach based on a single OK estimated orebody model as input to the SMILP model. No grade uncertainty was considered and so the cost of grade uncertainty was set to zero for this scenario. The production requirements included mining, processing and dyke material capacities boundaries and material quality requirements. The optimized production schedule generated in Scenario 1, was referred to as OK schedule. The MIP gap tolerance for this scenario was set at 5%.

The mine production schedule was limited by mining, processing, and dyke construction events. The validity of the schedule was based on its compliance to the operational constraints in Table 4-11 for all periods. The main focus of this experiment was to achieve a uniform processing feed throughout the mine life and generate a consistent production schedule with a maximized NPV while meeting the processing capacity requirements, and the quality and quantity requirements of dyke material needed for dyke construction.

The results of the production schedule based on the OK model are shown in Table 4-14, and Figure 4-20 to Figure 4-24. In the first year of the production schedule, it was observed that waste material was mined so as to get access to the ore. In the second year, the ore material became available and production ramps up from the third year to the eight year when the materials begin to deplete until the end of mine life. The material supply of overburden, interburden and TCS for dyke construction is maintained steadily throughout the life of mine. The overall NPV generated without dyke material cost was $1929.35 M while the overall NPV generated including dyke material cost was $1894.00 M.

Table 4-14: Production schedule results for case study Scenario 1

Period (Years)	Material mined (Mt)	Material processed (Mt)	Waste tonnage (Mt)	OB+IB dyke material tonnages (Mt)	TCS dyke material tonnages (Mt)	Average bitumen grade (%m)
1	31.55	10.00	19.55	2.00	2.00	12.76
2	31.40	12.00	17.40	2.00	2.00	12.36
3	31.40	14.00	15.40	2.00	2.00	12.41
4	31.40	14.00	15.40	2.00	2.00	10.95
5	31.40	14.00	15.40	2.00	2.00	11.42
6	31.40	14.00	15.40	2.00	2.00	10.73
7	31.40	14.00	15.40	2.00	2.00	10.52
8	31.40	14.00	15.40	2.00	2.00	10.05
9	31.40	13.00	16.40	2.00	2.00	8.92
10	4.40	2.41	0.00	2.00	2.00	7.78

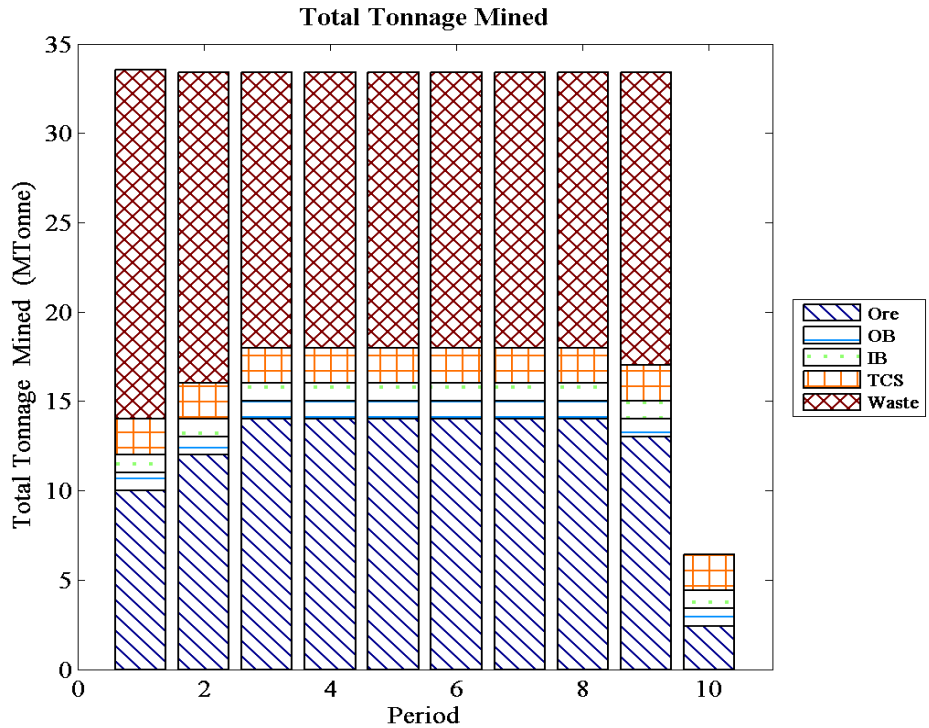

Figure 4-20: Schedules for Ore, OB, IB and TCS dyke materials, and waste for Scenario 1

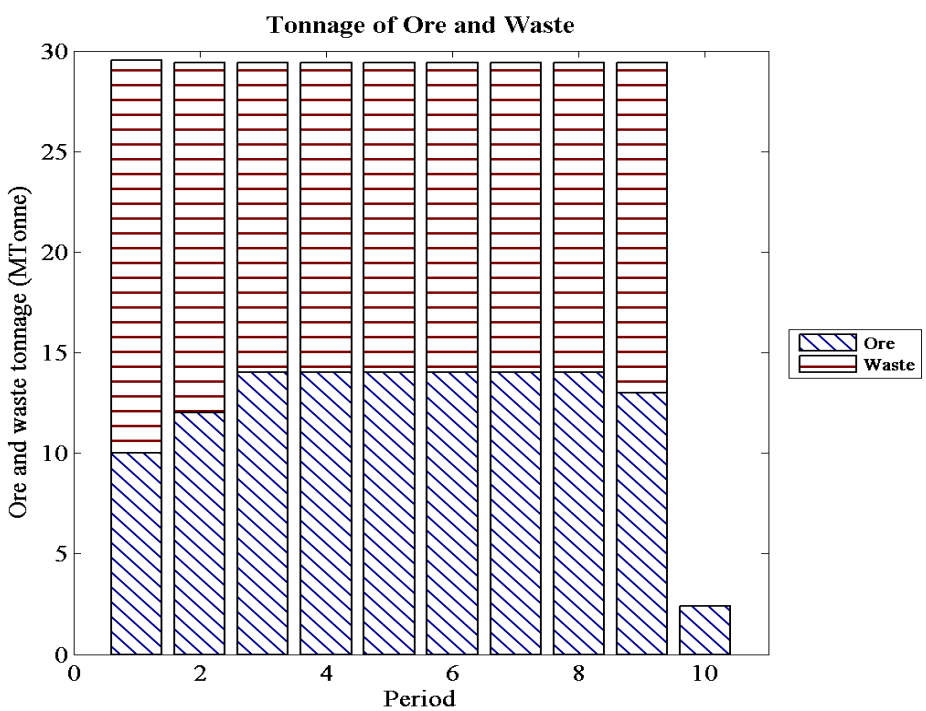

Figure 4-21: Ore and waste production schedules for Scenario 1

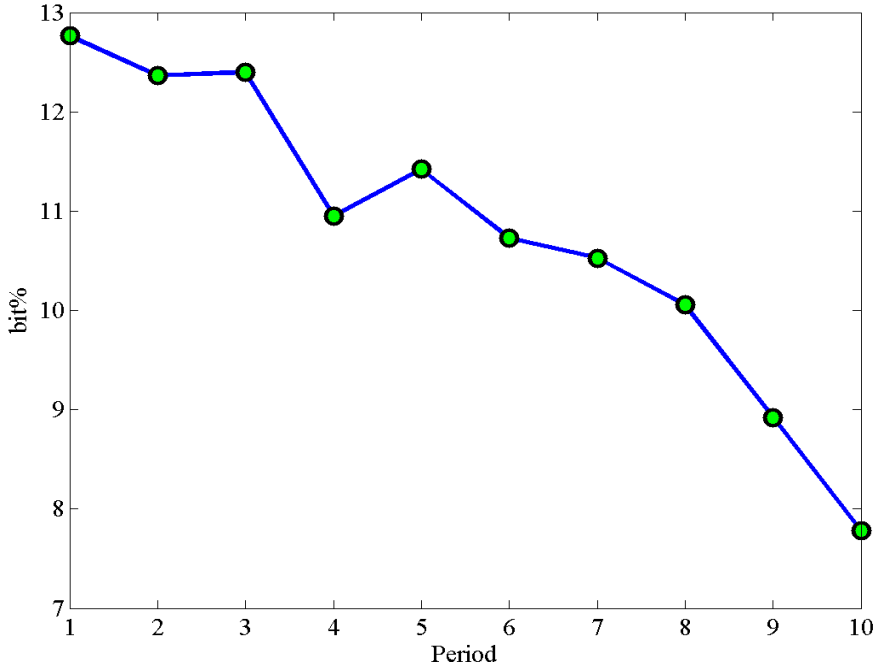

Figure 4-22: Average ore bitumen grades in all periods for Scenario 1

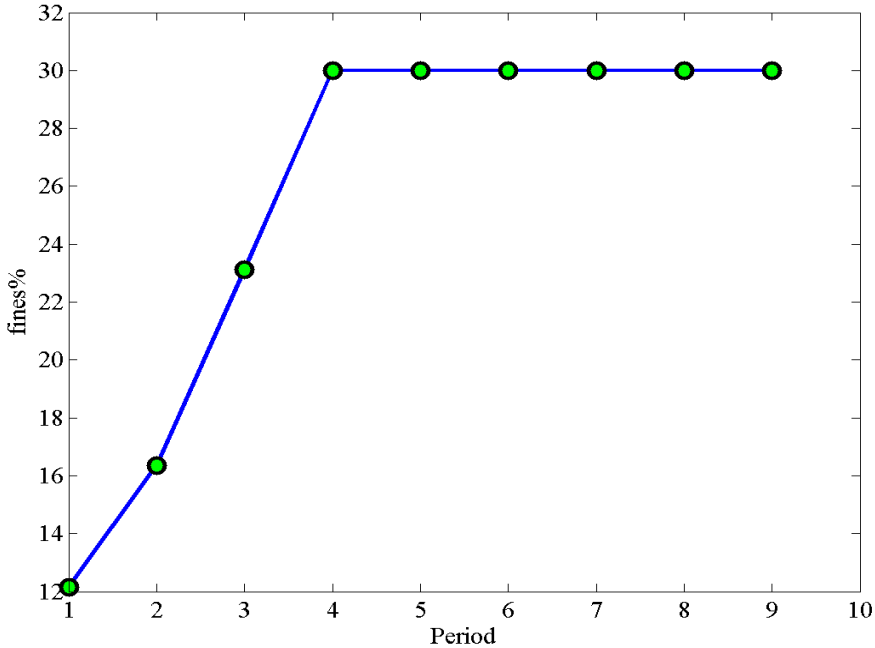

Figure 4-23: Average ore fines percent in all periods for Scenario 1

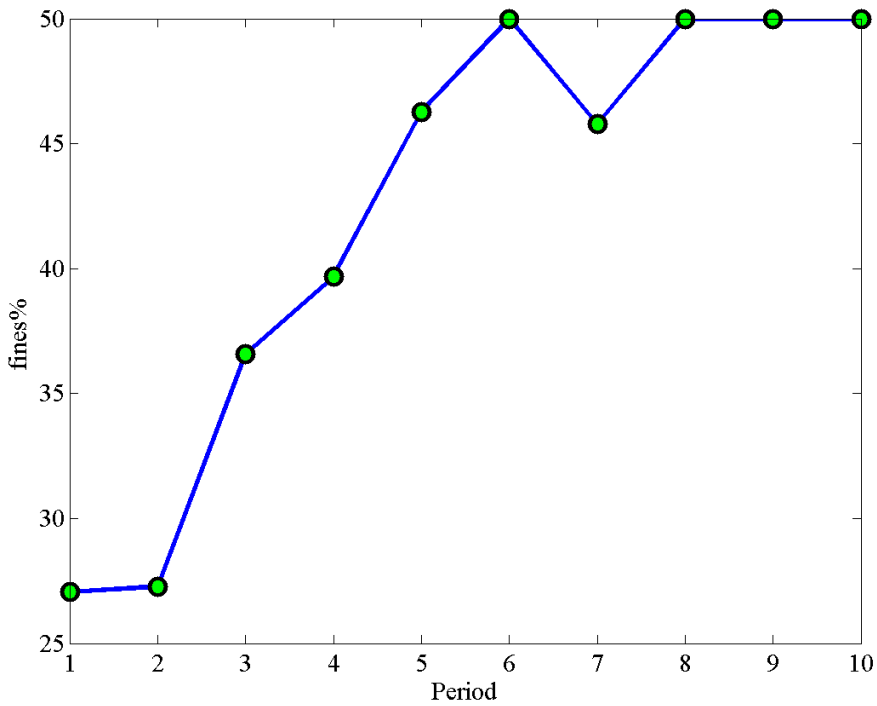

Figure 4-24: Average dyke material fines percent in all periods for Scenario 1

4.5.2 Discussion of Scenario 2 results: Case study with E-type block model

The E-type block model is the average block model of all the realizations generated. Similarly, this experiment was setup to achieve a uniform processing feed throughout the mine life. The yearly production schedules for mining, processing and dyke materials are shown in Table 4-15 and Figure 4-25 to Figure 4-29. Scenario 2 achieved the operational constraints and material quality and quantity requirements specified in Table 4-11 and provided a consistent production schedule for mining, ore processing and dyke construction throughout the mine life. The production schedule generated in Scenario 2 was referred to as E-type schedule and was generated with a MIP gap tolerance of 5% which was similar to the gap tolerance used in Scenario 1.

In the first year of the E-type production schedule, it was observed that waste material was mined to get access to the ore. In the second year, the processing plant operates at a maximum capacity as the ore material becomes available and begins to ramp up from the third year to the tenth year. The material supply of overburden, interburden and TCS for dyke construction was maintained steadily throughout the life of mine. The overall NPV generated without dyke material cost was $1973.87 M while the overall NPV generated including dyke material cost was $1938.52 M.

Table 4-15: Production schedule results for case study Scenario 2

Period (Years)	Material mined (Mt)	Material processed (Mt)	Waste tonnage (Mt)	OB+IB dyke material tonnages (Mt)	TCS dyke material tonnages (Mt)	Average bitumen grade (%m)
1	31.40	10.00	19.40	2.00	2.00	13.86
2	31.40	12.00	17.40	2.00	2.00	12.66
3	31.40	14.00	15.40	2.00	2.00	12.05
4	31.40	14.00	15.40	2.00	2.00	12.04
5	31.40	14.00	15.40	2.00	2.00	12.04
6	31.40	14.00	15.40	2.00	2.00	12.00
7	31.40	14.00	15.40	2.00	2.00	12.20
8	31.40	14.00	15.40	2.00	2.00	11.80
9	31.40	14.00	15.40	2.00	2.00	11.70
10	31.40	14.00	15.40	2.00	2.00	10.20
11	0.23	0.23	0.00	0.00	0.00	9.05

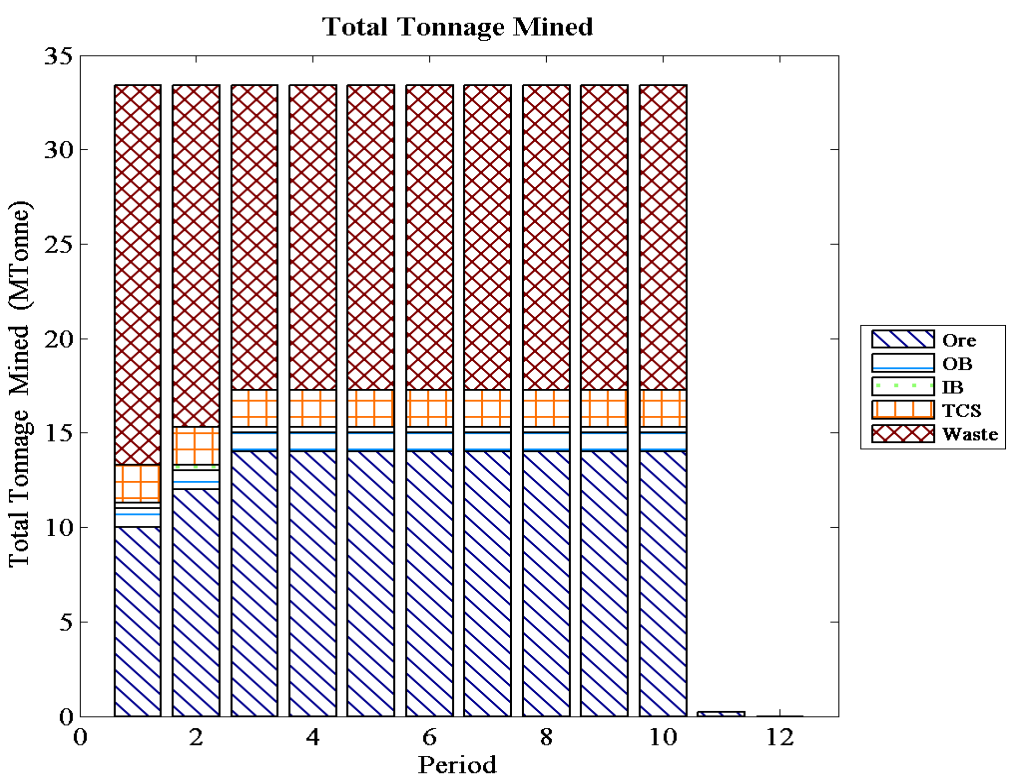

Figure 4-25: Schedules for Ore, OB, IB and TCS dyke materials, and waste for Scenario 2

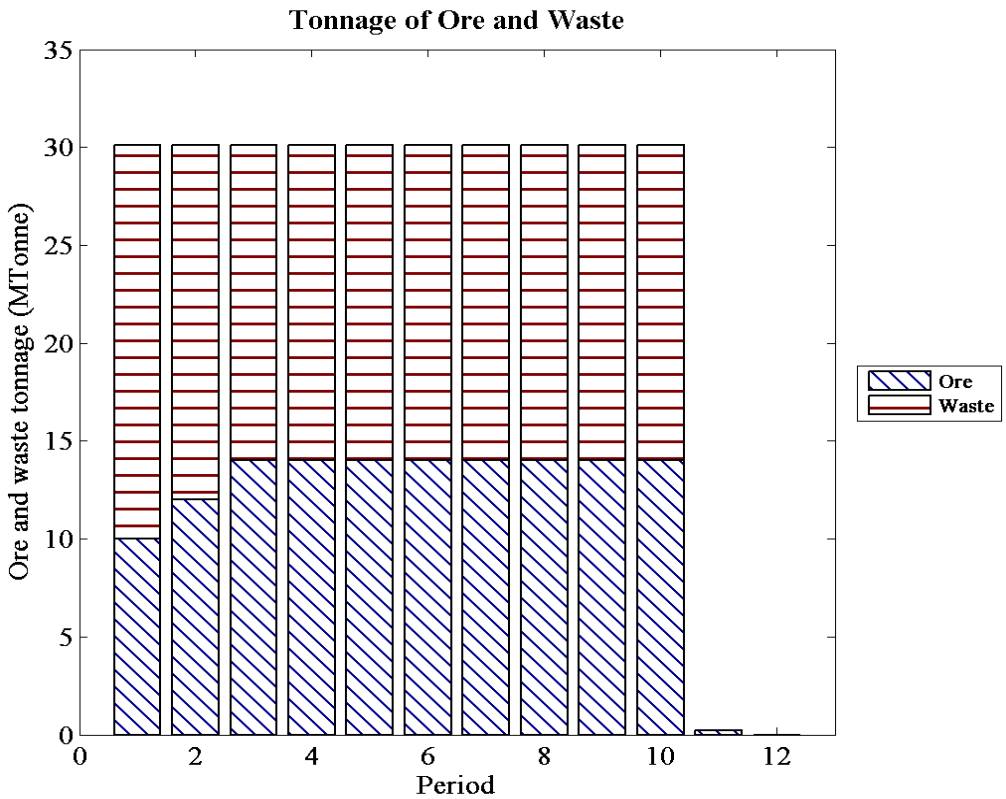

Figure 4-26: Ore and waste production schedules for Scenario 2

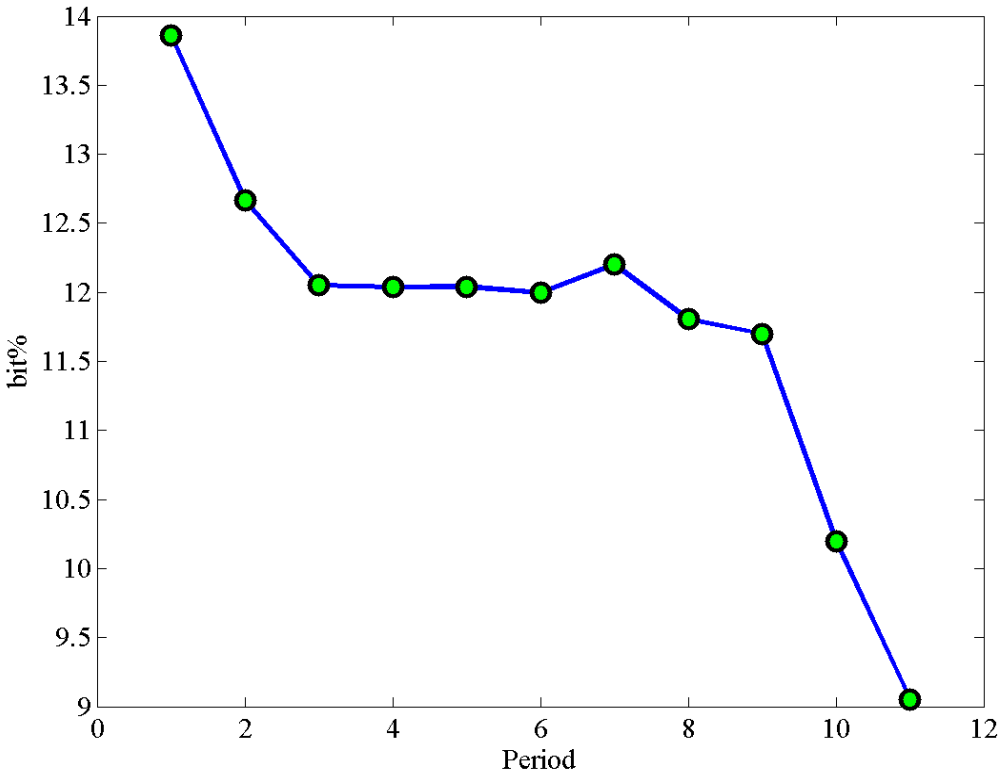

Figure 4-27: Average ore bitumen grades in all periods for Scenario 2

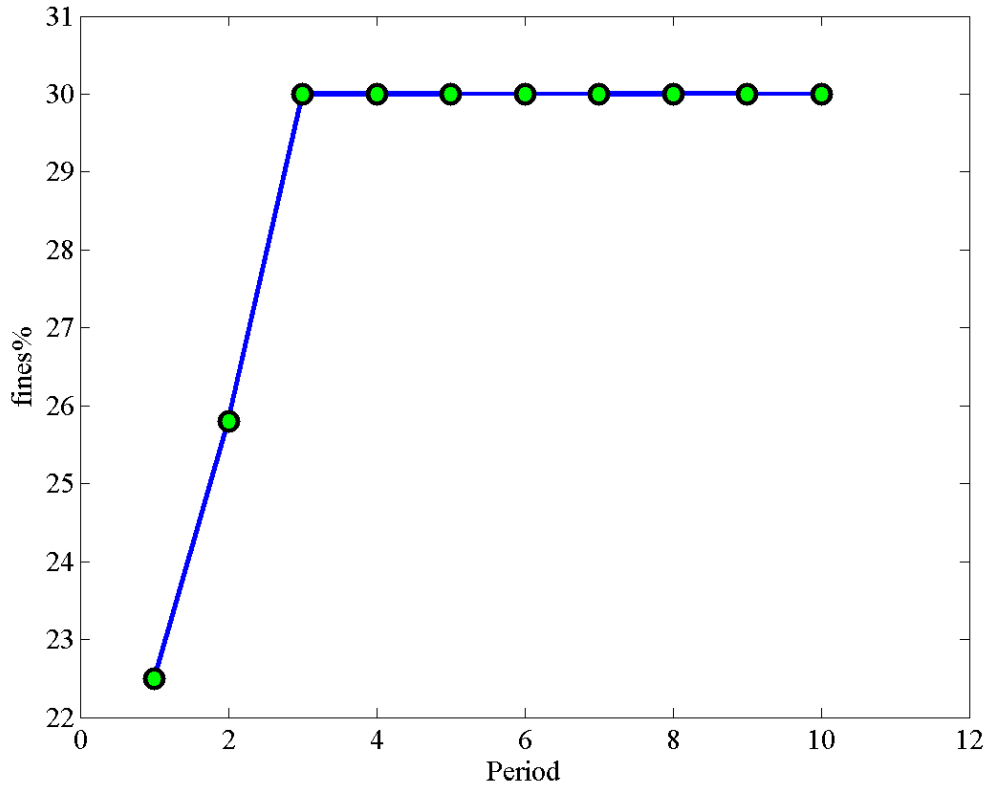

Figure 4-28: Average ore fines percent in all periods for Scenario 2

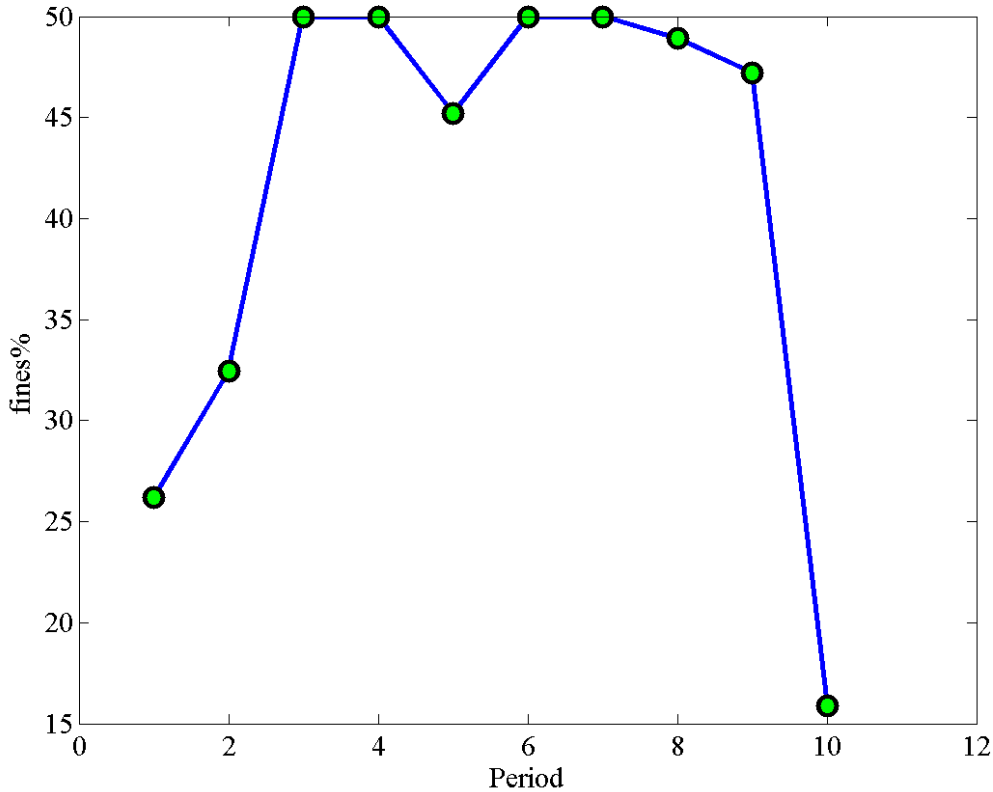

Figure 4-29: Average dyke material fines percent in all periods for Scenario 2

4.5.3 Discussion of Scenario 3 results: Case study with Stochastic block models (20 SGS realizations)

The third scenario features application of the SMILP framework with 20 equiprobable orebody realizations from SGS. Risk control parameters were implemented while maintaining the same technical and economic parameters. Uncertainty penalty cost parameters were implemented to manage the risk associated with deviations from the set ore production targets. This is achieved by simultaneously penalizing ore tonnage fluctuations and ore bitumen grade fluctuations for all realizations in the objective function. Higher penalty are applied in in the earlier periods in the objective function which forces the optimizer to minimize ore tonnage and ore grade deviations from the set target. This produces a smooth and practical schedule known as the SMILP schedule. The MIP gap tolerance was set at 5% as in Scenarios 1 and 2.

The overall NPV generated without dyke material cost was $2248.43 M while the overall NPV generated including dyke material cost was $2213.08 M. The yearly production schedule for Scenario 3 can be seen in Table 4-16, and Figure 4-30 to Figure 4-34. The SMILP schedule obtained was subsequently applied to the 20 individual SGS realizations to assess how they reconcile with the SMILP schedule results. Figure 13 to Figure 15 show how the results compare.

Table 4-16: Production schedule results for case study Scenario 3

Period (Years)	Material mined (Mt)	Material processed (Mt)	Waste tonnage (Mt)	OB+IB dyke material tonnages (Mt)	TCS dyke material tonnages (Mt)	Average bitumen grade (%m)
1	31.40	10.00	19.40	2.00	2.00	12.28
2	31.40	12.00	17.40	2.00	2.00	12.08
3	31.40	13.97	15.42	2.00	2.00	12.01
4	31.40	13.90	15.50	2.00	2.00	11.98
5	31.40	13.82	15.58	2.00	2.00	11.98
6	31.40	13.73	15.67	2.00	2.00	11.81
7	31.40	13.85	15.55	2.00	2.00	11.89
8	31.40	13.74	15.66	2.00	2.00	11.98
9	31.40	13.61	15.79	2.00	2.00	11.74
10	8.27	5.24	1.02	2.00	2.00	11.64

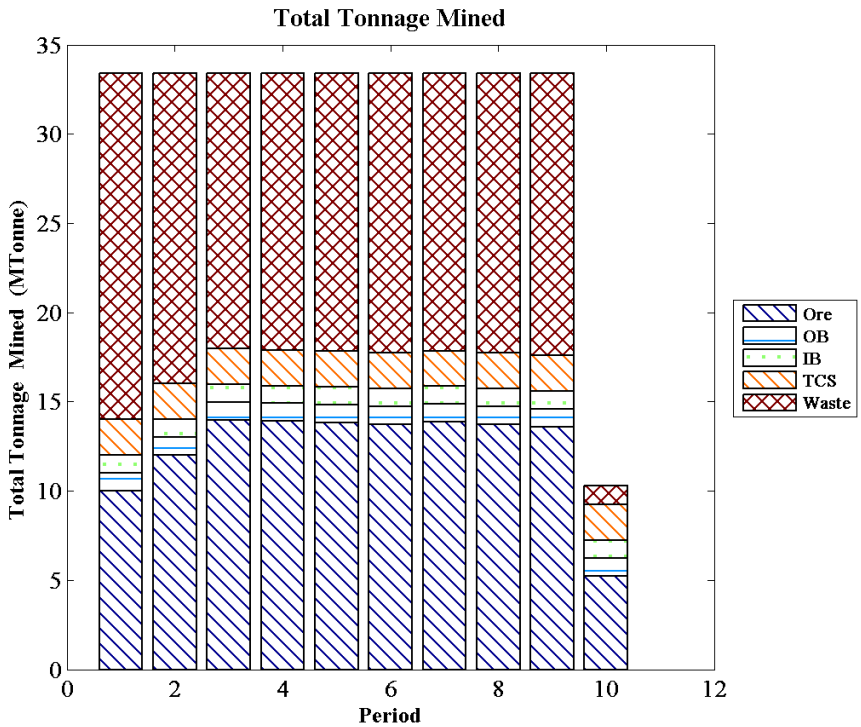

Figure 4-30: Schedules for Ore, OB, IB and TCS dyke materials, and waste for Scenario 3

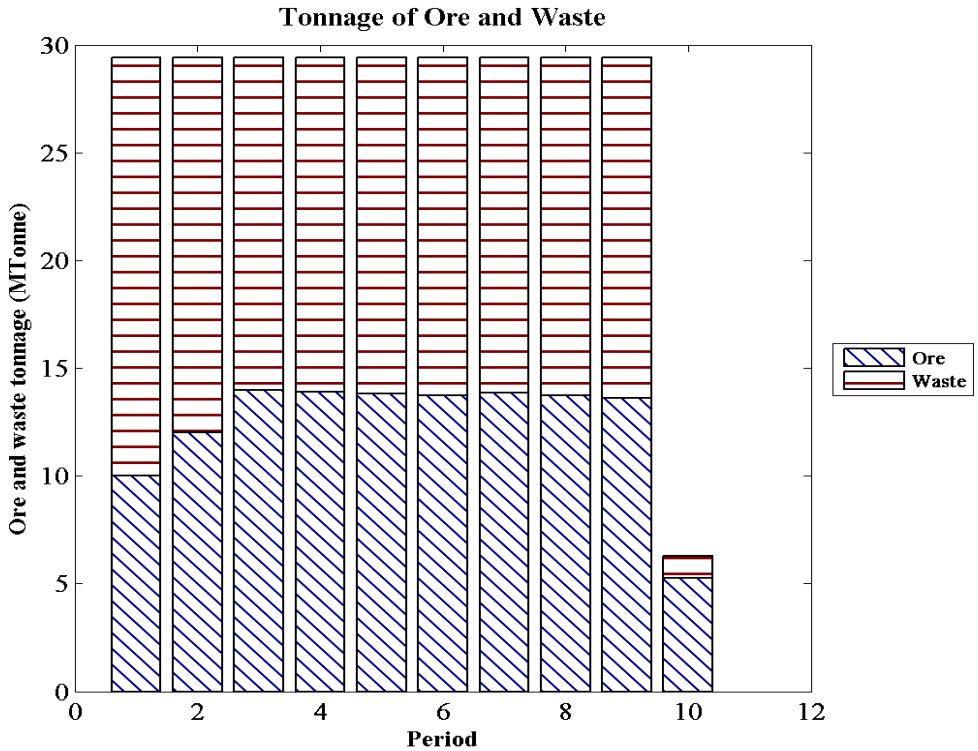

Figure 4-31: Ore and waste production schedules for Scenario 3

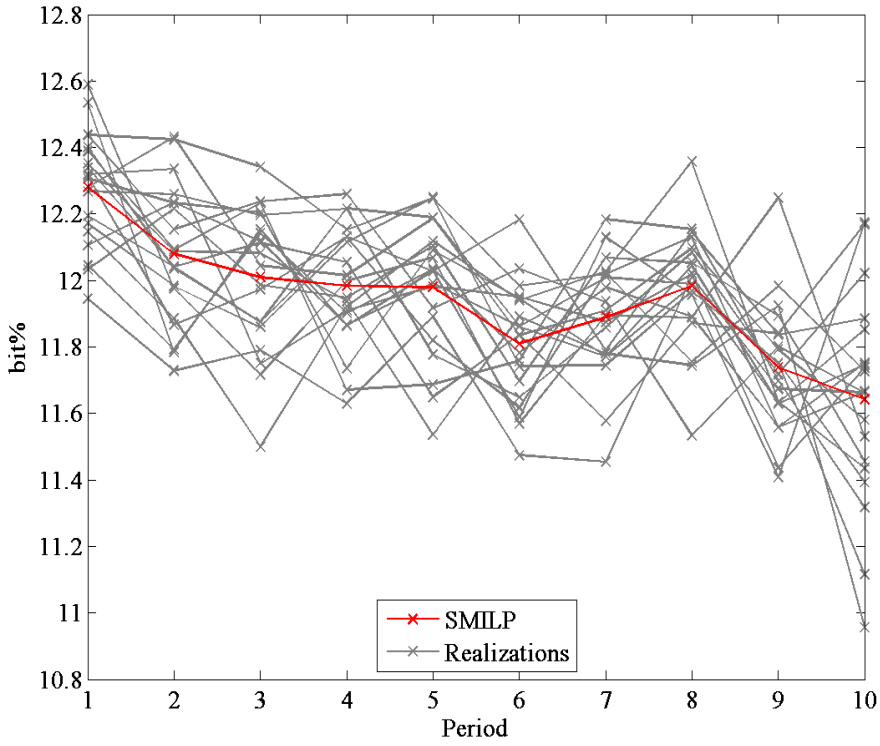

Figure 4-32: Average ore bitumen grades in all periods for Scenario 3, and applied to 20 SGS realizations

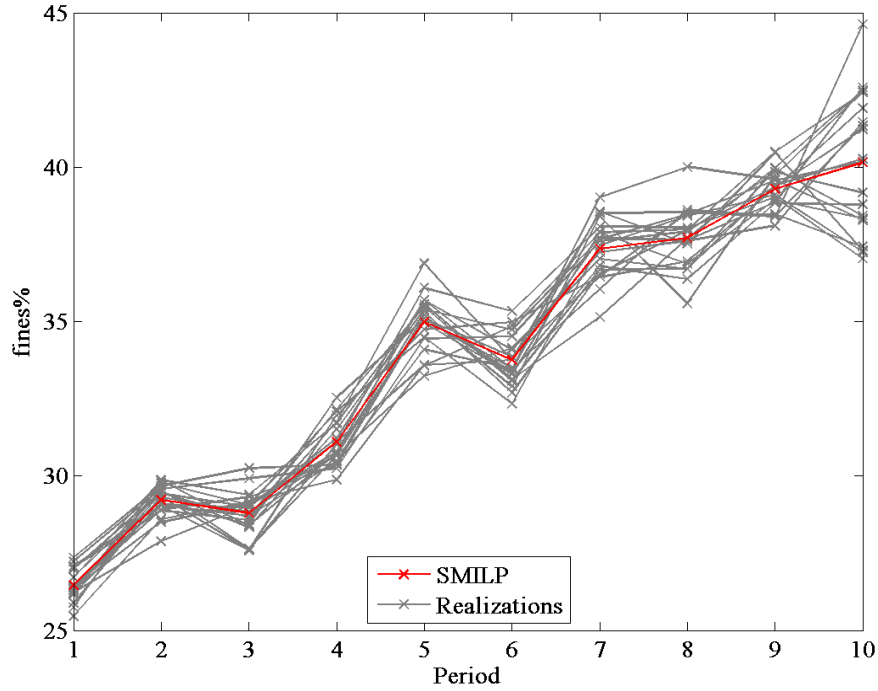

Figure 4-33: Average ore fines percent in all periods for Scenario 3, and applied to 20 SGS realizations

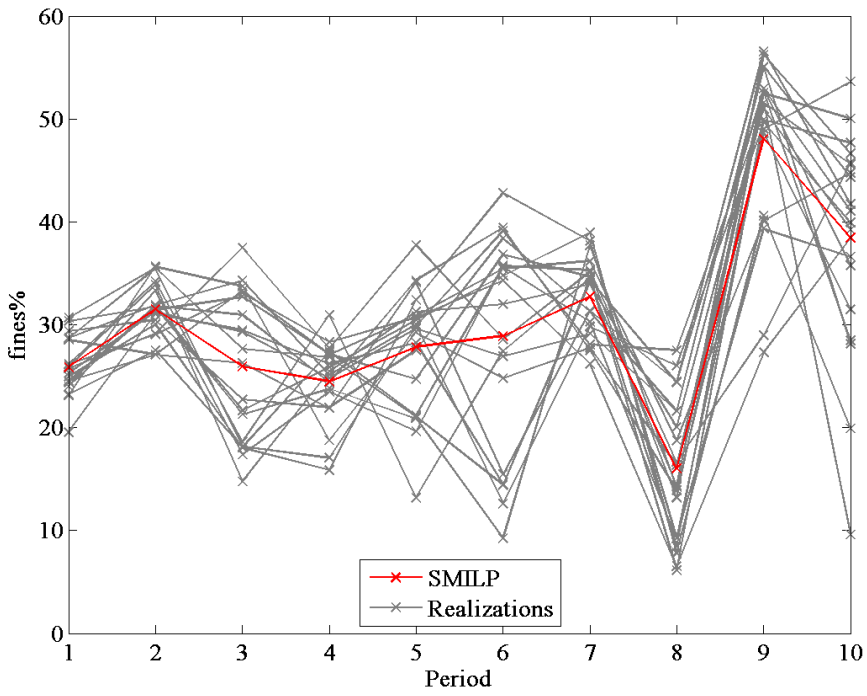

Figure 4-34: Average dyke material fines percent in all periods for Scenario 3, and applied to 20 SGS realizations

4.5.4 Comparative analysis of Case study

The optimal production schedules result for the OK schedule, E-type schedule and SMILP schedule were compared. The optimized SMILP schedule was generated based on the stochastic block model (20 SGS realizations) and a geological risk discount factor (GDR factor) to incorporate grade uncertainty into the optimization framework. The SMILP schedule results were subsequently applied to the 20 individual SGS realizations to assess how they reconcile with the three primary production schedules (OK, E-type and SMILP schedules). All three primary production schedules were compared based on the following areas of interest; cash flows, average ore bitumen grade, ore tonnage and waste management costs. Table 4-17 shows a summary of results for the comparison.

Table 4-17: Summary of material handling and performance measures for OK, E-type and SMILP schedules, and applied SGS realization schedules from the SMILP schedule

Parameters (Unit)	OK schedule	E-type schedule	SMILP schedule	SGS realizations schedule	
				Min	Max
Total tonnage mined (Mt)	287.16	314.23	290.87	290.87	290.87
Ore tonnage (Mt)	121.41	134.23	123.88	120.73	127.61
Waste tonnage (Mt)	145.75	160.00	146.99	143.48	150.41
OB and IB dyke material tonnage (Mt)	20.00	20.00	20.00	20.00	20.00
TCS dyke material tonnage (Mt)	20.00	20.00	20.00	18.74	21.57
NPV with dyke material cost ($M)	1894.00	1938.52	2213.08	1913.06	2443.94
NPV without dyke material cost ($M)	1929.35	1973.87	2248.43	1963.03	2496.34
CPU run time (hours)	4.13	10.51	11.70	-	-
MIP gap tolerance (%)			5.00		

4.5.4.1 Cash flow and NPV comparisons for Case study

The profitability of the mine plan was evaluated by comparing the cash flows from the production schedules and the realizations as shown in Figure 4-35. From Figure 4-35, the cash flow of the OK schedule in the second year was the highest compared to the other models but there was significant drop in cash flow for the remaining periods. The cash flow of the SMILP and E-type schedules remain uniform and consistent throughout the life of mine until the material becomes depleted in Periods 10 and 11. The NPV generated by the SMILP schedule was better than that of the E-type schedule and OK schedule by 14.29% and 16.85% respectively. This was because of the penalty parameter introduced in the objective function to minimize the risks of not meeting the production targets throughout the mine life. Table 4-18 shows the NPV results obtained from the OK, E-type and SMILP production schedules.

Table 4-18: NPV results obtained from the OK, E-type and SMILP production schedules

Schedule type	NPV with dyke material cost (NPV) ($M)	NPV without dyke material cost (NPVd) ($M)	NPV of models compared (%)	NPVd of models compared (%)
OK schedule (Conventional case)	1894.00	1929.35	-	-
E-type schedule	1938.52	1973.87	2.35	2.31
SMILP schedule	2213.08	2248.43	16.85	16.54

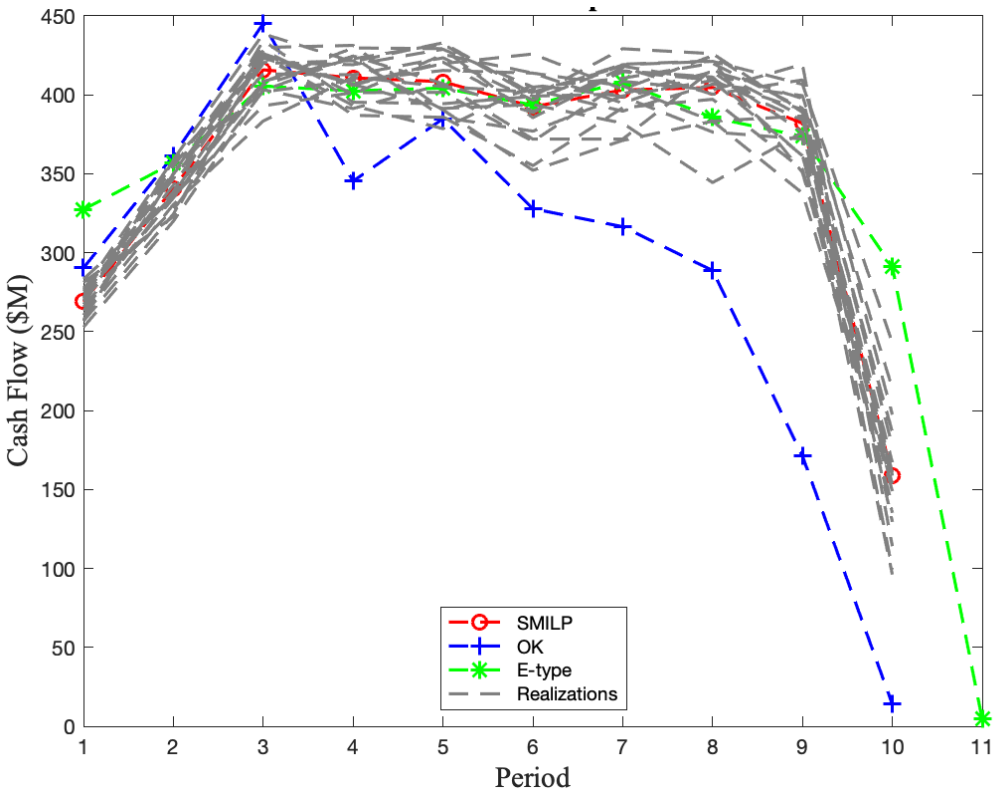

Figure 4-35: Cash flow comparisons for OK, E-type, SMILP and SGS realizations schedules

4.5.4.2 Ore tonnage comparisons for Case study

Figure 4-36 shows the comparison of ore tonnages sent to the plant for all the schedules. It is observed that the ore tonnages are approximately consistent in all periods with the exception of the last two years when the available material in the ultimate pit becomes depleted. The E-type schedule indicates that more ore tonnages are being sent to the plant as compared to both the OK and E-type schedules with a difference of 10.55% and 8.35% respectively. The E-type schedule is based on the average block values of all the realizations which is very sensitive to extreme values. Thus, based on the regulatory bitumen cutoff grade of 7 %m, majority of blocks that have grades close to the marginal cutoff grade get reclassified as ore after averaging and become available for processing. This results in higher ore tonnages being sent to the processing plant in the case of the E-type schedule as shown in Table 4-17.

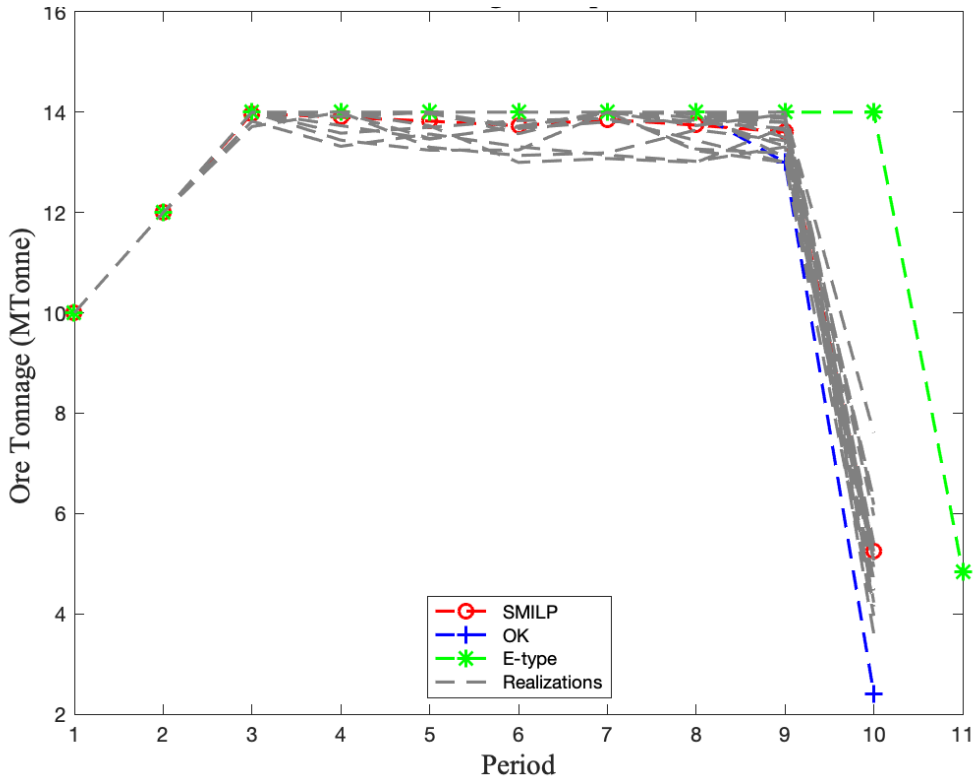

Figure 4-36: Processed ore tonnage comparisons for OK, E-type, SMILP and SGS realizations schedules

4.5.4.3 Average ore bitumen grade comparisons for Case study

The average ore bitumen grade for all three production schedules and that for the simulated realizations are shown on Figure 4-37. A rapid decline of the ore bitumen grade can be seen for the OK schedule which impacts the NPV of the overall mining project compared to the E-type and SMILP schedules. The SMILP schedule generated a balanced ore bitumen grade schedule during the entire mine life as compared to the other models. This is as a result of the penalty and geological risk discount rate parameters that was introduced in the objective function of the SMILP model to control the risk of not meeting production targets by minimizing deviations throughout the mine life while enforcing lower risk in the early productions years and deferring higher risk to later periods when more geological information becomes available.

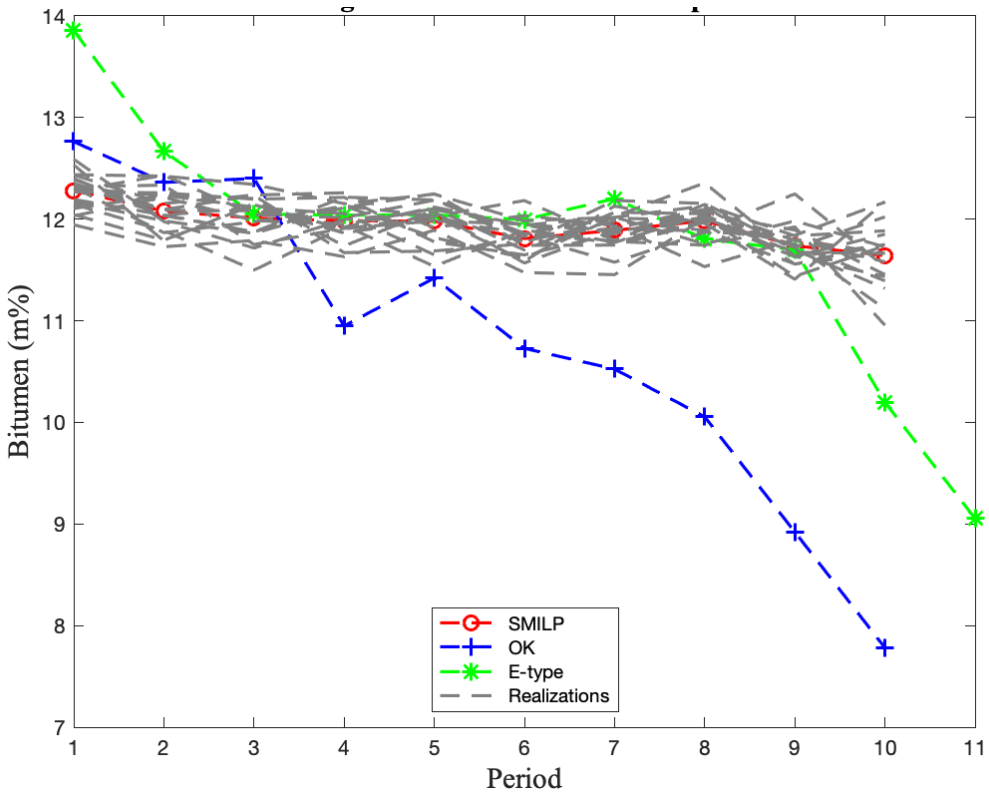

Figure 4-37: Average ore bitumen grade comparisons for OK, E-type, SMILP and SGS realizations schedules

4.5.4.4 Waste management tonnage and cost comparisons for Case study

The comparison of the waste management tonnes and costs in Table 4-19 and Figure 4-38 provide the risk profile for an uncertainty-based waste management strategy. For this case study, the waste management plan was compared with respect to the waste and dyke material tonnages including OB, IB and TCS dyke materials and its corresponding costs for each scenario. Since dyke material mining is an additional cost to the objective function, the optimizer only achieves the set minimum bounds and extracts the remaining dyke materials as waste. The E-type model yielded the most waste tonnage of 160.00 Mt as compared to the OK model that generated 145.75 Mt and SMILP model that generated 146.99 Mt. In terms of waste management costs, the E-type schedule yielded the highest total waste material cost which was 56.09 % more as compared to that of the OK schedule. The higher waste cost in the E-type schedule will have a negative economic impact on the life of mine since it requires more mining fleet and equipment to handle the large amount of waste. There was a reduction of 19.25% in the total waste material cost generated by the SMILP schedule compared to the OK schedule which is economically beneficial to the mine plan. The reduction of the total waste material tonnage and waste material

costs as observed in the SMILP production schedule is as a result of deferring waste mining to later years of mine life where possible. The consideration of grade uncertainty impacts directly the waste disposal plan ensuring that a sustainable waste management strategy is in place to avoid over- or under-design of the waste storage facilities. Insufficient design of waste storage facilities may result in missed environmental and financial opportunities.

Table 4-19: Waste management tonnage and cost comparisons for Case study

Paramater (Unit)		**Scenario 1 (OK schedule)**	**Scenario 2 (E-type schedule)**	**Scenario 3 (SMILP schedule)**
Dyke and waste material tonnages	Total dyke material tonnage (Mt)	40.00	40.00	40.00
	Comparison (%)	-	-	-
	Total waste material tonnage (Mt)	145.75	160.00	146.99
	Comparison (%)	-	9.77	0.85
Dyke construction and waste material costs	Total dyke material cost ($M)	35.35	35.35	35.35
	Comparison (%)	-	-	-
	Total waste material cost ($M)	484.27	755.92	391.05
	Comparison (%)	-	56.09	-19.25

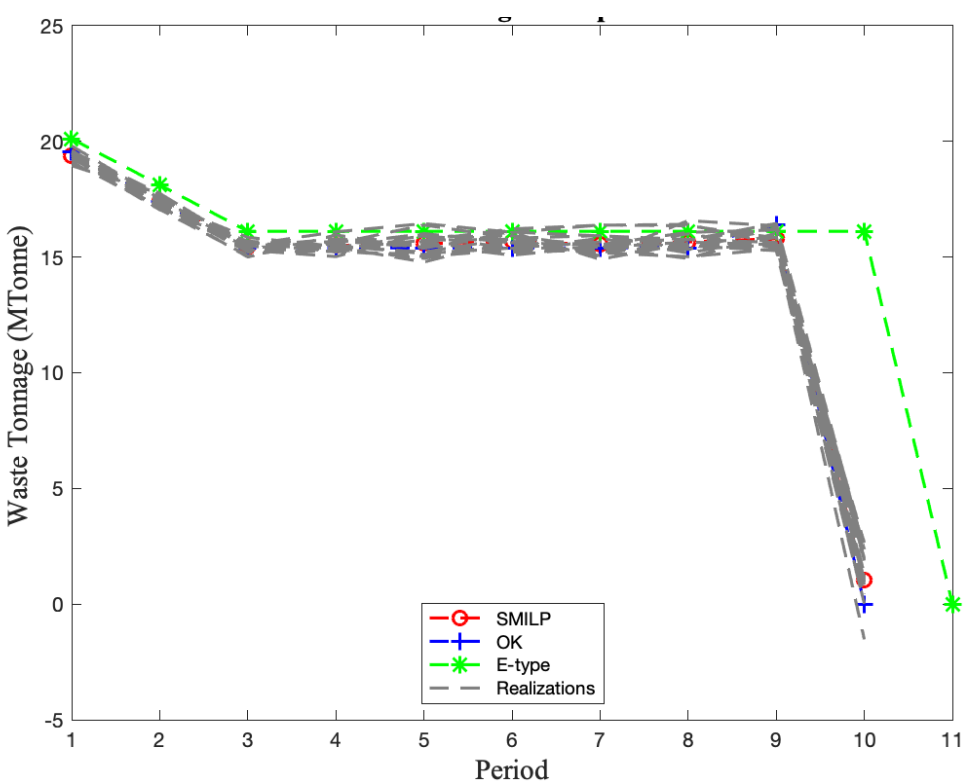

Figure 4-38: Waste material tonnage comparisons for OK, E-type, SMILP and SGS realizations schedules.

4.5.5 Application of OK, E-type and SMILP schedule results to realizations for Case study

The main purpose of implementing the SMILP framework is to minimize the risks associated with deviations in the operational targets during mining. In order to verify the uncertainty-based optimization framework as compared to the conventional approach, three random realizations were selected and the OK, E-type and SMILP schedule results were applied to each of the realization block models separately. This experiment was implemented by replacing the orebody models with each of the three randomly selected realizations to reproduce the OK, E-type and SMILP schedules. The three random realizations used for this experiment were the fifth (R5), ninth (R9) and nineteenth (R19) realizations. The resulting outputs compared were the net present value NPV, average ore bitumen grade and ore bitumen quantity. Table 4-20 shows a summary of the results obtained from the experiment. Table 4-21shows a similar comparison expressed in terms of percentages with the OK schedule as reference. It was observed that the SMILP schedule results generated a better NPV than the OK and E-type schedule results for all three realizations. In the case of average ore bitumen grade, the results were similar with up to 3% difference in ore bitumen quantity processed. These results are due to the architecture of the SMILP framework which places higher penalties for ore grade and ore tonnage deviations in the early years of mine life, improving the early years' cash flow and overall NPV of the mining project.

Table 4-20: Comparison of the OK, E-type and SMILP schedules generated from random SGS realizations

Realization #	Schedule type	Ore tonnage (Mt)	NPV ($M)	NPVd ($M)	Average ore bitumen grade (%m)	Ore bitumen Quantity (Mt)
R5	OK	117.77	2137.70	2173.00	11.89	14.05
	E-type	120.28	2342.60	2368.82	11.91	14.26
	SMILP	121.22	2419.36	2442.17	11.93	14.47
R9	OK	118.12	2186.96	2220.65	11.98	14.16
	E-type	119.76	2339.72	2367.32	11.95	14.30
	SMILP	121.20	2477.40	2499.52	11.97	14.54
R19	OK	117.77	2180.12	2214.38	11.99	14.17
	E-type	120.65	2400.33	2435.71	11.95	14.23
	SMILP	121.40	2457.68	2480.74	12.00	14.58

Table 4-21: Comparison in percentage of the OK, E-type and SMILP schedules generated from random SGS realizations

Realization #	Schedule type	Comparison (%)			
		Ore tonnage	NPV	Average ore bitumen grade	Ore bitumen Quantity
R5	OK	-	-	-	-
	E-type	2.13	9.59	0.17	1.50
	SMILP	2.93	13.18	0.34	2.98
R9	OK	-	-	-	-
	E-type	1.38	6.99	-0.25	0.98
	SMILP	2.60	13.28	-0.08	0.71
R19	OK	-	-	-	-
	E-type	2.44	10.10	-0.33	0.42
	SMILP	3.08	12.73	0.08	2.89

4.5.6 Comparative analysis of risk for Case study

In addition to the production scheduling results, the risk profiles of cash flows, ore tonnages and ore bitumen grades were evaluated and the results compared from Figure 4-39 to Figure 4-41. Risk profiles are generated using the tenth, fiftieth and ninetieth percentile (P10, P50 and P90) block models calculated from the equiprobable SGS realizations of the orebody. The spread of the ore bitumen grades and ore tonnages for these calculated percentiles provide an indication of the uncertainty range for the orebody. This experiment was implemented by replacing the orebody model of the SMILP schedule with the P10, P50 and P90 block models separately to generate the P10, P50 and P90 schedules.

The risk analysis was conducted by comparing the P10, P50 and P90 production schedules ore bitumen grades, cash flows and ore tonnages with that of the OK, E-type and SMILP schedules. It was observed that the SMILP schedule was similar to the P50 schedule demonstrating a strong indication of no over estimation or under estimation of the generated SMILP schedule cash flows, ore bitumen grades and ore tonnages. While the E-type and SMILP cash flow schedules consistently fall between that of the P10 and P90 schedules, the OK cash flow schedule for the most part falls below that of the P10 schedule.

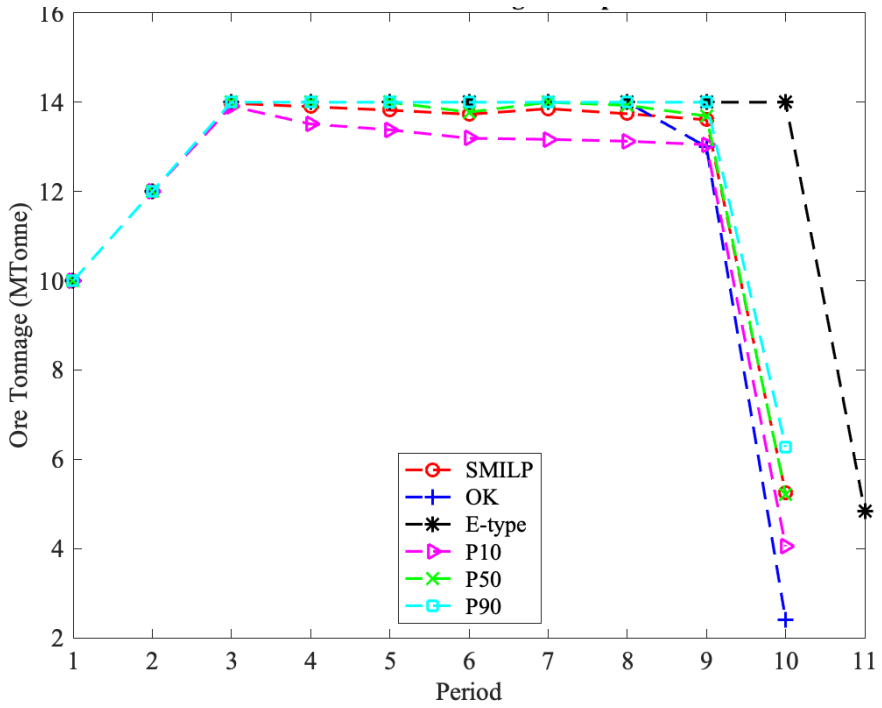

Figure 4-39: Risk profile for processed ore tonnages for P10, P50, P90, OK, E-type and SMILP schedules

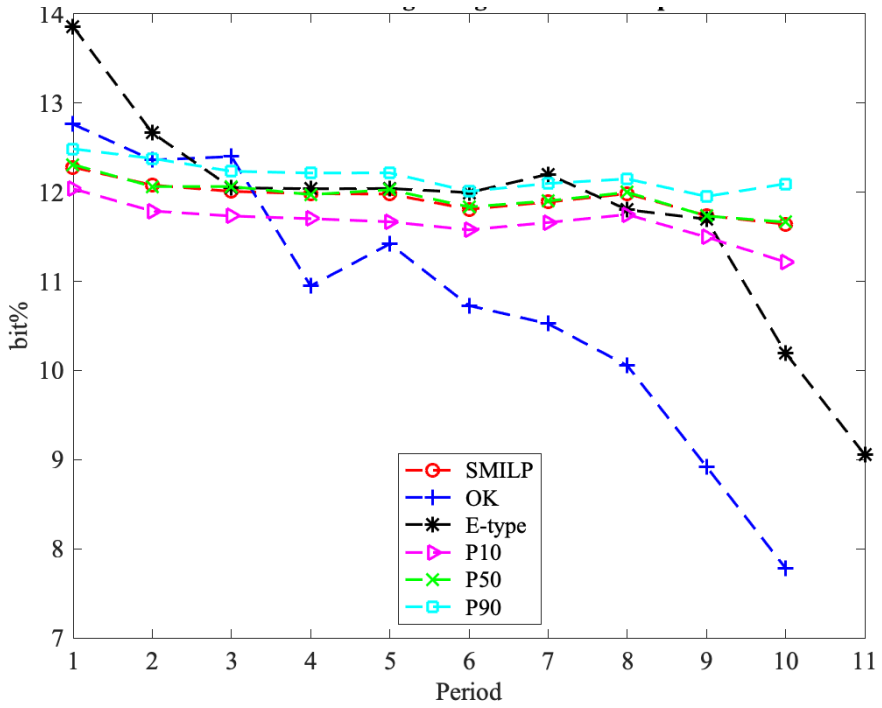

Figure 4-40: Risk profile for average ore bitumen grade for P10, P50, P90, OK, E-type and SMILP schedules

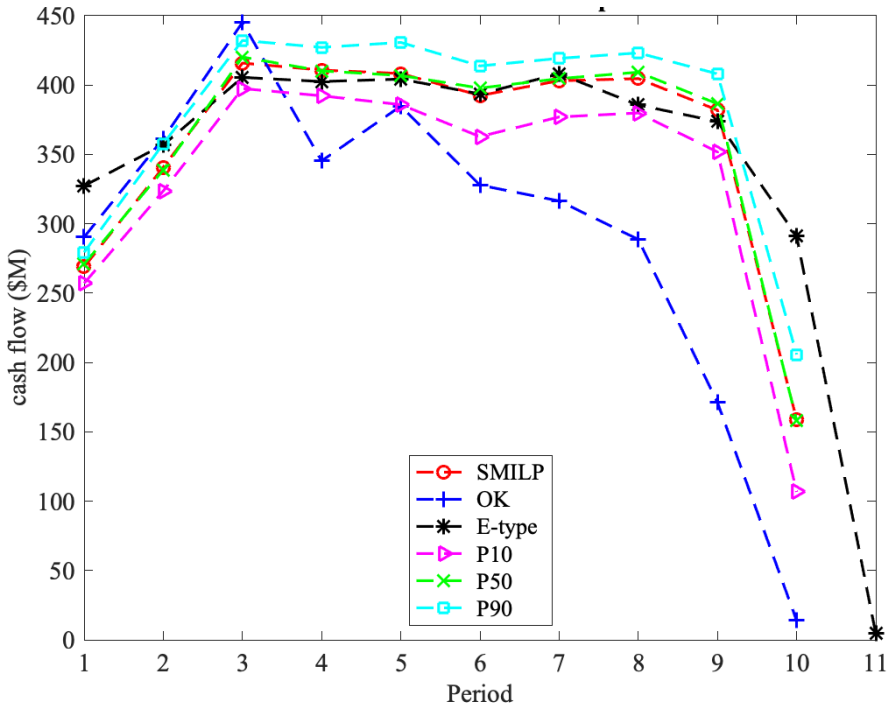

Figure 4-41: Risk profile for NPV for P10, P50, P90, OK, E-type and SMILP schedules

4.5.7 Sensitivity analysis of the SMILP schedule based on GDR parameter in Case study

A sensitivity analysis of the geological risk discount rate (GDR) parameter in the SMILP model was undertaken to assess its impact on NPV. The GDR parameter is associated with risk in the production schedule due to geological uncertainties. For this experiment, various SMILP runs were initiated by varying the GDR parameter while keeping all other economic and technical parameters unchanged. GDR parameters ranging from 0 % to 60% were considered as shown in Table 4-22 and Figure 4-42. From the results, it was observed that as the GDR parameter increases, the NPV becomes better. From the SMILP model objective function, increasing the GDR reduces the level of geological risk and hence an increased NPV of the mining project. Basically, increasing the GDR parameter allows less restriction on the objective function thus allowing the optimizer to generate an improved NPV. Also, increasing the GDR parameters result in more computational time since there is more restriction in the SMILP objective function. From the SMILP runs, it was observed that a GDR that falls within 10% to 20% represents an average geological risk while a GDR of 30% represents a high geological risk with the highest NPV.

Table 4-22: Sensitivity analysis based on GDR parameters

Parameter (unit)	GDR_0	GDR_5	GDR_{10}	GDR_{15}	GDR_{20}	GDR_{30}	GDR_{40}	GDR_{50}	GDR_{60}
NPV ($M)	2210.92	2211.03	2211.97	2212.95	2213.08	2213.63	2213.61	2213.60	2213.60
CPU runtime (hours)	10.07	10.33	10.46	11.46	11.70	12.36	12.40	12.49	12.44

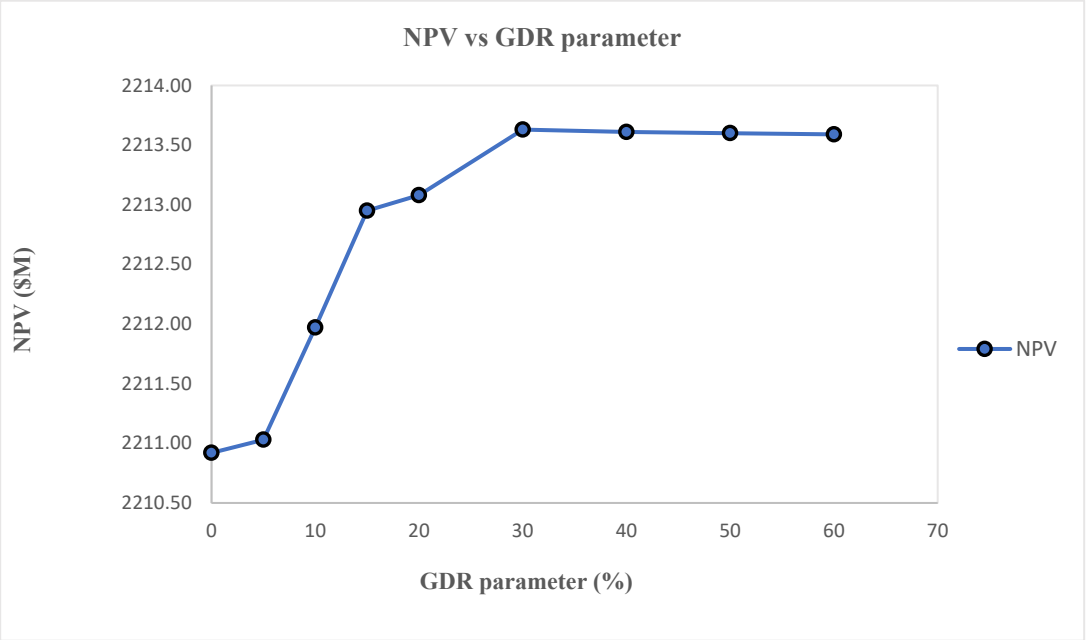

Figure 4-42: Sensitivity analysis of NPV vs GDR parameter

4.5.8 Sensitivity analysis of the SMILP schedule based on penalty parameters in Case study

In this experiment, sensitivity analysis was performed by varying the penalty parameters in the SMILP model to assess it impact on the SMILP schedule and NPV. For these sensitivity runs, a GDR parameter of 30% was considered. Results from the varying set of penalty parameters (P_1, P_2 and P_3) were compared to the reference set of penalty parameters (P_{base}) which were used in previous SMILP runs. Table 4-23 shows the set of P_1, P_2 and P_3 penalty parameters and the resulting SMILP schedules obtained. Figure 4-43 and Figure 4-44 show the respective cash flow and average ore bitumen grade comparisons based on the varying sets of penalty parameters. From the results, as the penalty costs increase, the overall net present value decreases. The increased penalty costs increase the cost of deviation from the set production targets, forcing the optimizer to generate a smooth ore tonnage and ore grade schedule at the expense of the NPV. On the other hand, as the penalty cost decreases, there is more flexibility

for the optimizer to generate an improved NPV. In terms of computation time, increasing the penatly costs results in less solution time and vice versa.

Table 4-23: Sensitivity analysis based on penalty parameters

Parameter (unit)	**P_1**	**P_{Base}**	**P_2**	**P_3**
Cost of shortage in ore production ($/tonne)	0.5	5	50	500
Cost of excess in ore production ($/tonne)	1	10	100	1000
Cost of shortage in ore grade (mill) ($/%m)	0.25	2.5	25	250
NPV ($M)	2234.70	2213.63	2171.90	2167.20
NPVd ($M)	2268.10	2246.62	2205.30	2200.50
Ore tonnage (Mt)	124.60	123.86	121.59	121.57
Average ore bitumen grade (%m)	11.97	11.94	11.92	11.91
CPU runtime (hours)	14.82	12.36	10.15	8.43

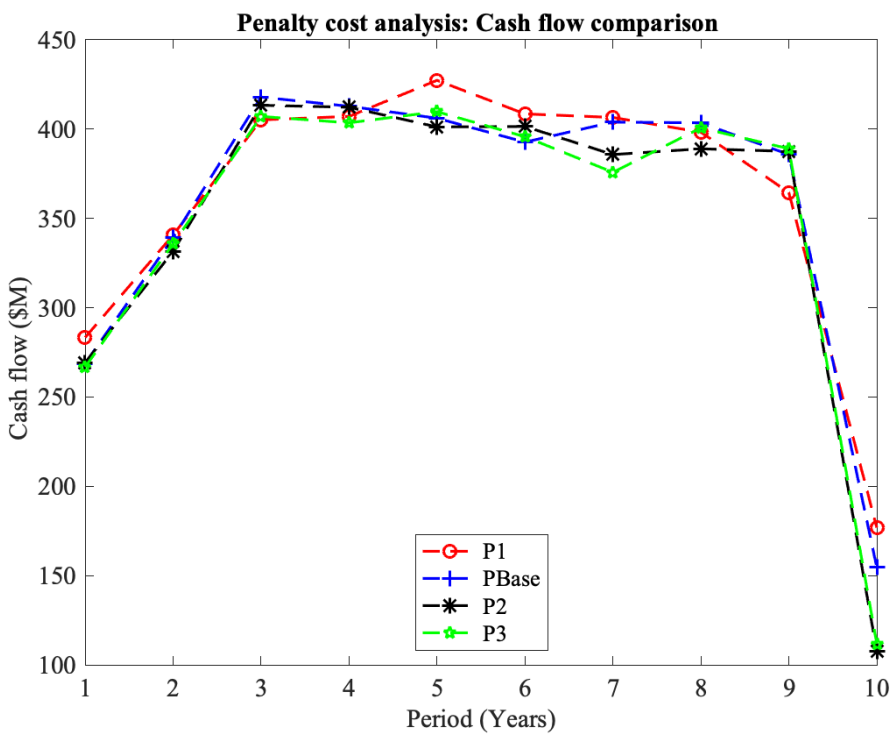

Figure 4-43: Cash flow comparisons using varying penalty cost parameters

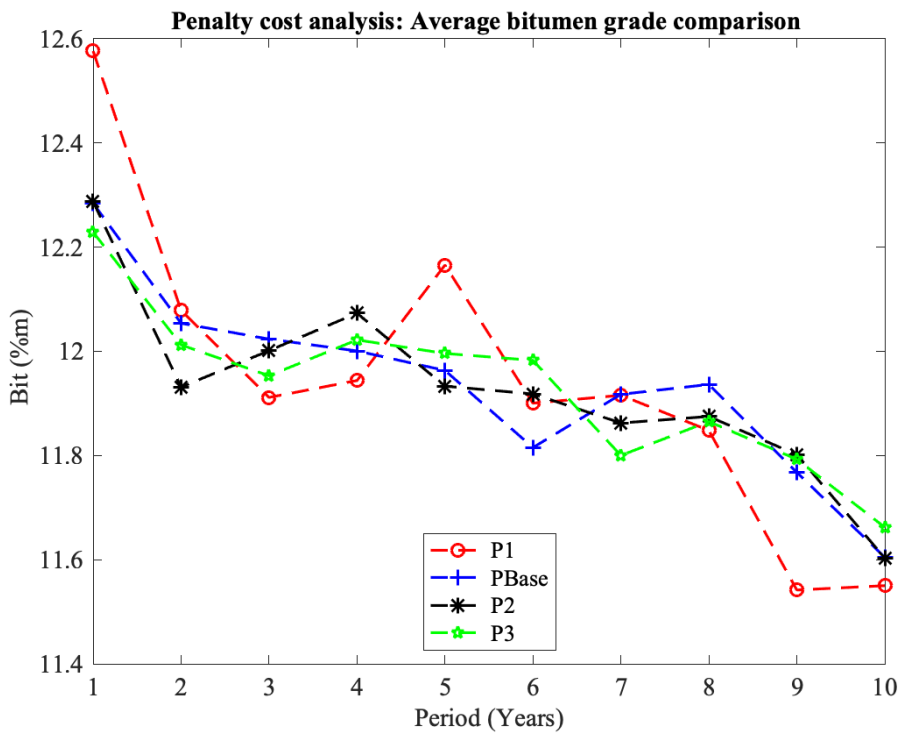

Figure 4-44: Average ore bitumen grade comparisons using varying penalty cost parameters

4.5.9 Conclusion for Case study

In this case study, three primary experimental scenarios were evaluated. Scenario 1 and Scenario 2 comprises of orebody models that is based on OK estimation and E-type estimation from SGS realizations. These scenarios are comprised of only a single estimated orebody model as input and resulted in the OK and E-type schedules. Scenario 3 is comprised of a set of equally probable orebody realizations from SGS that is used to define ore grade and ore tonnage uncertainty in oil sands deposits. 20 SGS realizations generated serve as the input orebody models and Scenario 3 resulted in the SMILP schedule. All three scenarios generated smooth schedules that satisfy the economic and technical constraints which was specified by the mine planner. However, Scenario 3 which was based on SGS realizations generates an uncertainty-based production schedule (SMILP schedule) which takes into consideration risks due to grade uncertainty and deviations from set production targets. The OK and E-type schedules do not asses the effect of grade uncertainty on the mining project. The SMILP framework accounts for geological risk by placing higher penalties for ore grade and ore tonnage deviations from production targets in the early years of mine life to defer production deviations to later years when more geological information becomes available to update the

block model and mine plan. By deferring geological risk to later years, the risk of not reaching production targets in the earlier years is minimized, thus creating a smoother and stable production schedule. The results in this case study demonstrates that the SMILP schedule generates 14.29% and 16.85% improvements in NPV compared to the E-type and OK schedules respectively.

CHAPTER 5

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 Summary of research

Open pit optimization and mine production scheduling algorithms have been developed by researchers focusing mainly on metallic deposits with little consideration for non-metallic deposits like oil sands resources. In the literature, there are some major limitations with existing planning and optimization techniques for oil sands deposits which has special requirements in terms of waste management. These include the following: (i) inadequate implementation of waste disposal planning in the optimization problem while considering grade uncertainty; and (ii) handling of stochastic variables as deterministic parameters during mine planning optimization. These limitations may lead to difficulties in the practical implementation of the mine plan as well as sustainability and regulatory challenges.

The development and implementation of the stochastic mixed integer linear programming (SMILP) framework in this research is designed for integrated stochastic oil sands mine planning and waste management in the presence of grade uncertainty. Thus, the application of the SMILP model allows the incorporation of uncertainties associated with ore grades and ore tonnages during mine planning and waste management. In this research, it has been demonstrated that the SMILP model has the capability of meeting the research objectives which include: (i) addressing the related domains of incorporating grade uncertainty and oil sands waste management through the generation of an uncertainty-based integrated production schedule; (ii) extracting ore and dyke material from a predefined final pit limit over the mine life while maximizing the net present value of the mining project, minimizing dyke construction costs, and minimizing geological risk cost; and (iii) evaluating the risk profile associated with the mine plan and its impact on plant processing, dyke construction and waste management.

A summary of the research methodology and developed framework is presented in Figure 5-1. MATLAB programming environment was used for implementation of the objective function, and technical and operational constraints of the SMILP framework. The components of the framework

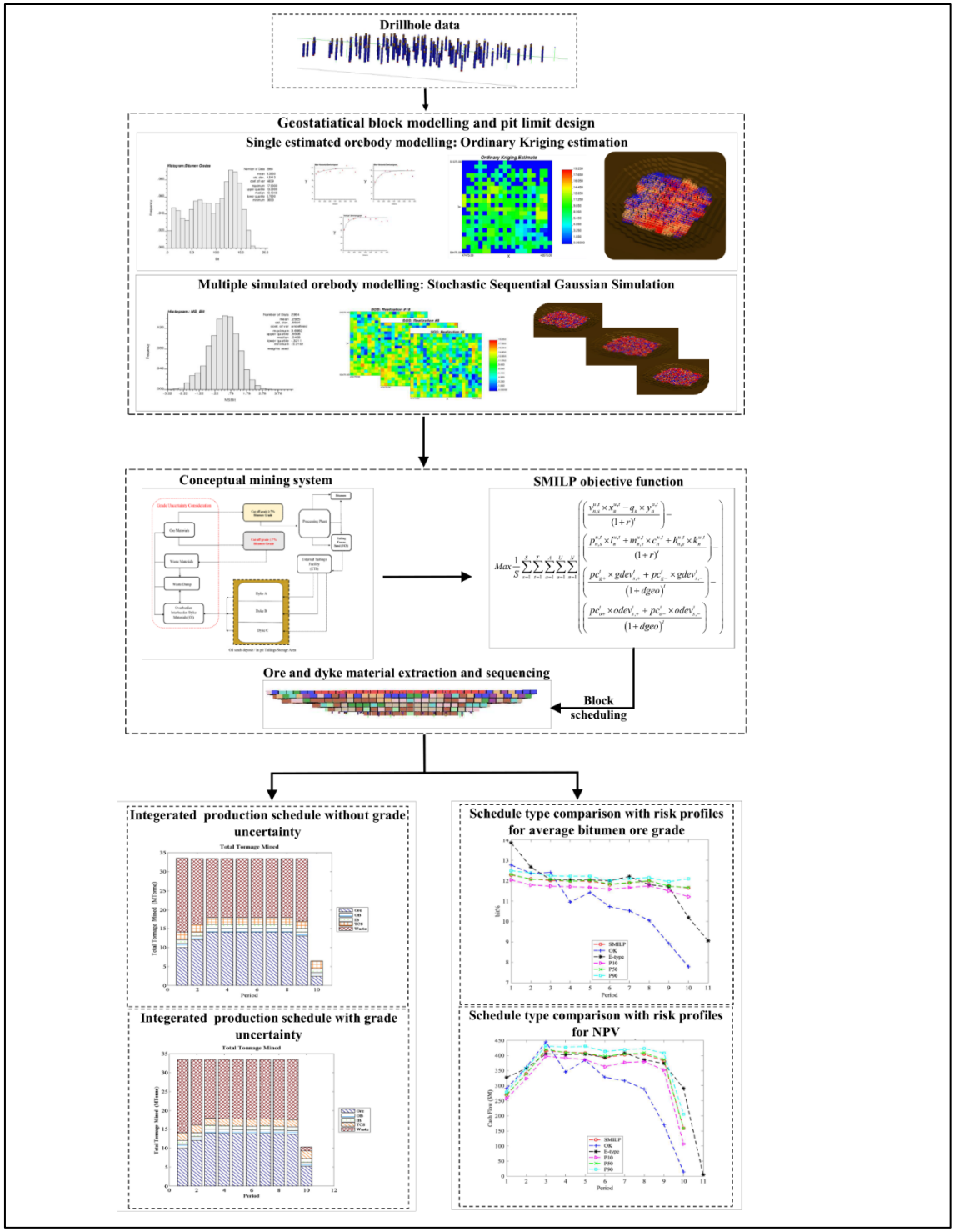

Figure 5-1: Summary of research methods and framework developed

interact with the block model through the input parameters which includes the economic, production and waste disposal parameters. For this research a large-scale optimization solver,

IBM/CPLEX, which uses a branch-and-cut algorithm to solve optimization problems was deployed.

Numerical experiments with oil sands datasets were used to demonstrate the impact of grade uncertainty in oil sands mine planning and waste management. A single estimated block model was first developed using Ordinary Kriging. This was followed by the simulation of 20 equiprobable realizations using Sequential Gaussian Simulation to asses and quantify the impact of geological uncertainty. The ultimate pit outline was determined using 3D LG algorithm in Whittle and the final pit limit was designed in Geovia GEMS to include practical and geotechnical parameters such as the berm width, bench height, and pit slope angle. The SMILP model was formulated and implemented based on the conceptual mining system while considering grade uncertainty in the mine plan. An oil sands case study with three primary scenarios were investigated. The results demonstrated that the implemented constraints were satisfied and the developed model worked as designed. Among other things, the case study scenarios were compared based on NPVs, production schedules, and bitumen grade profiles.

In Scenario 1, the block model data was based on Ordinary Kriging estimation. The SMILP implementation for this scenario did not include grade uncertainty. Thus, the cost of uncertainty was set to zero. This scenario generated a uniform and practical production schedule referred to as the OK schedule with an overall NPV of $1,894.00 M inclusive of dyke material costs. The second scenario was based on the average simulated block model obtained from post-processing the 20 realizations generated from Sequential Gaussian Simulation. Similarly, grade uncertainty was not considered. The second scenario also produced a uniform and practical schedule referred to as the E-type schedule with an overall NPV of $1,938.52 M inclusive of dyke material costs. The third scenario featured 20 SGS realizations block model data used to evaluate the impact of grade uncertainty and geological risks of not meeting production targets during mining. The cost of ore grade uncertainty and ore tonnage uncertainty was applied to the objective function of the SMILP model using penalty parameters to minimize the deviations from grade and tonnage targets during mining. This scenario generated a more uniform and practical production schedule referred to as the SMILP schedule with an overall NPV of $2,213.08 M.

In comparison, the SMILP schedule which was based on the developed SMILP framework generated an uncertainty-based integrated production schedule and waste management plan with

better financial profitability compared to the OK and E-type schedules. Additional numerical experiments and analysis was done by applying the three schedule results to each of three randomly selected realizations. The corresponding SMILP schedules generated from the realizations were consistently uniform and smooth compared to similar OK and E-type realization schedules. Since current industry standard software Whittle does not contain tools for uncertainty-based integrated mine planning and waste management, it cannot be used for validation of the proposed model. Instead, the practicality of the generated production and waste disposal schedules were considered as a measure for model validation.

5.2 Conclusions

During the course of this research, the literature review conducted has demonstrated that little attempt has been made to incorporate grade uncertainty into integrated production scheduling and waste management optimization for oil sands mining. This limitation affects the practicality and optimality of the generated mine plans. This research therefore pioneers the effort to employ a stochastic programming framework to contribute to the body of knowledge and provide a novel understanding in the area of uncertainty-based integrated mine planning and waste management optimization as applied in oil sands mining.

The SMILP theoretical framework was implemented and verified on real oil sands dataset. The research objectives stated in Chapter 1 have been successfully achieved within the research scope. The following conclusions were drawn from the research:

1. Grade uncertainty influences the ore tonnage and input ore grade uncertainties that is sent to the mill at different periods. Implementing the SMILP model contributes significantly to decrease the effect of grade uncertainty during mining.
2. Grade uncertainty has linear and nonlinear effects on the mining project. The linear effect impacts the input ore tonnage to the processing plant. This effect is due to the existence of a cut-off grade. If a realization for a block gets less simulated value than the cut-off grade, it is considered as a waste or dyke material block in that realization. Thus, grade uncertainty is transferred to ore tonnage uncertainty in that manner.
3. The SMILP model maximizes the net present value of the mining project, while minimizing the dyke construction cost and geological risk cost. This is achieved by placing higher penalties for ore grade and ore tonnage deviations from production targets in the

early years of mine life to defer production deviations to later years when more geological information becomes available to update the block model and mine plan.

4. The SMILP model simultaneously schedules ore and dyke materials in the presence of grade uncertainty. The resulting uncertainty-based integrated production schedule and waste management plan generates better financial profitability with a higher chance of success during implementation.
5. The proposed SMILP framework is verified in terms of both feasibility and risk assessment using the case study, and provides a systematic workflow towards promoting sustainable mining as directed by the ERCB Directive 074 regulation.

5.3 Contributions of MASc. research

This research has developed a stochastic mixed integer linear programing (SMILP) framework that deploys multiple material types, and different destinations and mining locations for integrated oil sands mine planning and waste management in the presence of grade uncertainty. The main contributions of this research are as follows:

1. This is an innovative effort in developing an integrated stochastic programming model that incorporates grade and tonnage uncertainties to simultaneously improve material handling and scheduling for processing plant and dyke construction with the objective of maximizing the NPV, minimizing the dyke construction cost, and minimizing the geological risk cost.
2. The research has developed a stochastic programming framework that further expand the boundaries of uncertainty-based integrated mine planning and optimization by generating de-risked production schedules with improved net present value compared to conventional mine planning approach.
3. The SMILP model framework provides a systematic workflow that evaluates the range of uncertainty associated with a mine plan through risk and sensitivity analysis thereby promoting sustainable oil sands production planning as directed by Directive 085 issued by the Alberta Energy Regulator (AER).

5.4 Recommendations for future research

Although the production scheduling and waste disposal planning workflow and models developed in this thesis have provided pioneering efforts for uncertainty-based integrated oil sands mine

planning optimization, there is still room for continuous improvement in the application of stochastic models for integrated mine planning in the oil sands industry. The following recommendations could improve and add to the body of knowledge in this research area.

1. More uncertain parameters other than grade should be incorporated in the optimization problem. This means the SMILP model should be extended to include stochastic economic variables like mining, processing, and dyke construction costs, and commodity price which changes with time.
2. Further investigation should be performed to determine the most suitable number of orebody realizations to use for stochastic optimization and its influence on the production schedule in the case of oil sands mining.
3. The SMILP model is computationally intensive and takes time to generate results. To make it more user friendly for mine planners, the efficiency of the solution process needs to be improved by reducing the solution space through pre-processing and hence the CPU runtime.
4. In addition to IBM CPLEX solver, the implementation of other fast and large-scale solvers based on metaheuristics like Genetic Algorithm should be investigated for stochastic mining-related optimization problems.

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