Forecasting COVID-19 with Gamma Model

by

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Abstract

COVID-19 is a highly contagiously atypical pneumonia attributed to a novel coronavirus. The global economy and people's lives have been tremendously affected by the COVID-19 pandemic since its outbreak in Wuhan, Hubei province, China. In this thesis, a non-linear model based on gamma distribution was built to verify the accuracy of the forecasting of the total confirmed cases of COVID-19 two weeks ahead. The daily growth in cases of COVID-19 for different countries was monitored and compared with the forecasted values. The verification of the performance of the non-linear Gamma distribution model has been verified by the non-linear regression. The data for the 19 countries with the most total confirmed COVID-19 cases as of June 22 was used. The data was sourced from the interactive web-based dashboard developed by the Center for System Science and Engineering (CSSE) at Johns Hopkins University. A web page has been developed to provide predictions generated by our models for individuals and public organizations to forecast the trends of COVID-19.

Keywords

Forecasting, forecasting verification, COVID-19, World Health Organization, WHO, Gamma distribution, non-linear regression, dashboard prototype, jQuery

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1 Introduction

1.1 COVID-19 Pandemic

In December 2019, a local outbreak of an atypical pneumonia occurred in Wuhan, the capital city of Hubei province of China [1, 2]. It was quickly discovered to be attributed to a novel coronavirus. The World Health Organization (WHO) has termed it as Coronavirus Disease 2019 (COVID-19) [3]. An epidemiological link to the Huanan Seafood Wholesale Market has been found as the source of the initial cluster of cases [4]. Some typical symptomatic manifestations have been observed in patients who are affected by COVID-19, including fever, cough, shortness of breath, headache, confusion, muscle ache, chest pain, diarrhea and nausea and vomiting [5, 6].

The annual period of mass migration called chunyun —associated with the Spring Festival holidays— coincided with the outbreak. This enabled the outbreak to spread rapidly to every province of China, with over 50,000 confirmed cases and 1,000 deaths cases reported domestically. While this was occurring, 603 cases were reported in other countries and regions outside of China [7, 8].

This ongoing public health emergency has surpassed the severity of the 2003 outbreak of severe acute respiratory syndrome (SARS), another atypical infectious pneumonia [9]. As such, a series of unprecedented intervention strategies have been implemented to contain the outbreak. For instance, cities have been quarantined, travel and public

gatherings strictly limited, public spaces shut down, and nationwide rigorous body temperature monitoring has been implemented.

Since COVID-19 is highly contagious, the number of confirmed cases in China increased significantly and rapidly. Due to this, on January 30, the WHO announced that the event already constituted a Public Health Emergency of International Concern [10]. The virus appeared quickly in countries outside of China due to international travel and transport, and the numbered of confirmed cases grew rapidly worldwide. All countries implemented strategies in order to contain the outbreak, including early detection, maintenance of social distance, mask-wearing, active surveillance, case management, and contact tracing. The global economy and people's lives have been impacted tremendously by the COVID-19 pandemic.

1.2 Gamma Distribution

Gamma distribution is a two-parameter family of continuous probability distributions. The gamma distribution is the maximum probability distribution [11]. Exponential distribution, Erlang distribution, and chi-squared distribution are specific types of gamma distribution, and are three different parameterizations for the utilization of gamma distribution. The first parameterization is composed of a shape parameter k and a scale parameter θ. The second parameterization is composed of a shape parameter $\alpha = k$ and an inverse scale parameter $\beta = 1/\theta$, called a rate parameter. Finally, the third parameterization is composed of a shape parameter k and a mean parameter $\mu = k\theta = \alpha/\beta$. In each of these three parameterizations,

all parameters are positive real numbers. In these three parameterizations, the first parametrization k and θ is especially widely utilized in econometrics and other applied fields. The implementation of gamma distribution is illustrated in [12].

A random variable X that exhibits gamma distribution with the shape K and scale θ as its parameterization is denoted as:

$$X \sim \Gamma(k, \theta) \equiv Gamma(k, \theta) \tag{1-1}$$

The probability density function of gamma distribution with the shape K and scale θ as its parameterization is:

$$f(x; k, \theta) = \frac{x^{k-1} e^{-\frac{x}{\theta}}}{\theta^k \Gamma(k)} \quad for x > 0 \quad and \quad k, \theta > 0 \tag{1-2}$$

$\Gamma(k)$ is the gamma function evaluated at k, which is:

$$\Gamma(k) = \int_0^\infty x^{k-1} e^{-x} dx \quad , \Re(k) > 0 \tag{1-3}$$

Figure 1 is taken from Wikipedia to illustrate the probability density function of gamma distribution with the shape K and scale θas its parameterization:

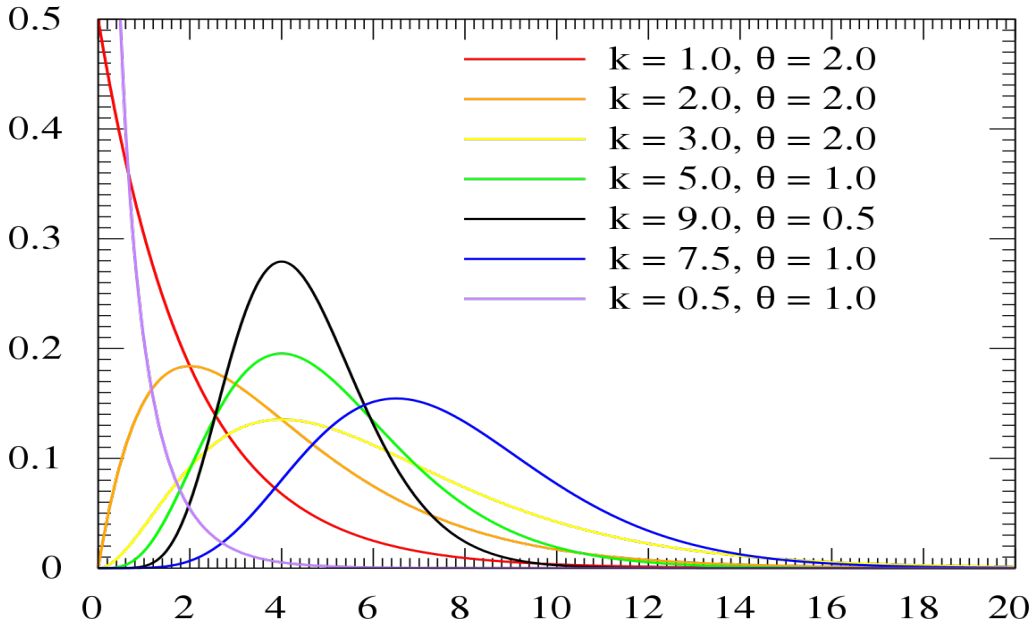


Figure 1: Probability density function plot of gamma distribution

The cumulative distribution function of gamma distribution with the shape K and scale θ as its parameterization is:

$$F(x;k,\theta) = \int_0^x f(u;k,\theta) = \frac{\gamma(k,\frac{x}{\theta})}{\Gamma(k)} \tag{1-4}$$

The $\gamma(k,\frac{x}{\theta})$ is the lower incomplete gamma function, which is:

$$\gamma(k,\frac{x}{\theta}) = \int_0^x t^{s-1}e^{-t}dt \tag{1-5}$$

Figure 2 is taken from Wikipedia to illustrate the cumulative distribution function of gamma distribution with the shape K and scale θ as its parameterization:

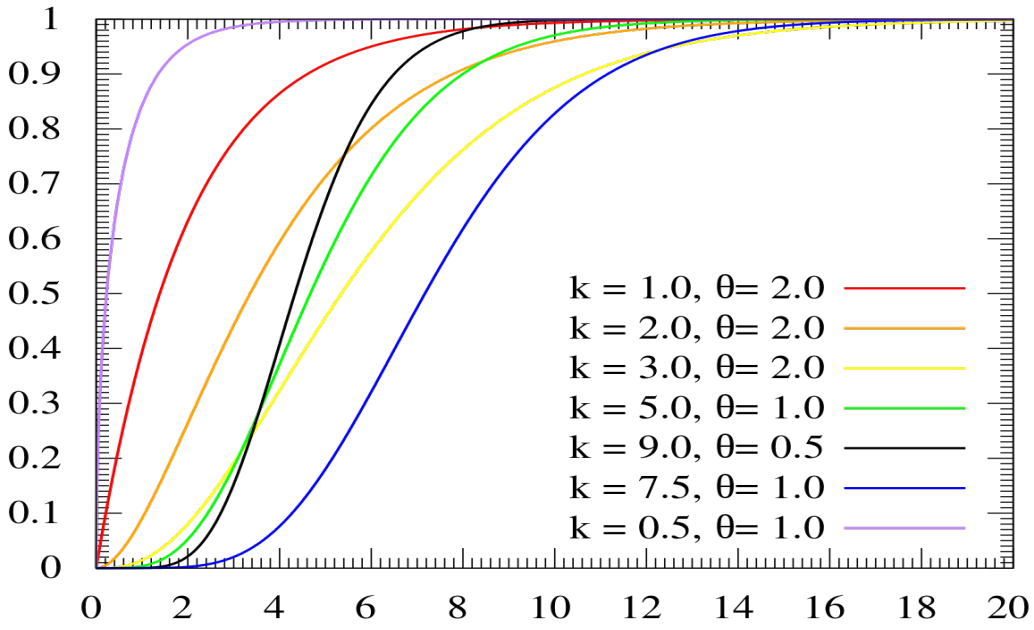


Figure 2: Cumulative distribution function plot of gamma distribution

Different parameterizations of gamma distribution have different implementations in various situations. In this thesis, we chose the shape K and scale θ as the parameterization.

1.3 Contributions

The main contribution of this thesis is that we proposed a model to predict the total confirmed and daily confirmed COVID-19 cases in different countries with relatively satisfactory accuracy. The core of our prediction algorithm is composed of gamma distribution and nonlinear regression verification of it. The model has two versions. The simpler version with lower consumption of computing power was implemented

to predict most of countries. The advanced version with a higher consumption of computing power was implemented to predict countries with more complex data. Furthermore, we developed a web service online to provide information, including the graph of real daily confirmed cases, the graph of real total confirmed cases, the graph of predicted daily confirmed cases, and the graph of predicted total confirmed cases. The corresponding predicted data over 14 days for the 19 countries that have the most amount of total confirmed cases was also shown. This information can be useful to individuals as well as public organizations. Public organizations will efficiently allocate medical resources and publish policies to prevent the spread of COVID-19. Individuals will get more information to help them make decisions about their plans during the COVID-19 pandemic, i.e. cancelling large gatherings, wearing masks, and/or limiting unnecessary travel. This thesis is of great importance because predictions of COVID-19 can increase the awareness of society-at-large.

2 Related Work

2.1 COVID-19 Pandemic the Growing Phase Forecasting

In the twenty-first century, human beings have experienced several serious public health events caused by pathogens shared with wild or domestic animals. These include severe acute respiratory syndrome coronavirus (SARS-CoV), Middle East Respiratory Syndrome coronavirus (MERS-CoV), and the influenza A (H1N1) virus [18]. In order to control and prevent outbreaks and epidemics of infectious diseases, numerous models have been designed within the epidemiologic field. Since the Outbreak of COVID-19, various prediction models have been implemented to aid in the comprehension of this virus.

It is important to identify the origin of the virus as well as its intermediate hosts. The development of genome sequencing technology has contributed a great deal towards the tracing of COVID-19. As is pointed out in[19], the noncoding flanks of the viral genome can be used to differentiate the recognized four Betacoronavirus subspecies through whole-genome sequence comparisons, which has implications for rapid classification of new viruses. The comparison between the numerous mammalian coronavirus sequences and the genomic sequence obtained from COVID-19 suggests that bats may be a natural reservoir for COVID-19 [20]; subsequently, Pangolins are potential intermediate hosts [21].

In the early stages of an outbreak, it is vital to gain an understanding of the trans-

missibility of the pathogen at hand. In the early stages of the COVID-19, the transmission chain of the virus was not clear. Therefore, many researchers adopted information from SARS and MERS, which are similar to COVID-19, to aid their understanding of it. The article [22] suggests that confirmed cases may have been roughly under-reported from the 1^{st} to the 15^{th} of January 2020, due to comparisons with SARS cases. The epidemic curve of COVID-19 in mainland China from December 1, 2019 to January 24, 2020 had also been built by the exponential growth Poisson process. They estimated the R_0 of COVID-19 to be 2.56, and the number of unreported cases to be 469, which has helped us understand what might have happened in the early stages of outbreak.

While super-spreading events remains rare, they can cause large and explosive transmission events and destroy hopes of successful prevention efforts. In the article [23], the researchers generated secondary cases for each primary case according to a negative-binomial offspring distribution with the mean R_0 and dispersion k. After 1000 stochastic simulations for each individual combination, their simulations suggested that very low values of k are less likely, and the establishment of sustained transmission chains from single cases cannot be ruled out. Therefore, the importance of screening, surveillance, and control efforts cannot be ignored.

There are numerous other predictions in the epidemiologic field which can help people understand COVID-19 from a medical perspective. Since this thesis focused on prediction of COVID-19 cases, we are not going to introduce other predictions within

the epidemiologic field.

2.2 Prediction of COVID-19 Cases

Since the outbreak of COVID-19, many researchers have built various models to predict the number of COVID-19 cases. These models have different advantages and disadvantages, and they provide different methods and perspectives to help people predict the number of cases.

In [24], the researchers successfully predicted that there would be over 1,000,000 COVID-19 cases outside of China by March 31^{st}, 2020. The data is sourced from the WHO situation report, in order to avoid risking data from an individual country being biased or politically motivated. The model is based on assumption exponential growth of the pandemic. The coefficients of exponential growth are estimated by nonlinear regression analysis and the least squares method. The coefficient a is estimated at 10.4791 and the coefficient b is estimated at 0.161. The model is simple and easily explained. The error is 1.29%, which is acceptable for a short-term prediction model.

In [25], the researchers built a model to predict the trend of COVID-19 in Wuhan, Beijing, Shanghai and Guangzhou, four of the biggest cities in China. SEIR (Susceptible, Exposed, Infectious, Recovered) and neural networks (NNs) were used to build the model. The data was composed of real-time data of COVID-19 in addition to population mobility data. The time series data for COVID-19 by location, in-

cluding the number of confirmed cases, deaths, recovered cases, and newly diagnosed cases, was extracted from the website Tencent News. The population migration data was extracted from "Baidu migration", which is an open-source big data project. According the results of the model, the researchers make the conclusion that the COVID-19 epidemic in China has been effectively contained, and that the national medical service is going to recover by April. However, they stated that the potential for asymptomatic virus carriers still cannot be ignored.

The researchers in [26] built a model to predict the trend of the COVID-19 in China based on the public health interventions. A modified Susceptible-Exposed-Infections-Removed (SEIR) model along with artificial intelligence (AI) were used to estimate the curve of the epidemic. The data sources were composed of data from COVID-19 and SARS. The epidemiological data from COVID-19 was extracted from the National Health Commission of China. The 2003 SARS epidemic data between April and June 2003 across China was retrieved from the SOHU News website. The SARS epidemic data was then used to train the AI model. According to the results of the model, the researchers predicted that the peak of the epidemic in China would be February and that a gradual decline would appear in April. They also predicted that a second epidemic peak might appear in Hubei province in the middle of March, if the quarantine of Hubei province was suspended. Lastly, they predicted that if the implementation of prevention measures in China were to have been delayed by five days, the size of the epidemic would have been three times larger.

The researchers in [27] built a model for the early warning of some notable infectious diseases in China. Real-time recurrent learning (RTRL) and EKF (Kalman filter) were used to build the model. The numbers of new confirmed cases of eight types of infectious diseases, reported per month between January 2004 and December 2015, were extracted from the Center for National Public Health Scientific Data and the National Health Commission websites. The model from this article can effectively provide early warnings for some infectious diseases.

2.3 Approach Used in This Thesis

In this thesis, the core part of our model was selected as gamma distribution with shape K and scale θ for parameterization. Furthermore, two gamma distributions were combined to handle data from countries with complex patterns of daily reporting. In order to calculate the coefficients of our model, nonlinear regression was used to fit the given data, which was the total confirmed cases and daily confirmed cases of different countries. During the process, the least square method was used to figure out the most fitted coefficients. More specific information about our model will be introduced in the chapter 4.

3 Dataset

3.1 Data Sources

Since the outbreak of COVID-19, different countries and public organizations have collected and reported COVID-19 data, the WHO [28] and the Chinese Center for Disease Control and Prevention [29] being two of those entities. It was difficult to find a dataset that was able to aggregate all of these data sources.

On January 22, 2020, an interactive web-based dashboard [30] was developed by the Center for System Science and Engineering (CSSE) at Johns Hopkins University in Baltimore, MD, USA. It was able to visualize and track reported cases of COVID-19 in real time. The dashboard is available online at [31]. It illustrates the location and number of confirmed COVID-19 cases, deaths, and recoveries for all affected countries, as well as provides a global hot spot map of COVID-19 and other statistical information related to COVID-19. It is a user-friendly tool for individuals, researchers, and public health authorities to learn about the COVID-19 pandemic. COVID-19 cases are reported at different levels depending on the region and country. Cases are reported at the provincial level in China, at the city level in the USA, Australia and Canada, and at the country level in many other countries and regions. The home page of the online dashboard is shown in figure 3:

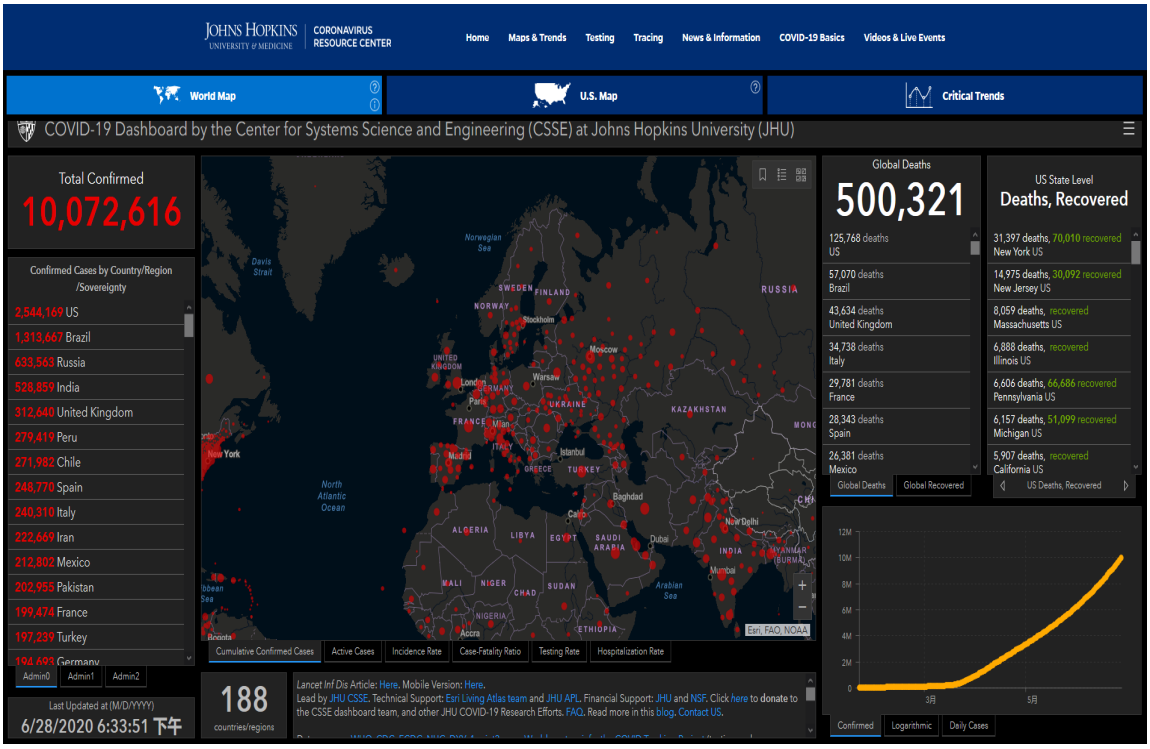


Figure 3: COVID-19 Dashboard by CSSE at JHU

The data collection period was divided into two parts. From January 21, 2020 to January 31, all collection and processing of data was done manually. The data was typically updated twice daily. After February 1, 2020, a semi-automated system of collection was adopted. Before the online dashboard data was updated, the case numbers were checked with the centers for disease control and prevention in China, Taiwan, and Europe, as well as the Hong Kong Department of Health, the Macau Government, the WHO, state-level health authorities in the U.S., the US CDC, the government of Canada, the Australian Government Department of Health, and various state or territory health authorities. Furthermore, the dashboard is particularly

effective at capturing the timing of the first reported cases of COVID-19 in new countries and regions. Due to all of this, we were confident that this dashboard is a suitable data source for this thesis. More importantly, the data from this dashboard is freely available at GitHub [32].

Our original data was downloaded using an R package: 'RCurl'. RCurl is a free and open source R Package developed to transfer data. The commands from 'RCurl' we used to download our original data directly from GitHub is as follows:

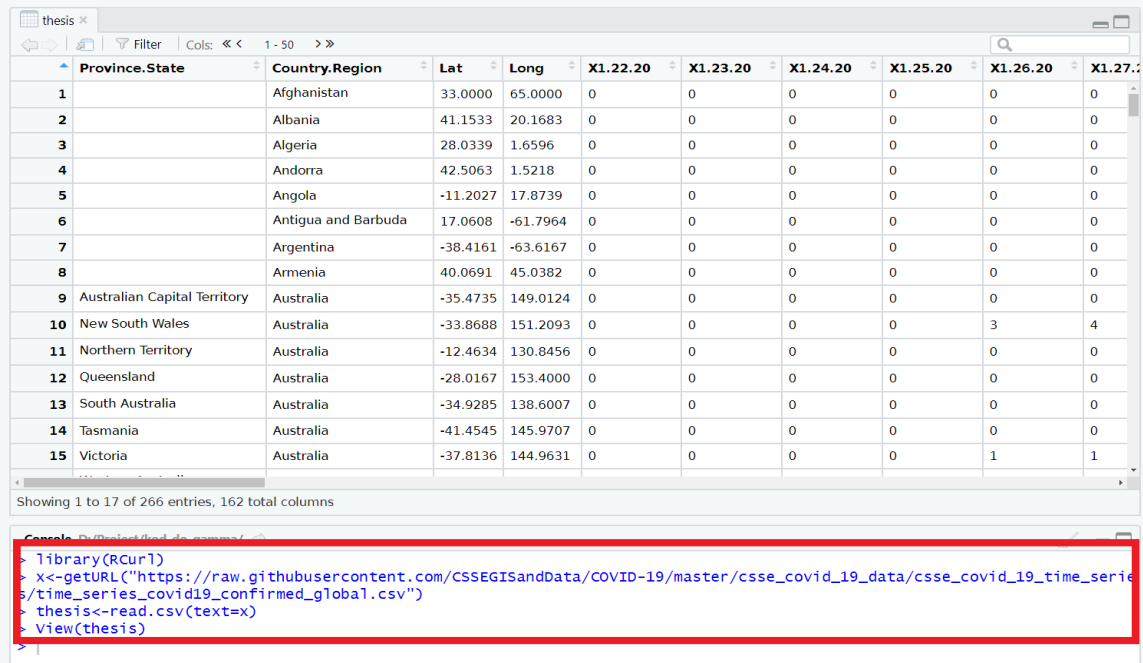


Figure 4: Commands of 'RCurl' for downloading original data

All of the data processing and analysis in this thesis were implemented by R [33]. R is a programming language and free software environment for statistical computing and

graphics supported by the R Foundation for Statistical Computing [34, 35, 36]. The R language is widely used in statistics, data mining, and statistical software. In the scholarly research field, the applications of R increase substantially [37]. The official R software environment with command line interface is a GNU package which is developed mainly in C, Fortran, and R itself. In this thesis, we used RStudio, which is an integrated development environment for R. R is an implementation of the S programming language and was created by Ross Ihaka and Robert Gentleman at the University of Auckland, New Zealand. Since R is an open source programing language, it contains almost all of the popular algorithm packages used in statistics and data analysis.

3.2 Data Identification

Since the data source is continuously updating, we adopted the data from June, 21, 2020 to use for this thesis. The dataset illustrated in this thesis is composed of 266 observations with 156 variables. The construction of the original data is as shown in table 1:

Table 1: Construction of Original Data

Name	Data Type
Province/State	VARCHAR(50)
Country/Region	VARCHAR(50)
Lat	FLOAT
Long	FLOAT
1/22/20	INT
1/23/20	INT
1/24/20	INT
...	INT
6/21/20	INT

The name of the first row is 'Province/State', which describes the location of this observation at the provincial or state level. The name of the second row is 'Country/Region', which describes the location of this observation at the country or regional level. The name of the third row is 'Lat', which describes the latitude value of this observation. The name of the fourth row is 'Lon', which describes the longitude value of this observation. The names of the other rows are the dates, from January, 22, 2020 to June,21,2020. These rows describe the total confirmed cases of COVID-19 at this specific location on each corresponding date.

As we mentioned before, in the online COVID-19 dashboard, cases are reported at different levels depending on the different region and country. In the original

data, observations were divided into three constructions. The first type of data construction was designed for China and Canada. A sample of the first type of data construction is as shown in table 2:

Table 2: Sample of the First Type of Data Construction

Province/State	Country/Region	Lat	Long	1/22/20	1/23/20	6/20/20	6/21/20
Anhui	China	31.8257	117.2264	1	9	991	991
Beijing	China	40.1824	116.4142	14	22	821	830
Chongqing	China	30.0572	107.874	6	9	582	582
Fujian	China	26.0789	117.9874	1	5	363	363
Gansu	China	37.8099	101.0583	0	2	151	151
Guangdong	China	23.3417	113.4244	26	32	1634	1634
Guangxi	China	23.8298	108.7881	2	5	254	254
Guizhou	China	26.8154	106.8748	1	3	147	147
Hainan	China	19.1959	109.7453	4	5	171	171
Hebei	China	39.549	116.1306	1	1	344	346
Heilongjiang	China	47.862	127.7615	0	2	947	947
Henan	China	33.882	113.614	5	5	1276	1276
Hong Kong	China	22.3	114.2	0	2	1128	1131
Hubei	China	30.9756	112.2707	444	444	68135	68135
Hunan	China	27.6104	111.7088	4	9	1019	1019
Inner Mongolia	China	44.0935	113.9448	0	0	238	238
Jiangsu	China	32.9711	119.455	1	5	653	653
Jiangxi	China	27.614	115.7221	2	7	932	932
Jilin	China	43.6661	126.1923	0	1	155	155
Liaoning	China	41.2956	122.6085	2	3	153	154

In this first type of data construction, each observation describes the total confirmed number of COVID-19 cases in one province of one country on the corresponding date. However, this is not designed to describe the total number of confirmed COVID-19 cases in the whole country. Hence, in order to get results for the whole country, we needed to sum the results of all the observations from the first type of data construction.

The second type of data construction was designed for the United Kingdom and France. A sample of the second type of data construction is as shown in table 3:

Table 3: Sample of the Second Type of Data Construction

Province/State	Country/Region	Lat	Long	1/22/20	1/23/20	6/20/20	6/21/20
Bermuda	United Kingdom	32.3078	-64.7505	0	0	146	146
Cayman Islands	United Kingdom	19.3133	-81.2546	0	0	195	195
Channel Islands	United Kingdom	49.3723	-2.3644	0	0	570	570
Gibraltar	United Kingdom	36.1408	-5.3536	0	0	176	176
Isle of Man	United Kingdom	54.2361	-4.5481	0	0	336	336
Montserrat	United Kingdom	16.7425	-62.1874	0	0	11	11
Anguilla	United Kingdom	18.2206	-63.0686	0	0	3	3
British Virgin Islands	United Kingdom	18.4207	-64.64	0	0	8	8
Turks and Caicos Islands	United Kingdom	21.694	-71.7979	0	0	12	14
Falkland Islands (Malvinas)	United Kingdom	-51.7963	-59.5236	0	0	13	13
	United Kingdom	55.3781	-3.436	0	0	303110	304331

In the second type of data construction, most of the observations describe the total confirmed number of COVID-19 cases in one state of one country on the corresponding date. There is also a metric designed to describe the total number of confirmed COVID-19 cases for the whole country. Thus, in order to get the results for the whole country, we simply need to find the cell that is aligned with the value for 'Country/Region', and contains the given country's name.

The third type of data construction is designed for all other countries. A sample for the second type of data construction is as shown in table 4:

Table 4: Sample of the Third Type of Data Construction

Province/State	Country/Region	Lat	Long	1/22/20	1/23/20	6/20/20	6/21/20
	Azerbaijan	40.1431	47.5769	0	0	12238	12729
	Barbados	13.1939	-59.5432	0	0	97	97
	Costa Rica	9.7489	-83.7534	0	0	2127	2213
	Diamond Princess	0	0	0	0	712	712
	Egypt	26	30	0	0	53758	55233
	Fiji	-17.7134	178.065	0	0	18	18
	Germany	51	9	0	0	190670	191272
	Haiti	18.9712	-72.2852	0	0	5077	5077
	Iran	32	53	0	0	202584	204952
	Japan	36	138	2	2	17725	17780
	"Korea, South"	36	128	1	1	12421	12438
	Kuwait	29.5	47.75	0	0	39145	39650
	Lebanon	33.8547	35.8623	0	0	1536	1587
	Malaysia	2.5	112.5	0	0	8556	8572
	Nepal	28.1667	84.25	0	0	8605	9026
	Oman	21	57	0	0	28566	29471
	Peru	-9.19	-75.0152	0	0	251338	251338
	Qatar	25.3548	51.1839	0	0	86488	87369
	Russia	60	90	0	0	576162	583879
	Singapore	1.2833	103.8333	0	1	41833	42095
	Turkey	38.9637	35.2433	0	0	186493	187685
	Uganda	1	32	0	0	763	770
	Vietnam	16	108	0	2	349	349
	Zambia	-15.4167	28.2833	0	0	1430	1430

In the third type of data construction, there is only one metric designed to describe the number of total confirmed COVID-19 cases for each country. Thus, we can easily get the results for the whole country by finding the cell that aligns with the value of 'Country/Region', and that contains the desired country's name.

3.3 Data Preprocessing

Analyzing data that has not been carefully screened for problems can produce misleading results. Thus, the representation and quality of data is the first priority

before running an analysis [38]. Data cleaning, data integration, data reduction, data transformation, and data discretization are common methods of data preprocessing [39]. Even though the original data of this thesis is relatively clean and well-organized, it is still unable to be used directly as data for our prediction model. There are three main data preprocessing tasks in this thesis: constructing data for the prediction model, saving the results of the prediction model, and constructing data for data visualization.

3.3.1 Data Construction for Prediction Model

As we mentioned in chapter 3.2, the original data observations were divided into three constructions. Therefore, we needed to deal with the different constructions in order to generate the data for our prediction model. The codes for accomplishing this task is as follows:

```
country <- df %>% filter(Country.Region == country_names[j])
if (country_names[j]%in% c("China", "Canada")) {
  country <- country[,-c(1:4)]
  country <- apply(country, 2, sum)} else if (country_names[j] %in% c("United Kingdom", "France")){
    country <- country %>% filter(Province.State=="")
    country <- country[,-c(1:4)]} else country <- country[,-c(1:4)]

    country <- as.vector( t(country))

    ind <- which(country != 0)
    country <- country[ind]
    x <- 1:length(country)
    train_data<-data.frame(x=x[-1],y=diff(country))
```

Figure 5: Codes to Generate Data for Prediction Model

At the beginning of the code, the subset of the original data is generated based on the name of the country or region. Then, according to the different data constructions of different countries, the value for total confirmed COVID-19 cases on different dates in

the corresponding country are extracted to a vector named 'country'. Then, a vector named 'x' is created with the same length as that of the vector named 'country'. These two vectors are then aggregated into a data frame.

3.3.2 Saving the Result of Predction Model

After the prediction model is generated, the prediction value from the prediction model and the real value should be saved properly for the next step of data processing: data visualization. The codes to accomplish this are as follows:

```
data_diff <- data_cum <- NULL
day <- as.Date("22-01-2020", format="%d-%m-%Y")+0:(length(country)-1)+min(ind)-1
total <- as.vector(country)
data_cum <- data.frame(day, total, type=rep("real", length(country)))
data_diff <- data.frame(day=day[-1], growth=diff(total), type=rep("real", length(country)-1))

t <- 0:180
s <- min(ind)-1

wsp_gam <- coef(mod_gam)
wsp_list<-c(wsp_gam[1],wsp_gam[2],wsp_gam[3])

data_cum <- rbind(data_cum, data.frame(day = as.Date("22-01-2020", format="%d-%m-%Y")+t+s,
                                       total=wsp_gam[1]*pgamma(t, shape = wsp_gam[2],
                                                               scale =  wsp_gam[3]),
                                       type=rep("scaled Gamma", length(t))))


data_diff <- rbind(data_diff, data.frame(day = as.Date("22-01-2020", format="%d-%m-%Y")+t-1+s,
                                         growth=wsp_gam[1]*dgamma(t, shape = wsp_gam[2],
                                                                  scale =  wsp_gam[3]),
                                         type=rep("scaled Gamma", length(t))))
```

Figure 6: Codes to Save the Result of Prediction Model

At the end of these codes, a data frame named 'data_cum' is generated to save the real and predicted values for the number of total confirmed COVID-19 cases on different dates of the corresponding country. Another data frame named 'data_diff' is generated to save the real and predicted values for the number of daily confirmed

COVID-19 cases on different dates of the corresponding country. A sample for a subset of the 'data_cum' frame is as follows:

Table 5: Subset of One 'data_cum'

date	total	type
2020-01-22	1.0000	real
2020-01-23	1.0000	real
2020-01-24	2.0000	real
2020-01-25	2.0000	real
2020-01-26	5.0000	real
2020-01-27	5.0000	real
2020-01-22	0.0000	scaled Gamma
2020-01-23	0.0000	scaled Gamma
2020-01-24	0.0000	scaled Gamma
2020-01-25	0.0001	scaled Gamma
2020-01-26	0.0012	scaled Gamma
2020-01-27	0.0069	scaled Gamma
2020-01-28	0.0286	scaled Gamma
2020-01-29	0.0947	scaled Gamma
2020-01-30	0.2649	scaled Gamma
2020-01-31	0.6514	scaled Gamma
2020-02-01	1.4475	scaled Gamma
2020-02-02	2.9631	scaled Gamma
2020-02-03	5.6686	scaled Gamma
2020-02-04	10.2449	scaled Gamma
2020-02-05	17.6408	scaled Gamma

The attribute named 'type' is used to indicate whether the observation is the real value, or the predicted value calculated by the model. The attribute named 'total' represents the real or predicted total number of confirmed COVID-19 cases for the corresponding date. A sample of the subset for the 'data_diff' frame is as follows: In this frame, the attribute named 'type' is likewise used to indicate whether the observation is the real value or the predicted value calculated by the model. The

Table 6: Subset of One 'data_diff'

date	growth	type
2020-01-23	0	real
2020-01-24	1	real
2020-01-25	0	real
2020-01-26	3	real
2020-01-27	0	real
2020-01-28	0	real
2020-01-21	0	scaled Gamma
2020-01-22	0	scaled Gamma
2020-01-23	0	scaled Gamma
2020-01-24	0.0003	scaled Gamma
2020-01-25	0.0023	scaled Gamma
2020-01-26	0.0108	scaled Gamma
2020-01-27	0.0372	scaled Gamma
2020-01-28	0.1046	scaled Gamma
2020-01-29	0.254	scaled Gamma
2020-01-30	0.5507	scaled Gamma
2020-01-31	1.0924	scaled Gamma
2020-02-01	2.0164	scaled Gamma
2020-02-02	3.5071	scaled Gamma
2020-02-03	5.8027	scaled Gamma
2020-02-04	9.2011	scaled Gamma

attribute named 'growth' represents the real or predicted number of daily confirmed COVID-19 cases on the corresponding date.

3.3.3 Data Construction for Data Visualization

We used an R package named 'Plotly' to visualize the results from our model. 'Plotly' is a free and open source R graphing library that makes interactive graphs suitable for public use [40]. In order to utilize this package, we reconstructed the data to meet the data structure requirement of 'Plotly'. The codes used for this are shown

in figure 7:

```
################################################################## plotly cum
real_cum<-data_cum[which(data_cum$type=='real'),]
gamma_cum<-data_cum[which(data_cum$type=='scaled Gamma'),]
c<-NA
for (i in 1:(length(gamma_cum$day)-length(real_cum$day)-1)) {
  c<-c(c,NA)
}
real_cum_data<-c(real_cum$total,c)
plot_cum_data<-data.frame(day=gamma_cum$day,scaled=gamma_cum$total,real=real_cum_data)
################################################################## plotly daily
real_diff<-data_diff[which(data_diff$type=='real'),]
gamma_diff<-data_diff[which(data_diff$type=='scaled Gamma'),]
d<-NA
for (i in 1:(length(gamma_diff$day)-length(real_diff$day)-1)) {
  d<-c(d,NA)
}
real_diff_data<-c(real_diff$growth,d)
plot_diff_data<-data.frame(day=gamma_diff$day,scaled=gamma_diff$growth,real=real_diff_data
```

Figure 7: Codes to Construct Data for Data Visualization

At the end of these codes, a data frame named 'plot_cum_data' is generated to plot the curve of the real or predicted values for the total number of confirmed COVID-19 cases for each corresponding country. Additionally, a data frame named 'plot_diff_data' is generated to plot the curve of the real or predicted values for the number of daily confirmed COVID-19 cases for each corresponding country. A sample of the subset for 'plot_cum_data' and 'plot_diff_data' are as follows:
The attribute named 'scaled' represents the predicted daily or total confirmed COVID-19 cases for the corresponding date. The attribute named 'real' represents the real daily or total number of confirmed COVID-19 cases for the corresponding date. The reason that there are some 'NA' values in the attribute named 'real' is that we didn't

Table 7: Example of One 'plot_cum_data' or 'plot_diff_data'

date	scaled	real
2020-06-08	2072305.243	1961781
2020-06-09	2092653.896	1979868
2020-06-10	2112629.573	2000702
2020-06-11	2132231.892	2023590
2020-06-12	2151460.848	2048986
2020-06-13	2170316.804	2074526
2020-06-14	2188800.469	2094058
2020-06-15	2206912.89	2114026
2020-06-16	2224655.433	2137731
2020-06-17	2242029.77	2163290
2020-06-18	2259037.861	2191052
2020-06-19	2275681.943	2222579
2020-06-20	2291964.516	2255119
2020-06-21	2307888.323	2279879
2020-06-22	2323456.345	NA
2020-06-23	2338671.779	NA
2020-06-24	2353538.028	NA
2020-06-25	2368058.69	NA
2020-06-26	2382237.54	NA
2020-06-27	2396078.522	NA
2020-06-28	2409585.735	NA

have the real value for that corresponding date when we generated this data frame.

3.4 Data Inaccuracy

As we mentioned, the data source for the article [24] is the WHO situation report, in order to mitigate the risk of data from an individual country being misreported due to bias or politically motivation, reducing the overall accuracy of our data. Furthermore, according to CNN [41], the Center for Disease Control and Prevention announced that for every COVID-19 infection reported, there are ten more that have

gone undiagnosed. This implies that some level of inaccuracy is inevitable for any of the data collected about COVID-19. In another words, it is relatively impossible to find a data source that will represent the real state of COVID-19 with 100% accuracy.

4 Nonlinear Algorithmic Models

4.1 Basic Analysis of Original Data

A basic analysis of the original data is a necessary precursor for the next steps of analysis and for building the prediction model. First, we found the nineteen countries with the highest number of total confirmed COVID-19 cases on June, 22. The results are as follows:

Figure 8: Total Confirmed Cases of Top 19 Countries on June,22

According to the figure 8, it is clear that the US had the highest number of total confirmed COVID-19 cases on June, 22 by a significant margin. Brazil had the second highest number of total confirmed COVID-19 cases, with close to half of the cases of the US. The raw data for total confirmed COVID-19 cases of these countries

are as follows:

Table 8: Total Confirmed Cases of Top 19 Countries on June,22, 2020

Country	June,22
US	2280969
Brazil	1083341
Russia	591456
India	425282
United Kingdom	305803
Peru	251338
Spain	246272
Chile	242355
Italy	238499
Iran	207525
France	197008
Germany	191668
Turkey	187685
Pakistan	181088
Mexico	180545
Bangladesh	115786
Canada	103078
Qatar	87369
China	84573

It is helpful for us to identify these top 19 countries, as it allows us to assume that there is enough valid data to build and train the prediction model using the data of these countries. To verify that the data from these 19 countries has enough valid values and diversified distributions, we plotted a graph of the total confirmed cases and the daily growth in cases for the 10 countries with the most representative data distribution. This graph is as follows:

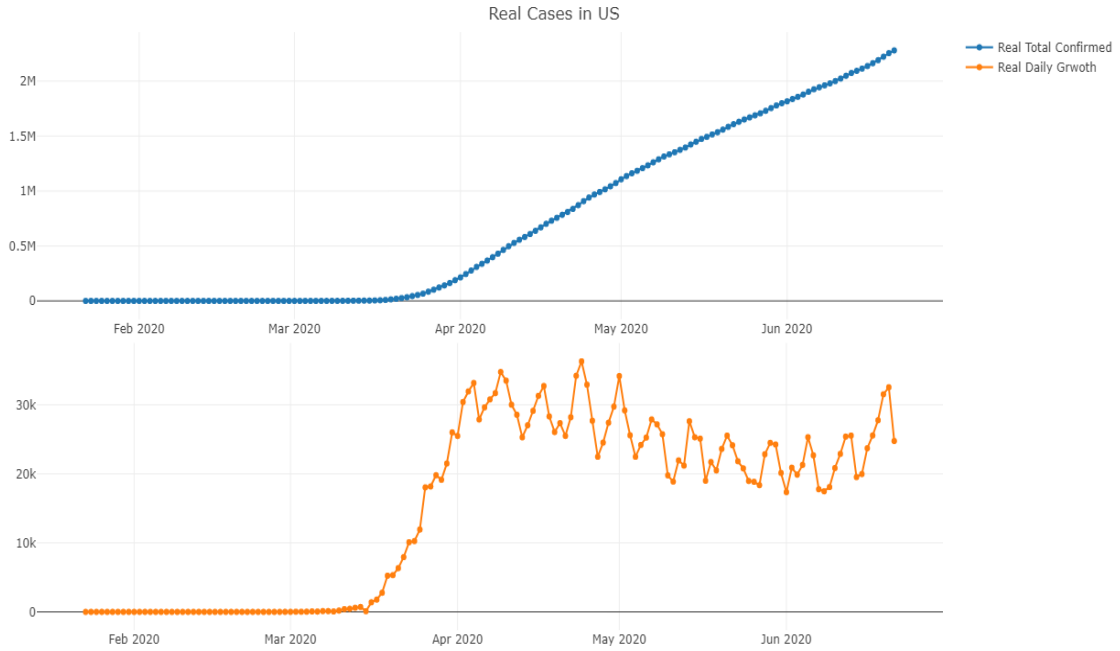


Figure 9: Real Daily Growth and Total Confirmed Cases in the US

Figure 9 illustrates the trends for daily growth in number of confirmed cases, as well as the cumulative total number of COVID-19 cases in the US. As is shown, the total number of confirmed COVID-19 cases in the US began to sharply increase in late March, and the shape of the graph is close to a line with a positive slope. The daily growth in number of COVID-19 cases in the US peaked for the first time in early April, and then it started to fluctuate violently. Although the fluctuation in daily growth was drastic, the lowest value was still just under 20,000, an extremely high number. The graph indicates that the COVID-19 epidemic in the US was getting worse as of June 22.

Brazil had the second-highest total number of confirmed COVID-19 cases as of June, 22. The graph of total confirmed cases and daily growth in cases in Brazil is as follows:

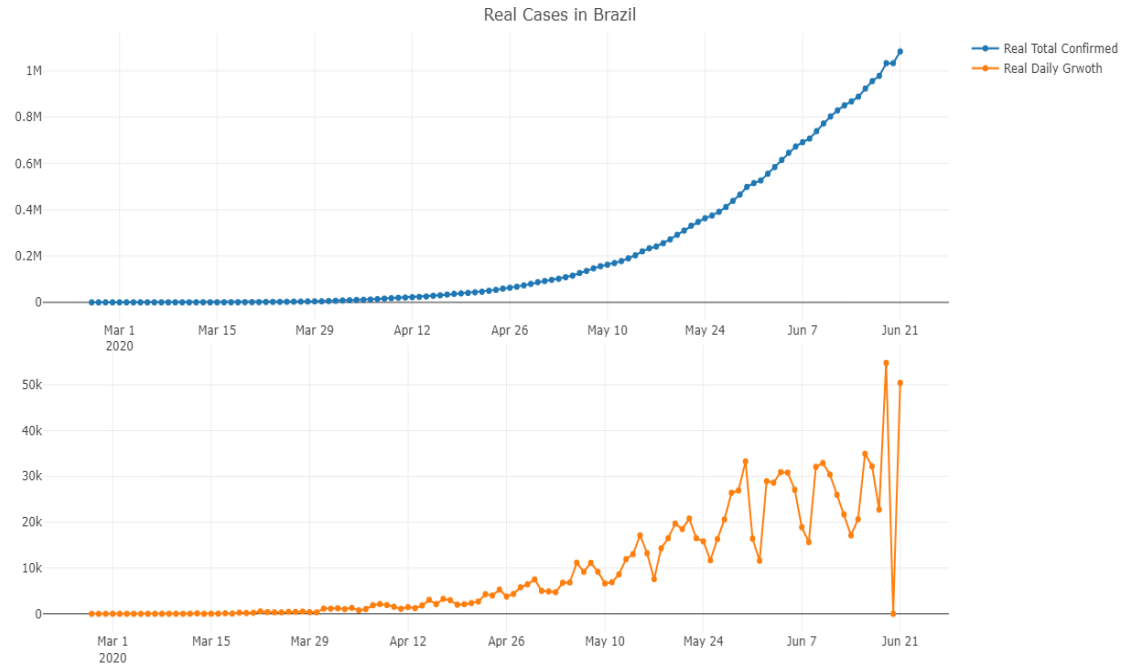


Figure 10: Real Daily Growth and Total Confirmed Cases in Brazil

Figure 10 illustrates the trends of daily growth in number of cases, as well as the cumulative total number of COVID-19 cases in Brazil, shown over time. As is shown, the total number of confirmed COVID-19 cases in Brazil started to increase gradually on April 12th, and the shape of the graph is close to exponential growth. The daily growth in number of COVID-19 cases in Brazil started to gradually increase on April 12th, with increasingly volatile fluctuations, and an upward trend on the whole. The graph indicates that the COVID-19 epidemic in Brazil was getting worse as of June 22.

Russia had the third-highest total number of confirmed COVID-19 cases as of June, 22. The graph showing number of total confirmed cases and daily growth in number cases in Russia is shown below:

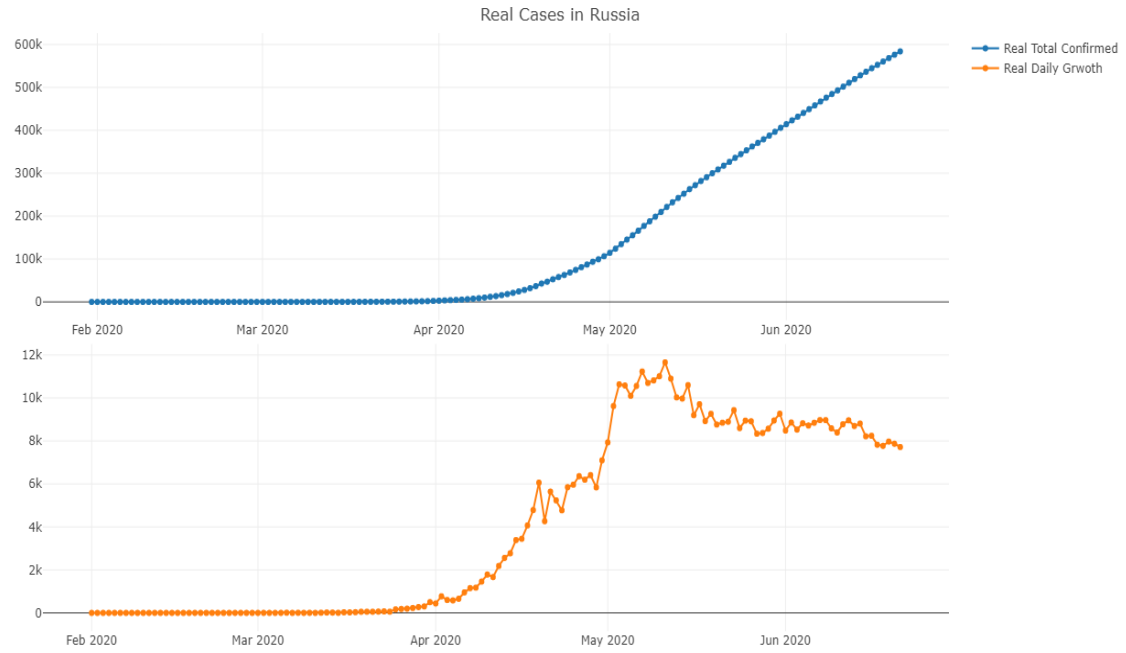


Figure 11: Real Daily Growth and Total Confirmed Cases in Russia

Figure 11 illustrates the trends of daily growth in number of cases as well as the cumulative total number COVID-19 cases in Russia. As is shown by the graph, the total number of confirmed COVID-19 cases in Russia started to increase gradually in early April, the shape of the graph being close to exponential growth. The daily growth in number of COVID-19 cases in Russia started to gradually increase in early April with slight volatility, peaking in early May. Although the curve fluctuated

slightly, it exhibited a downward trend after it peaked. The graph indicates that the COVID-19 epidemic in Russia began to improve after peaking.

Furthermore, the graph of the total number of confirmed cases and daily growth cases in India is shown below:

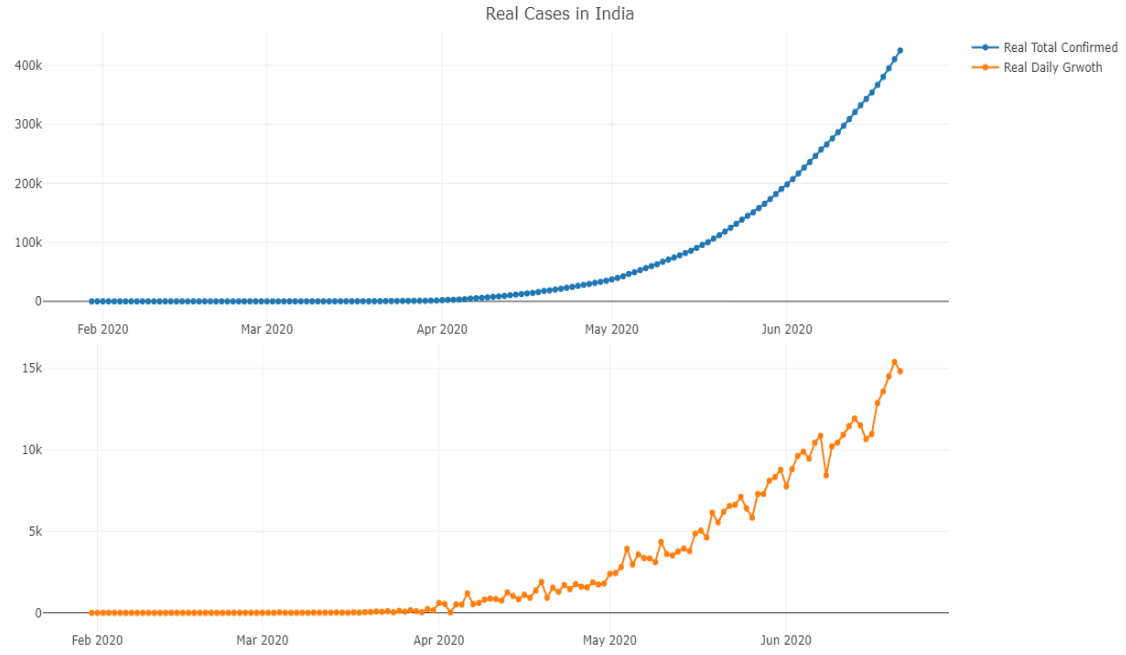


Figure 12: Real Daily Growth and Total Confirmed Cases in India

Figure 12 illustrates the trends in daily growth of COVID-19 cases, as well as the cumulative total number of COVID-19 cases in India. As is shown by the, the total number of confirmed COVID-19 cases in India started to increase gradually in early April, with the shape of the graph close to exhibiting exponential growth. The daily growth in number of COVID-19 cases in India started to gradually increase in early

April, and continued growing afterwards with slight volatility. Although the curve fluctuated slightly, it showed a continuous upward trend. The graph indicates that the COVID-19 epidemic in India was getting worse as of June 22.

The graph showing total number of confirmed cases and daily growth in number of cases for the United Kingdom is shown below::

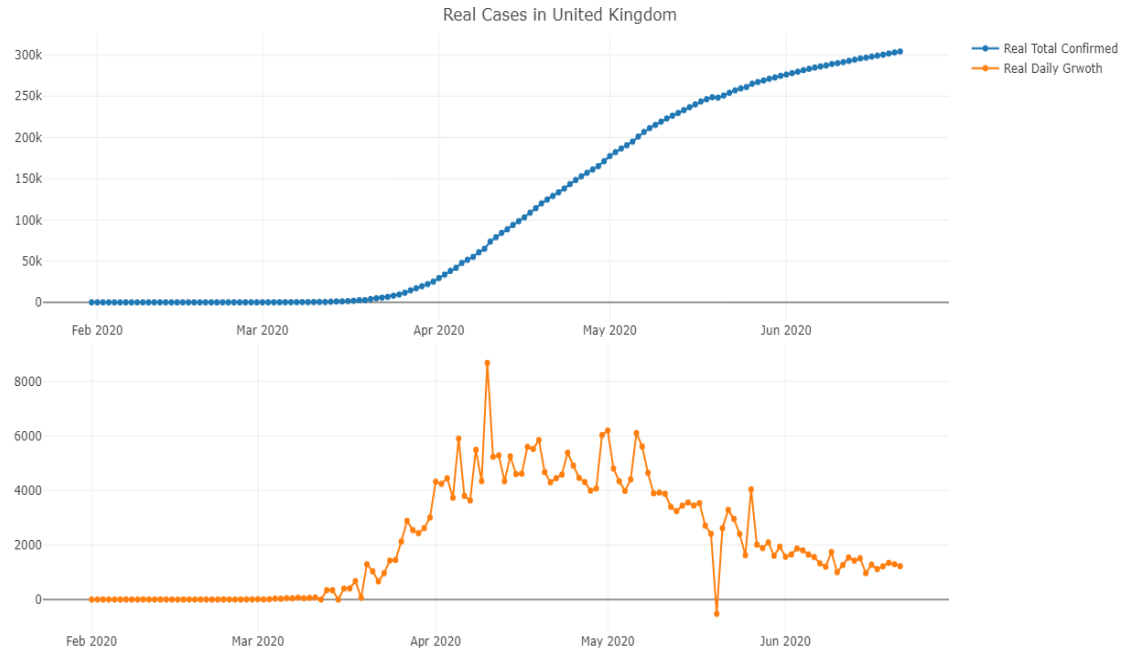


Figure 13: Real Daily Growth and Total Confirmed Cases in the United Kingdom

Figure 13 illustrates the trends in daily growth of COVID-19 cases, as well as the cumulative total number of COVID-19 cases, for the United Kingdom. As is shown by the graph, the total number of confirmed COVID-19 cases in the United Kingdom started to increase gradually in late March, with the shape of the graph close to ex-

hibiting exponential growth. After that, the curve for the total number of confirmed cases flattens slightly after June. The daily growth in number of COVID-19 cases in the United Kingdom started to sharply increase in the middle of March with slight volatility, peaking shortly thereafter towards the middle of April. Although the curve initially fluctuated violently, it showed a downward trend after peaking. The graph indicates that the COVID-19 epidemic in the United Kingdom has been improving after its peak.

The graph showing total number of confirmed cases and daily growth in number of cases in Peru is shown below:

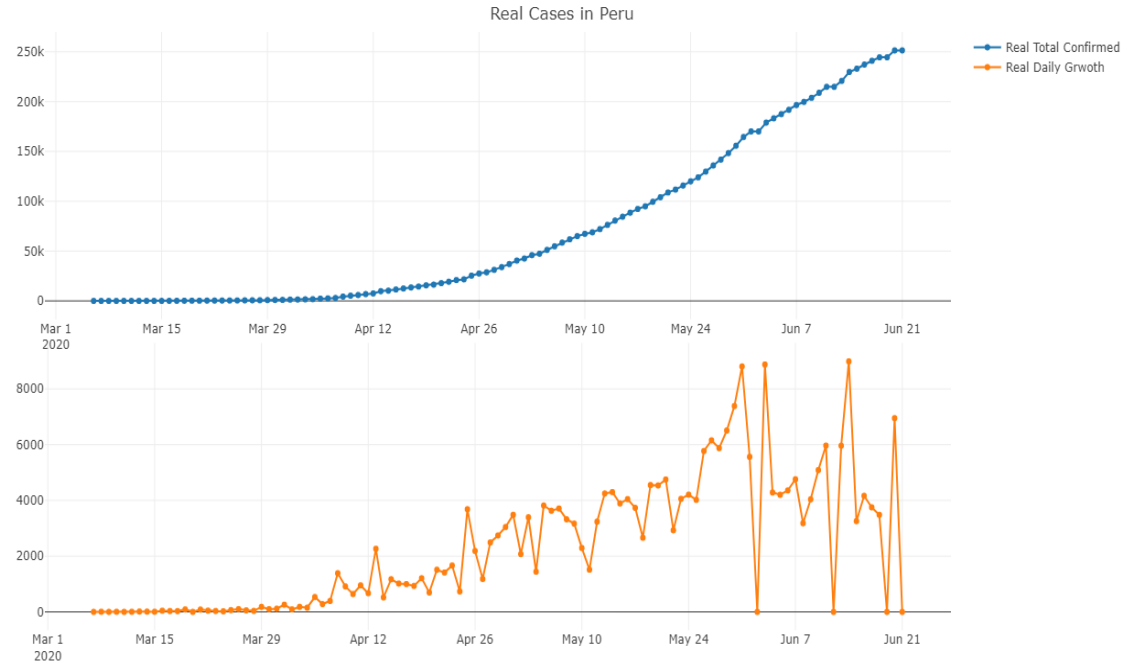


Figure 14: Real Daily Growth and Total Confirmed Cases in Peru

Figure 14 illustrates the trends in daily growth in number of COVID-19 cases, as well as the cumulative total number of COVID-19 cases in Peru. As is shown by the graph, the total number of confirmed COVID-19 cases in Peru started to increase gradually in late March, with the shape of the graph close to exhibiting exponential growth. Furthermore, the daily growth in number of COVID-19 cases in Peru started to sharply increase in the middle of March with volatility. The curve fluctuated with increasing volatility, showing a continuous upward trend. The graph indicates that the COVID-19 epidemic in the Peru has been getting worse.

The graph showing total number of confirmed COVID-19 cases, as well as daily growth in number of cases for Spain is shown below:

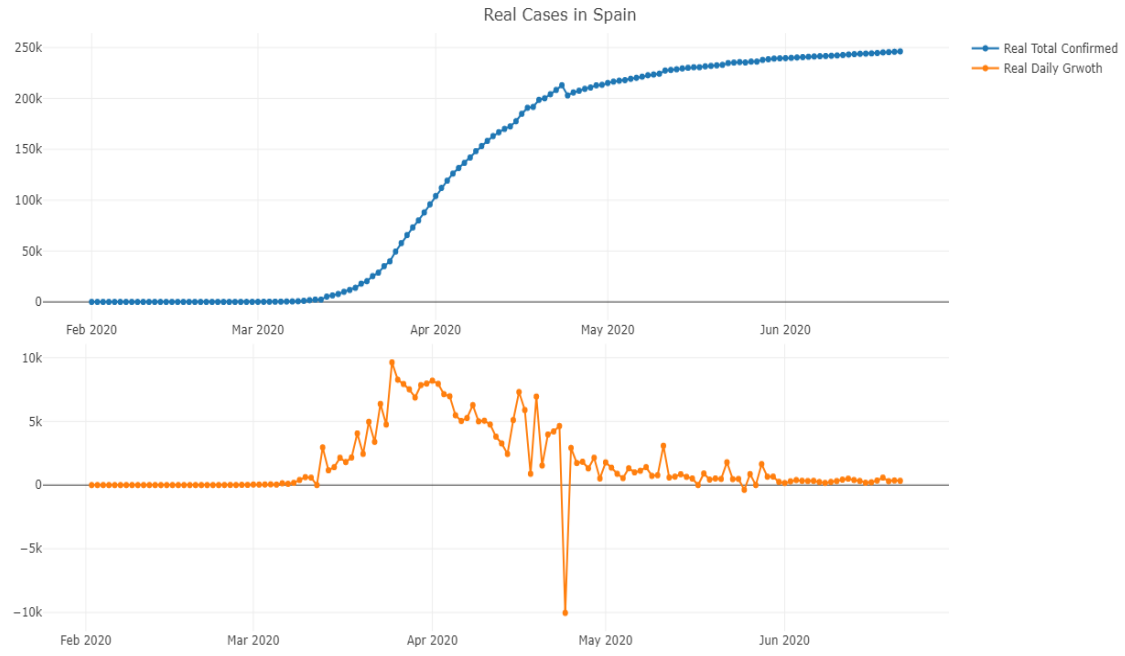


Figure 15: Real Daily Growth and Total Confirmed Cases in Spain

Figure 15 illustrates the trends in daily growth in numbers of COVID-19 cases, as well as the cumulative total number of COVID-19 cases in Spain. As is shown by the graph, the total number of confirmed COVID-19 cases in Spain began to increase sharply in mid-March, with the shape of the graph close to exhibiting exponential growth. Afterwards, the curve for the total number of confirmed cases begins to slightly flatten after May. The daily growth in number of COVID-19 cases in Spain began to sharply increase in the middle of March with slight volatility, with its peak in late March. Although the curve fluctuated violently, it showed a downward trend after peaking. There was an obvious outlier in late April, the reason for which is unclear. The graph indicates that the COVID-19 epidemic in Spain has been getting better after peaking.

The graph showing the total number of confirmed cases and the daily growth in number of cases in Chile is shown below:

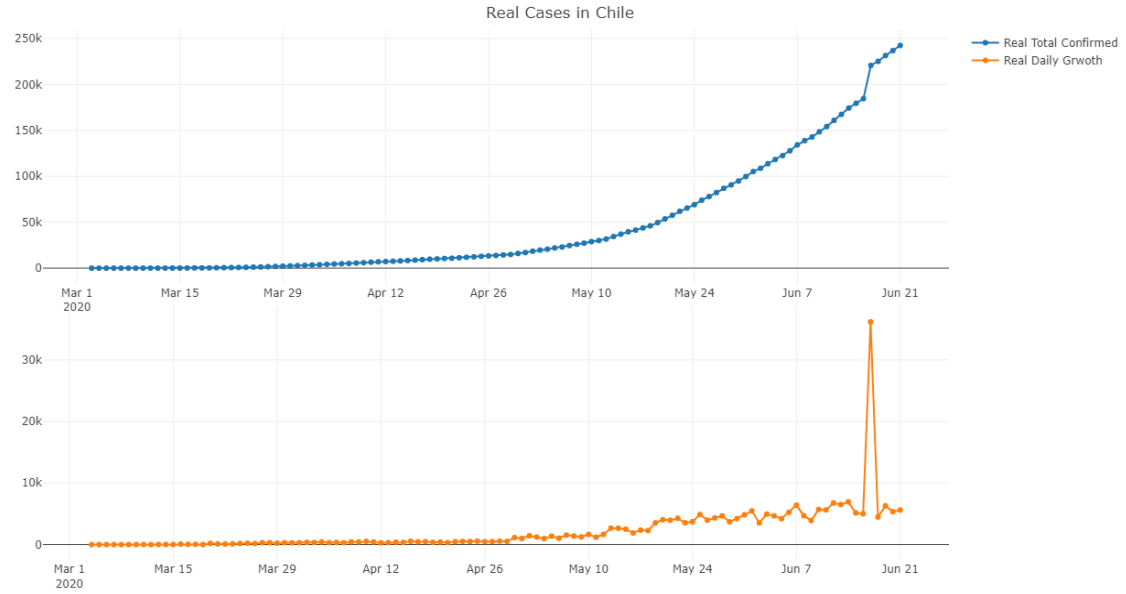


Figure 16: Real Daily Growth and Total Confirmed Cases in Chile

Figure 16 illustrates the trends in daily growth in number of COVID-19 cases, as well as the cumulative total number of COVID-19 cases in Chile. As is shown by the graph, the total number of confirmed COVID-19 cases in Chile began to increase gradually in the mid-April, with the shape of the graph close to exhibiting exponential growth. The daily growth in number of COVID-19 cases in Chile started to gradually increase at the end of April with some volatility. Although the curve fluctuated slightly, it stayed relatively level around 5,000. There was an obvious outlier in late June, the reason which is also unclear. The graph indicates that the COVID-19 epidemic in Chile has been getting slightly worse.

The graph showing total number of confirmed cases and the daily growth in number

cases for Italy is shown below:

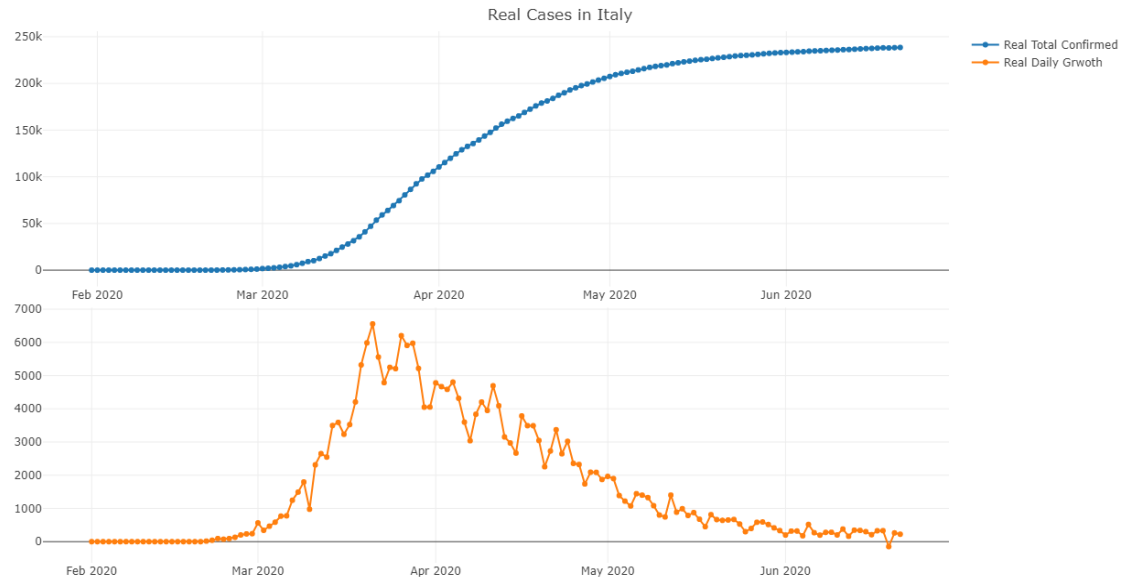


Figure 17: Real Daily Growth and Total Confirmed Cases in Italy

Figure 17 illustrates the trends in daily growth in number of COVID-19 cases, as well as the cumulative total number of COVID-19 cases in Italy. As is shown by the graph, the total number of confirmed COVID-19 cases in Italy began to increase sharply in the middle of March, with the shape of the graph close to exhibiting exponential growth. The curve for the total number of confirmed cases begins to flatten in early May. The daily growth in number of COVID-19 cases in Italy started to sharply increase in the middle of March with slight volatility, peaking in the middle of April. Although the curve fluctuated violently, it showed a downward trend and finally hovered around zero. The graph indicates that the COVID-19 epidemic in Italy has been getting better after peaking.

Lastly, the graph showing total number of confirmed cases and daily growth in number cases for Iran is shown below:

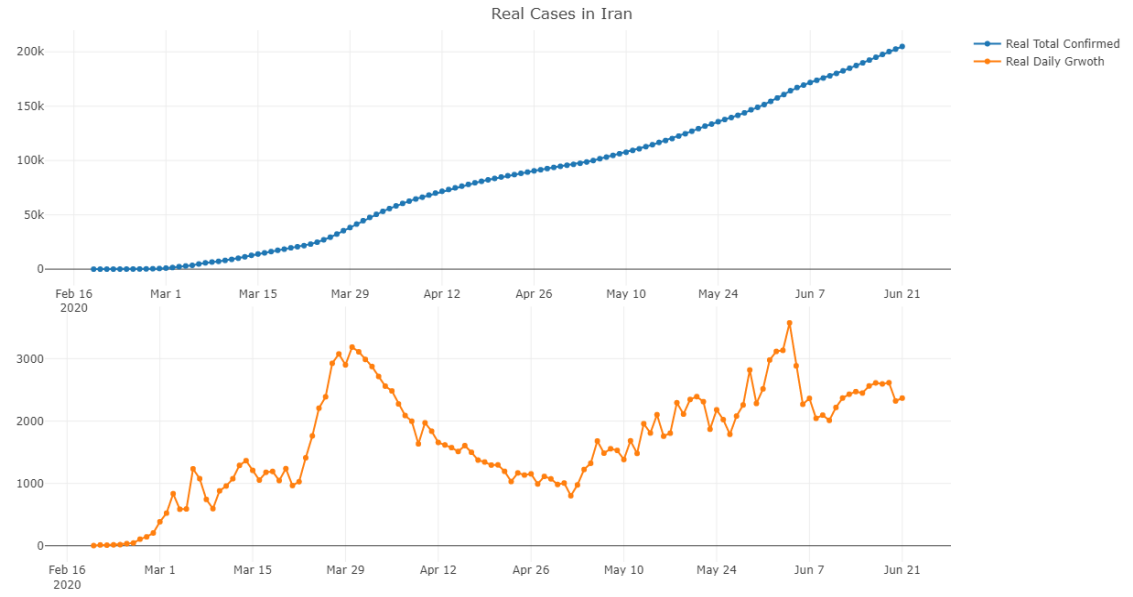


Figure 18: Real Daily Growth and Total Confirmed Cases in Iran

Figure 18 illustrates the trends in daily growth in number of COVID-19 cases, as well as the cumulative total number of COVID-19 cases in Iran. As is shown by the graph, the total number of confirmed COVID-19 cases in Iran started to increase sharply in late February, with the shape of the graph resembling a line with a positive slope. Notably, it is apparent that there were two peaks in the curve for daily growth in number of COVID-19 cases in Iran. This value peaked for the first time in late March, and then gradually decreased until early May. After early May, the daily growth in number of COVID-19 cases sharply increased once again, and peaked for a second time in early June. After the second peak, daily growth in COVID-19 cases

remained at a stable level, and didn't show a downward trend. The graph indicates that Iran had two peaks in the number of COVID-19 cases from February to June, and that the COVID-19 epidemic in Iran has still been getting worse.

According to the basic analysis of the original data for the 10 countries with the highest total number of cases, we can come to some conclusions. First, the trends of total number of confirmed COVID-19 cases for different countries are generally similar, and is either close to exponential growth or a line with a positive slope. Using this, it is possible to build a model with a simple statistical core to estimate the total number of confirmed COVID-19 cases in different countries. Second, the distributions of the daily growth in number of COVID-19 cases in different countries are extremely different. Thus, it is comparatively more difficult to build a model with a simple statistical core to estimate the daily growth in number of COVID-19 cases in different countries. Third, the shape of the curve for the daily growth in number of COVID-19 cases in Iran is totally different from the other countries, since it has two peaks. Therefore, estimating the daily growth in number of COVID-19 cases in Iran will probably require a more complex model.

4.2 Basic Gamma Model

After the analysis of the original data in the previous part, we started to consider the mathematical core of our prediction model. We first compared the graphs showing daily growth in number of COVID-19 cases in different countries, and the graphs showing the total number of confirmed COVID-19 cases in different countries, with

the plots for probability density functions and cumulative distribution functions of various traditional statistical models. This was done in order to find some similar patterns between them. After communicating with professor Sobocinski [42], we decided to compare the two sets of graphs with plots of probability density function and cumulative distribution functions of gamma distribution. The comparison between the plots of daily growth in number of COVID-19 cases in five countries with the plots of probability density function of gamma distribution is as follows:

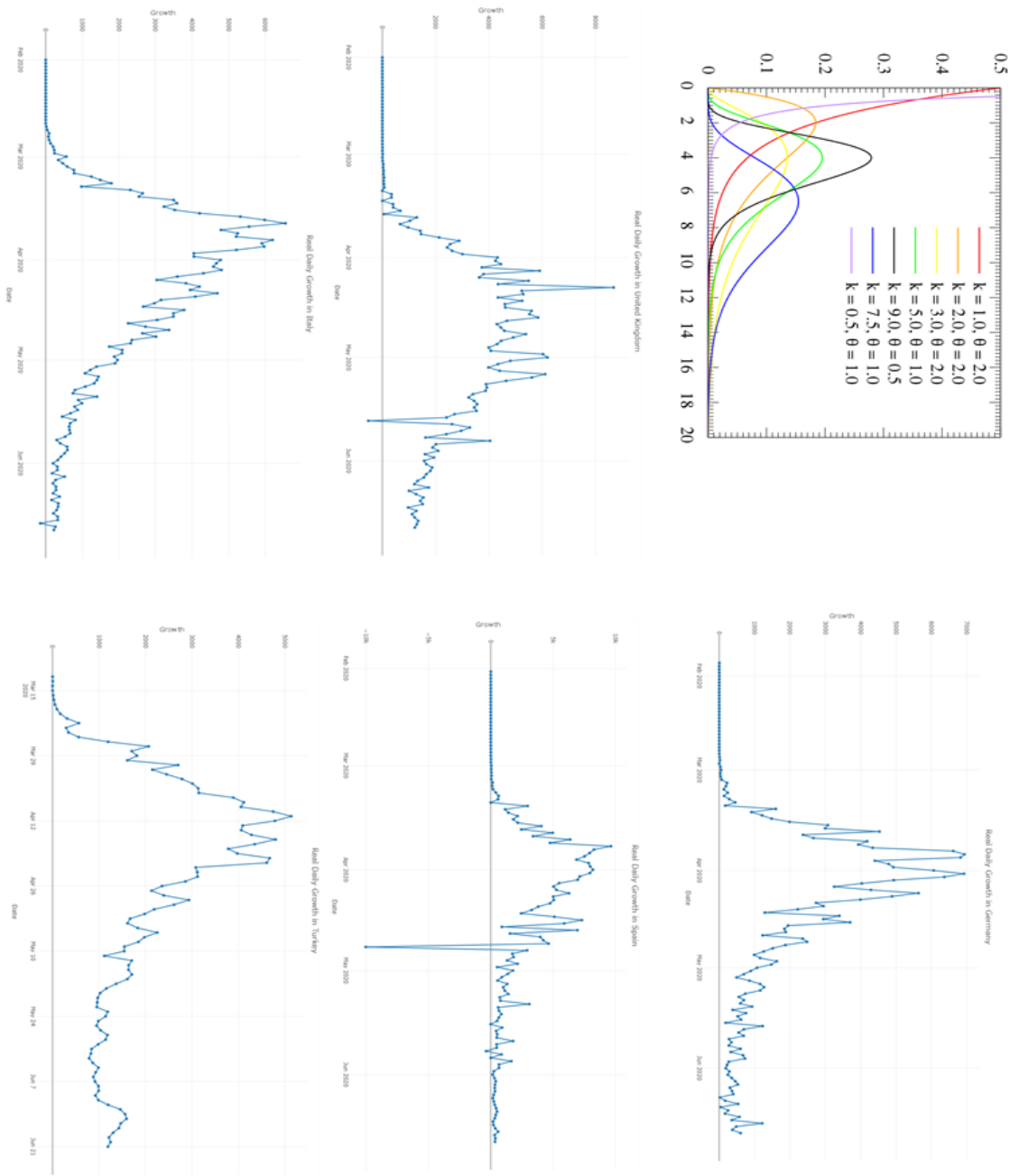


Figure 19: PDF of Gamma Distribution compared with Daily Growth

This figure illustrates the probability density function of gamma distribution and the plots of daily growth in number of COVID-19 cases in the United Kingdom, Spain, Italy, Germany, and Turkey. It is apparent that the plot of probability density function of gamma distribution is extremely similar to the plots showing daily growth in COVID-19 cases in these countries. This may indicate that they show similar patterns.

The comparison between the plots showing the total number of confirmed COVID-19 cases in five countries with the plots of cumulative distribution function of gamma distribution is as follows:

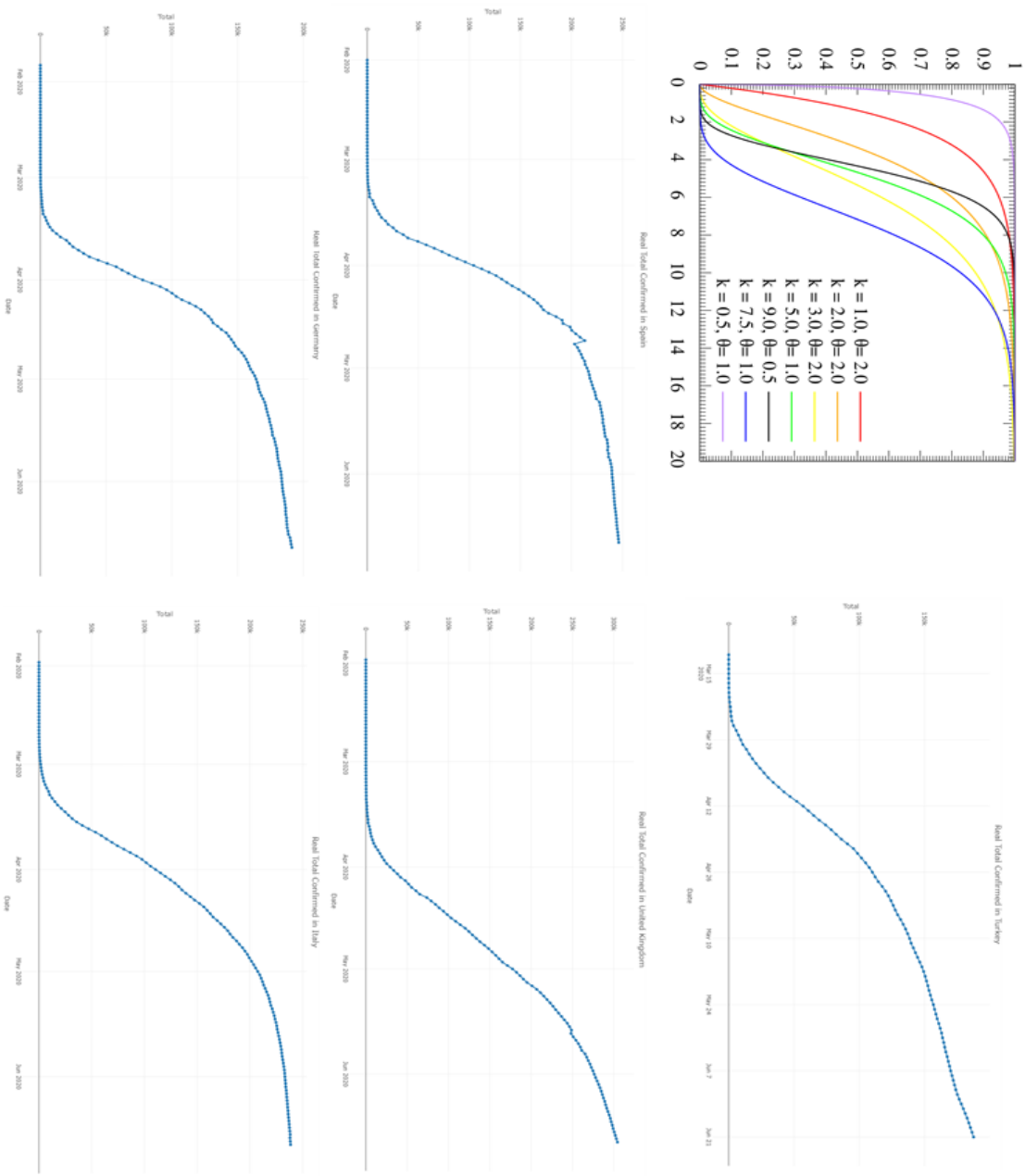


Figure 20: CDF of Gamma Distribution compared with Total Confirmed

This figure illustrates the cumulative distribution function of gamma distribution and the plots showing the total number of confirmed COVID-19 cases in the United Kingdom, Spain, Italy, Germany, and Turkey. From this, we can see that the plot for cumulative distribution function of gamma distribution is quite similar with the plots showing total confirmed cases in these countries. This may also imply that the two exhibit similar patterns. Based on these comparisons, we decided to use the gamma distribution as the mathematical core for our prediction model.

The process for our Basic Gamma Prediction Model is as follows:

Basic Gamma Prediction

(1) Load data of COVID-19 dashboard by CSSE into the data frame.

(2) Data preprocess to generate trained data.

(3) Use R package *nls* and R function *gamma*() to fit the non-linear model to data by computing a, b, and, c:

$$y = f(x) = \frac{a * x^{(b-1)} * e^{-\frac{x}{c}}}{c^b * gamma(b)}$$

(4) Compute the prediction of total confirmed cases by the R function *pgamma*() and plot the curve:

$$prediction = a * pgamma(x, shape = b, scale = c)$$

(5) Compute the prediction of daily growth cases by the R function *dgamma*() and plot the curve:

$$prediction = a * dgamma(x, shape = b, scale = c)$$

In the first step of this process, the original data is loaded as a data frame into the R environment in order to preprocess it for the next step.

In the second step, the original data is reconstructed to the form as the data for

regression.

In the third step, the R function $gamma(b)$ is the gamma function evaluated at b, which is:

$$gamma(b) = \int_0^\infty x^{b-1} e^{-x} dx \quad , \Re(b) > 0 \qquad (4-1)$$

In order to estimate the coefficients of a, b and c in the gamma distribution, the R function nls was utilized. According to [33], the Nonlinear Least Squares (nls) calculates the nonlinear (weighted) least-squares estimates of the coefficients in a nonlinear model. An nls object is a type of fitted model object. It has methods for the generic functions anova, coef, confint, deviance, df.residual, fitted, formula, log-Lik, predict, print, profile, residuals, summary, vcov, and weights.

In the third step, we considered a non-linear model of the following form:

$$y_i = f(x_i; a, b, c) + \varepsilon_i, i = 1, ..., n \qquad (4-2)$$

with the function $f(.)$ of the form:

$$f(x) = \frac{a * x^{(b-1)} * e^{-\frac{x}{c}}}{c^b * gamma(b)} \qquad (4-3)$$

From [43], we estimated the parameters a, b, and c by applying the non-linear least squares method, in which the residual sum of squares was minimized.

$$S_n(a,b,c) = \sum_{i=1}^{n} [y_i - f(x_i; a, b, c)]^2 \qquad (4-4)$$

In this formula, the y_i represents the number for the daily increase in COVID-19 cases in one country.

It is usually necessary to provide the initial values of the parameters in a non-linear model when people are using nls, in order to avoid a convergence failure, which is when the function evaluation limit is reached without convergence. As such, we needed to estimate the initial values for parameters a, b, and c. The formulas and variable relations, which helped us to estimate the initial values of parameters a, b, and c, are as follows:

$$a = -ln(\sigma\sqrt{2\pi}) - \frac{\mu^2}{2\sigma^2} + ln(k+\Delta) \qquad (4-5)$$

$$a_\Delta = -ln(\sigma\sqrt{2\pi}) - \frac{\mu^2}{2\sigma^2} \qquad (4-6)$$

$$ln(k+\Delta) = a - a_\Delta \qquad (4-7)$$

$$b = \frac{\mu}{\sigma^2} - 1 \qquad (4-8)$$

$$c = -(\frac{1}{2}\sigma^2) \qquad (4-9)$$

Here, μ is the expected value of the original data and σ is the standard deviation for the original data.

In the fourth step of the process, the R function $pgamma(x, shape = b, scale = c)$ is the distribution function of gamma distribution where the value of the shape is b and value of the scale is c. This is shown below:

$$F(x; b, c) = \int_0^x f(u; b, c) = \frac{\gamma(b, \frac{x}{c})}{\Gamma(b)} \qquad (4-10)$$

Here the $\gamma(b, \frac{x}{c}$ is the lower in complete gamma function:

$$\gamma(b, \frac{x}{c}) = \int_0^x t^{s-1} e^{-t} dt \qquad (4-11)$$

In the fifth step, the R function $dgamma(x, shape = b, scale = c)$ is the probability density function of gamma distribution, where the value for shape is b and the value for scale is c. This is shown below:

$$f(x; b, c) = \frac{x^{(b-1)} e^{-\frac{x}{c}}}{c^b * \Gamma(b)} \qquad (4-12)$$

Where $\Gamma(b)$ is the gamma function evaluated at b, which is:

$$\Gamma(b) = \int_0^\infty x^{b-1} e^{-x} dx \quad , \Re(b) > 0 \qquad (4-13)$$

4.3 Application of Gamma Distribution

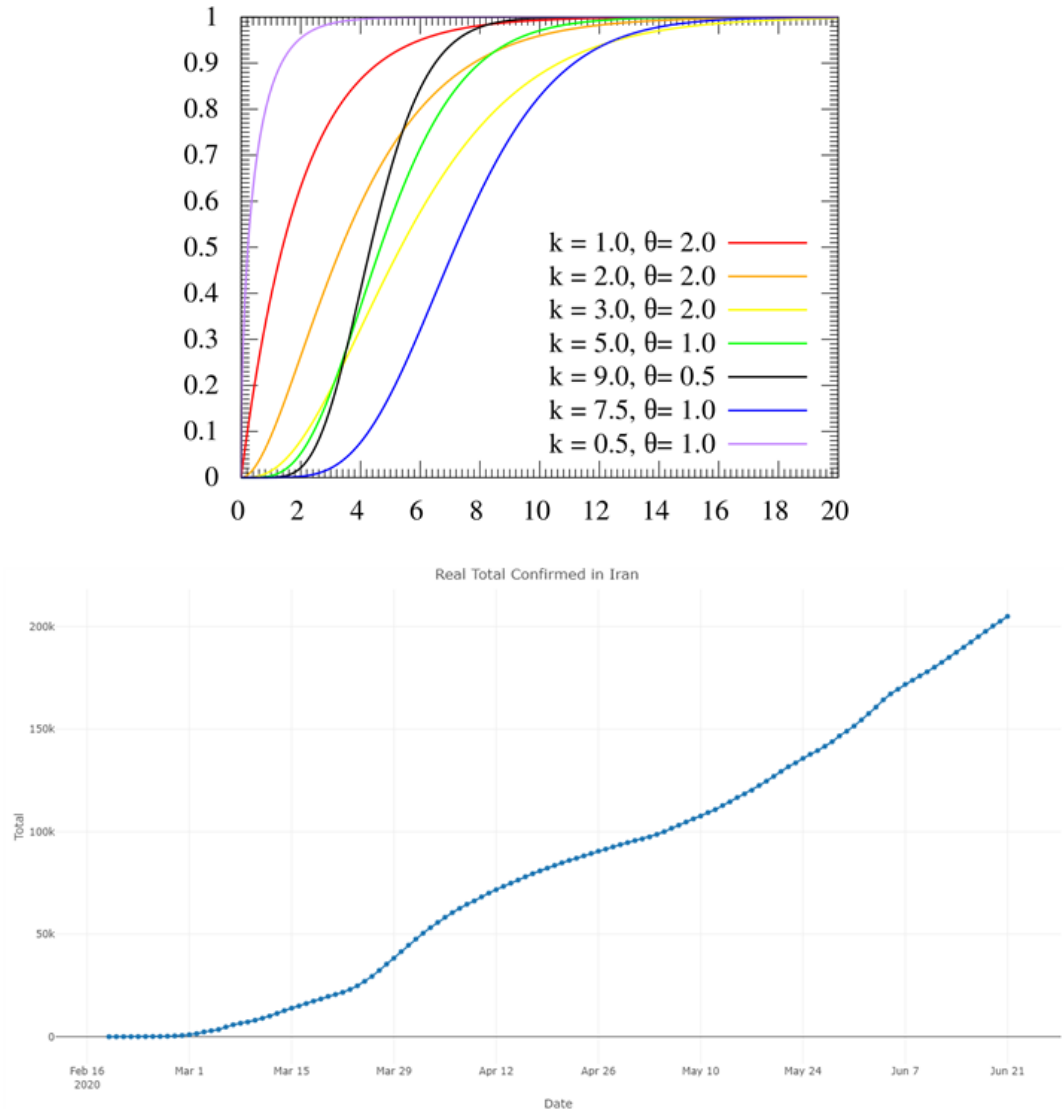


Figure 21: CDF of Gamma distribution compared with Total Confirmed in Iran

As is shown in the figure 21, the total number of confirmed COVID-19 cases in Iran started to increase sharply in late February, and the shape of the graph is similar to a line with a positive slope. It is apparent that the shape of the curve for total number of confirmed COVID-19 cases in Iran is not similar to the shape of the curve for the cumulative distribution function of gamma distribution.

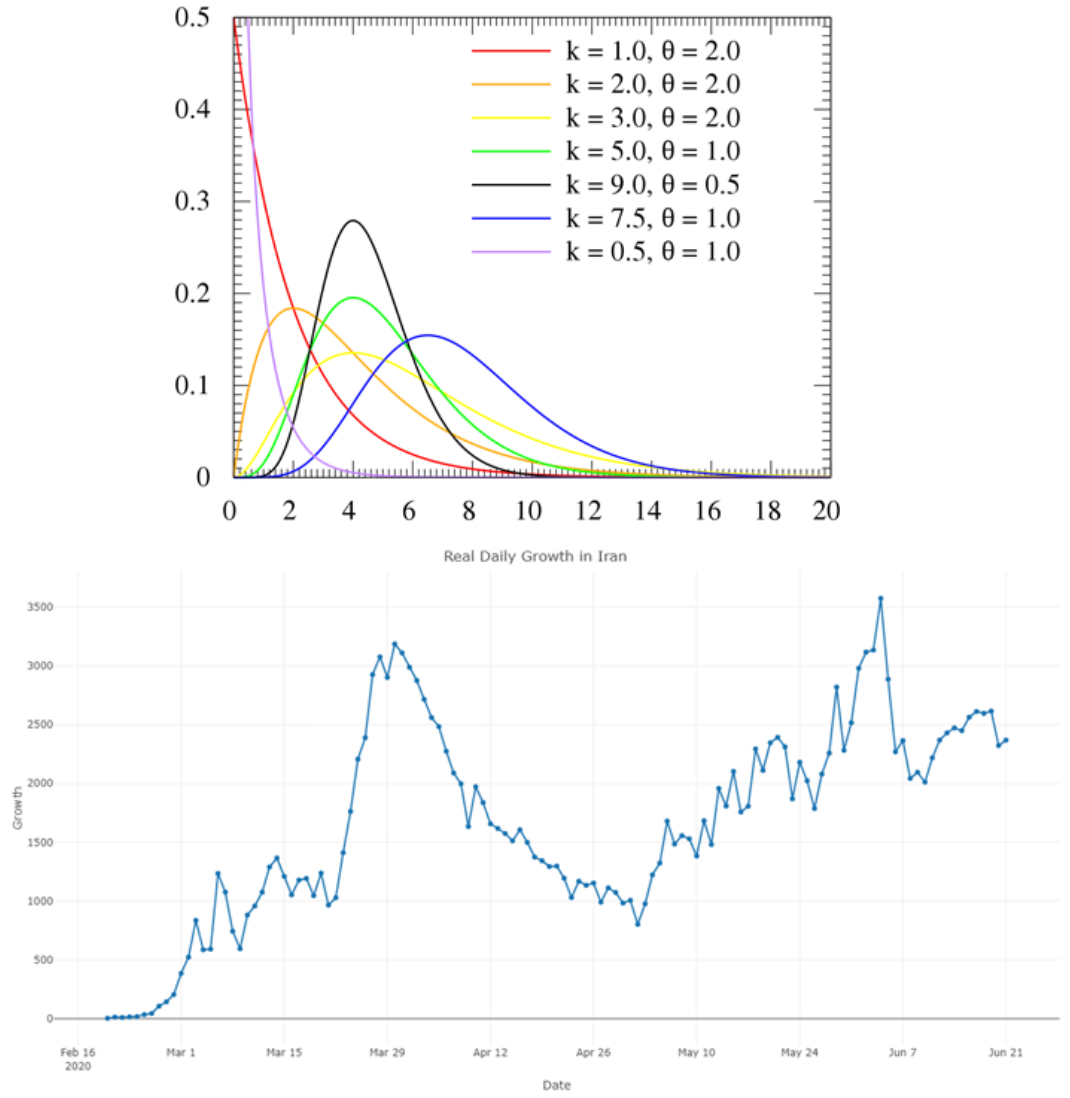


Figure 22: PDF of Gamma Distribution Compared with Daily Growth in Iran

As is shown in the figure 22, there were two peaks in the graph of daily growth in number of COVID-19 cases in Iran. The first peak was in late March, and then the daily growth in cases gradually decreased until early May. After early May, the daily

growth in cases began to sharply increase again and peaked for the second time in early June. It is apparent that this curve for Iran is totally different from the shape of the curve of probability density function of gamma distribution.

The comparison of the two plots therefore implies that it is very likely that the basic gamma prediction model that we proposed in chapter 4.2 will not work well for the data from Iran. The basic gamma prediction model for this data is as follows:

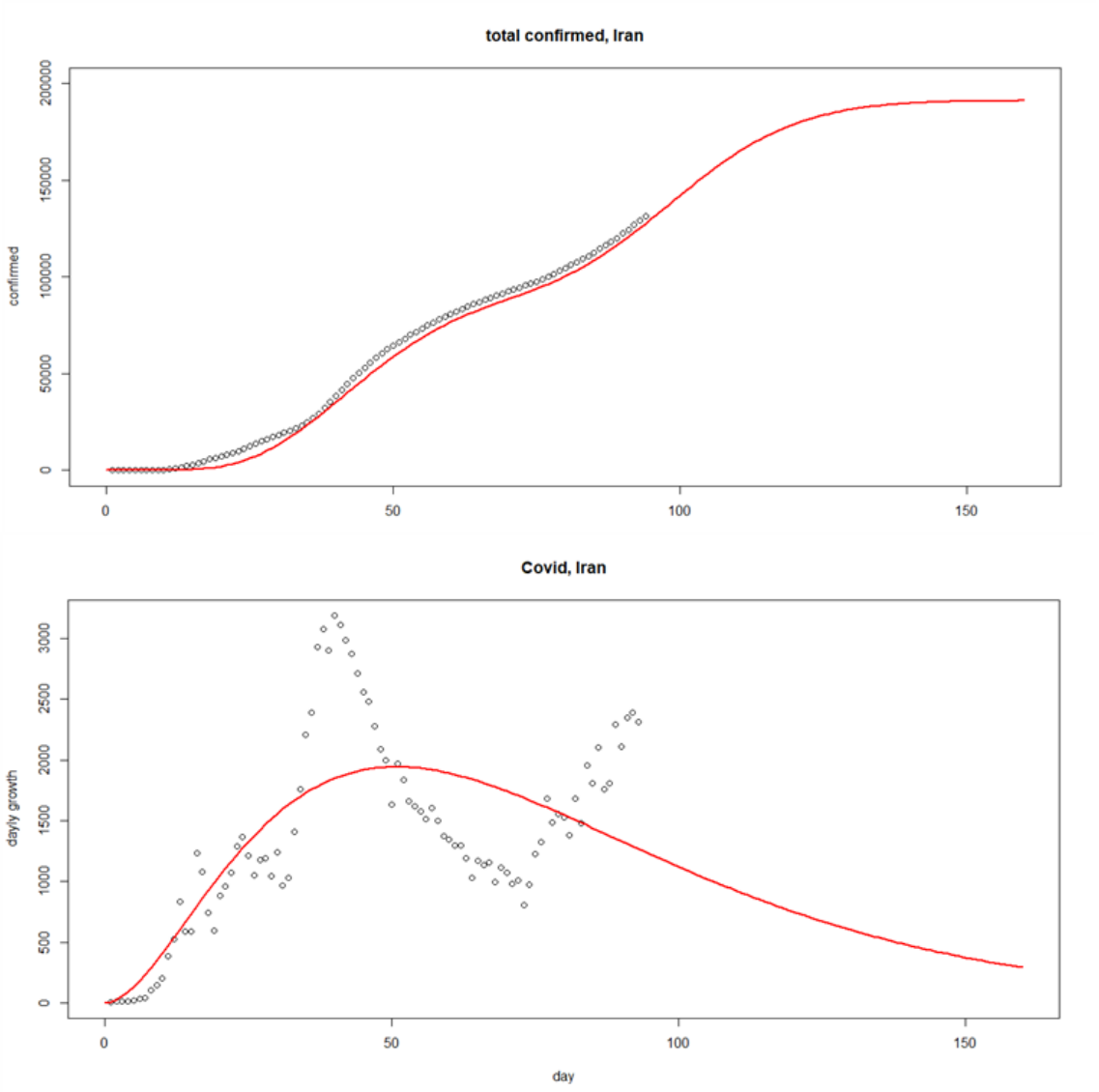


Figure 23: PDF of Gamma Distribution Compared with Daily Growth in Iran

In the figure 23, the hollow dots show the distribution of the real total number of confirmed cases and the daily growth in number of COVID-19 cases in Iran, respectively. The curves illustrate the results from the prediction of these two values

produced by our basic gamma prediction model.

As is shown by the first plot, the prediction curve for the total number of confirmed COVID-19 cases in Iran generally fits the real data. The trend for the predicted total number of cases curve shows roughly the same distribution as the real data. However, as is shown by the second plot, the prediction curve for daily growth in number of COVID-19 cases in Iran wholly does not fit the real data. The trend for the prediction of daily growth in cases is not the same as the distribution of the real data. In order to improve the accuracy of the basic gamma prediction model on the data for Iran, we designed a prediction model by a combination of two Gamma distributions.

The process for application of gamma distribution is as follow:

Application of Gamma distribution

(1) Load data of COVID-19 dashboard by CSSE into the data frame.

(2) Data preprocess to generate trained data.

(3) Use the R package *nls* and R function *gamma*() to fit the non-linear model to data by computing a, b, c, d and e:

$$y = f(x) = \frac{a}{2} * (\frac{x^{(b-1)} * e^{-\frac{x}{c}}}{c^b * gamma(b)} + \frac{x^{(d-1)} * e^{-\frac{x}{e}}}{e^d * gamma(d)})$$

(4)Compute the prediction of daily growth cases by the non-linear model and plot the curve:

$$daily - growth - prediction = \frac{a}{2} * (\frac{x^{(b-1)} * e^{-\frac{x}{c}}}{c^b * gamma(b)} + \frac{x^{(d-1)} * e^{-\frac{x}{e}}}{e^d * gamma(d)})$$

(5) the prediction of total confirmed cases by the R function and plot the curve:

$$total - confirmed - prediction = cumsum(daily - grwoth - prediction)$$

In the first step of this process, the original data is loaded as a data frame into the R environment in order to preprocess it for the next step.

In the second step, the original data is reconstructed to the form as the data for regression.

In the third step, the R function $gamma(b)$ is the gamma function evaluated at b. This is shown below:

$$gamma(b) = \int_0^\infty x^{b-1}e^{-x}dx \quad , \Re(b) > 0 \tag{4-14}$$

In order to assume the coefficients of a, b, c, d, and e in the nonlinear model, the R function nls was utilized. As stated before, according to [33], the Nonlinear Least Squares (nls) calculates the nonlinear (weighted) least-squares estimates of the coefficients in a nonlinear model.

In the third step, we considered a non-linear model of the following form:

$$y_i = f(x_i; a, b, c, d, e) + \varepsilon_i, i = 1, ..., n \tag{4-15}$$

Here is the function $f(.)$ of the form:

$$f(x) = \frac{a}{2} * (\frac{x^{(b-1)} * e^{-\frac{x}{c}}}{c^b * gamma(b)} + \frac{x^{(d-1)} * e^{-\frac{x}{e}}}{e^d * gamma(d)}) \tag{4-16}$$

According to [43], we estimated the parameters a, b, c, d, and e by applying the non-linear least squares method, in which the residual sum of squares is minimized.

$$S_n(a, b, c, d, e) = \sum_{i=1}^{n}[y_i - f(x_i; a, b, c, d, e)]^2 \tag{4-17}$$

Here, the y_i represents the value for the daily growth in number of COVID-19 cases in Iran.

In the fifth step, the R function $cumsum(daily - grwoth - prediction)$ is meant to cumulatively sum the predicted value of daily growth in number of COVID-19 cases, in order to produce the predicted value of total number of confirmed COVID-19 cases in Iran.

The curve for the daily growth in number of COVID-19 cases in Iran, as predicted by the application of gamma distribution, is shown below:

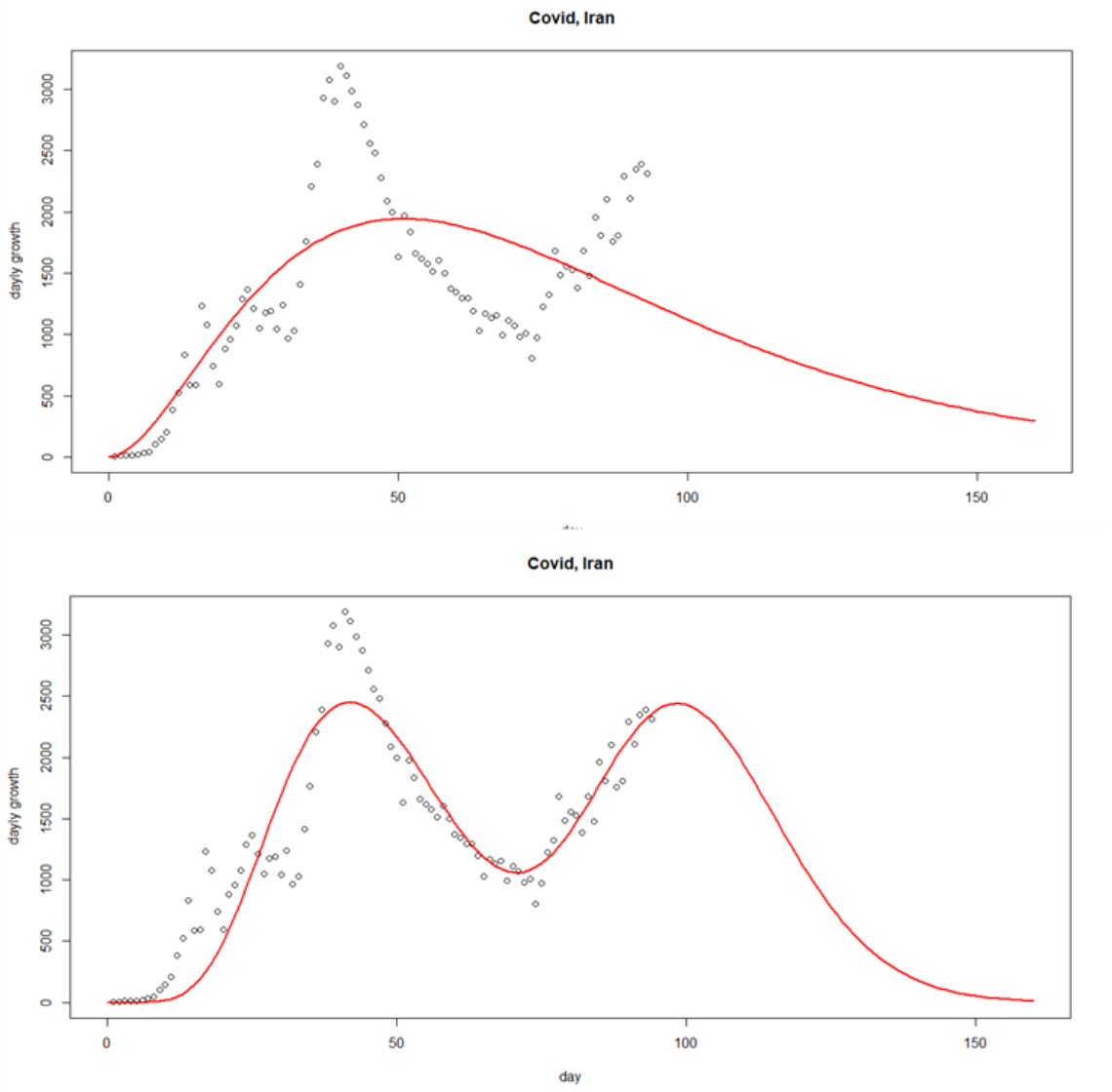


Figure 24: Basic Gamma compared with Application Gamma

In the figure, the hollow dots illustrate the real daily growth in number of COVID-19 cases in Iran. The red curves illustrate the prediction results for this value as produced by our prediction model. The top curve was produced by the basic gamma

prediction model, and the plot below was produced by the application gamma prediction model. As is shown by the second plot, the predicted curve for daily growth in number of COVID-19 cases in Iran almost completely fits the real data for this value. The trend of the daily growth prediction curve is similar the distribution of the real data. This means that the accuracy of the application gamma prediction model has been significantly improved for the data from Iran.

4.4 Analysis of Models

We found that the nonlinear algorithmic models were able to make a good prediction after being trained. A good prediction is defined as having an acceptable level of inaccuracy between the prediction value and the actual value. In this section, we evaluated the performance of our prediction models using the data from the 10 countries with the most total confirmed COVID-19 cases on June, 22. We forecasted the total confirmed number of COVID-19 cases for the 14 days following 22 June. The inaccuracies between the predicted number of cases and the real number of cases over those 14 days were calculated. The degree of fit between the prediction curve and the original data was illustrated, as well as some statistical information. Furthermore, the estimated value of the weights of the model were also computed. These analyses helped us to evaluate the performance of our models and find the methods to improve them.

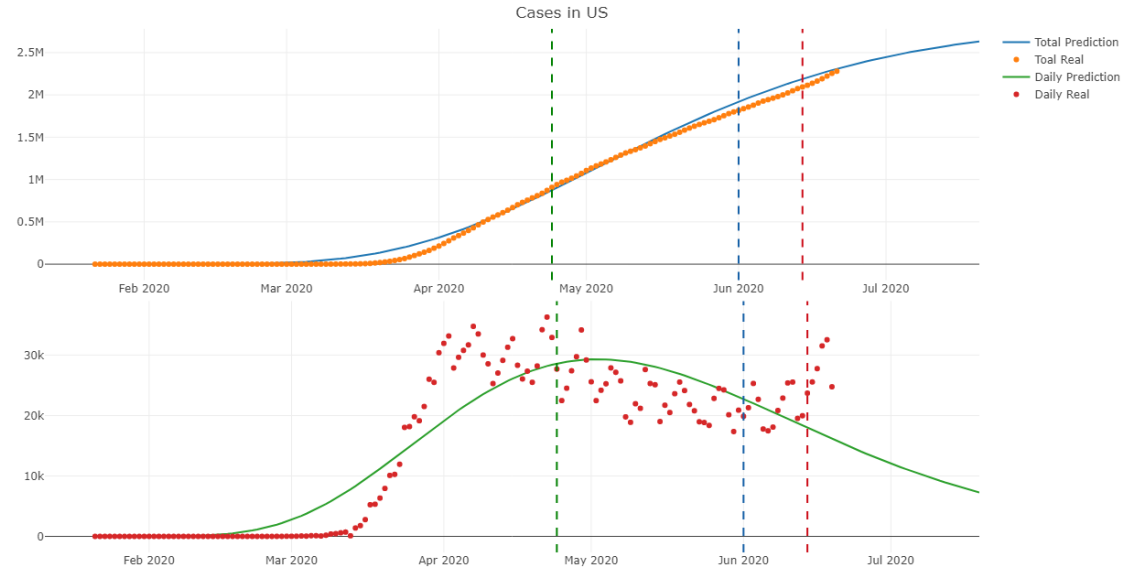


Figure 25: Prediction Result of Basic Gamma Model of the US

Figure 25 illustrates the prediction results of our basic gamma model for the US. The blue curve in the top graph illustrates the prediction results for total number of confirmed COVID-19 cases in the US using the basic gamma prediction model. The green curve illustrates the prediction results for daily growth in number of COVID-19 cases in the US. The orange dots illustrate the real data for total confirmed COVID-19 cases in the US, and the red dots illustrate the real data for daily growth in number of COVID-19 cases in the US. The blue vertical dashed line represents the date when the mean of daily growth in number of COVID-19 cases appears, which is June 1. The red vertical dashed line illustrates the date when the median of daily growth in COVID-19 cases appears, which is June 14. The green vertical dashed line illustrates the date when the maximum daily growth in COVID-19 cases appears, which is April 24.

As is shown by the graph, the predicted curve of total number of confirmed COVID-19 case in the US aligns with the that of the real data in the US. The predicted curve of daily growth in number of COVID-19 cases in the US mostly reflects the trends shown by the real data in the US. The residual standard error of the model for the US data is 5161 on 158 degrees of freedom. The values of the coefficients a, b, and c for the model were estimated at 2840341, 8.166, and 14.275, respectively. The raw prediction values of total number of confirmed cases and the inaccuracy rate of the basic gamma prediction model based on the US data is as follows:

Table 9: Prediction Inaccuracy Rate of the US

Type	6-23	6-24	6-25	6-26	6-27	6-28	6-29	6-30	7-01	7-02	7-03	7-04	7-05	7-06
Real	2347491	2382426	2422299	2467554	2510259	2549294	2590668	2636414	2687588	2742049	2795361	2841241	2891124	2936077
Prediction	2323456	2338671	2353538	2368058	2382237	2396078	2409585	2422763	2435616	2448147	2460363	2472268	2483865	2495162
Inaccuracy Rate (%)	1.02	1.84	2.84	4.03	5.10	6.01	6.98	8.10	9.37	10.71	11.98	12.98	14.08	15.01

As is shown by table, the inaccuracy between the prediction value and real value for the first day is only 1.0239%. With the range of prediction improved, the inaccuracy rate of the basic gamma prediction model increases to 15.0171% for the US data. However, the inaccuracy remains below 7% until the seventh day. This implies that the short-term prediction accuracy of the basic gamma prediction model is satisfactory for the US data, and that the long-term prediction accuracy is mostly acceptable.

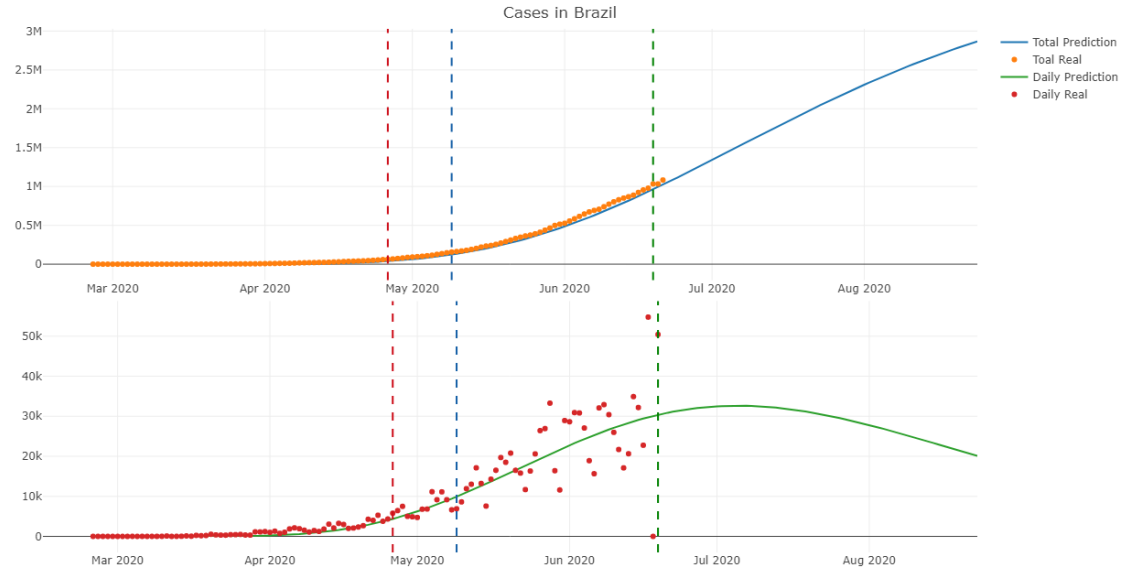


Figure 26: Prediction Result of Basic Gamma Model of Brazil

Figure 26 illustrates the prediction results of our basic gamma prediction model for Brazil. The blue curve of the top graph illustrates the prediction results for the total number of confirmed COVID-19 cases in Brazil, by our basic gamma prediction model. The green curve illustrates the prediction results for daily growth in number of COVID-19 cases. The orange dots illustrate the real data for total number of confirmed COVID-19 cases, and the red dots illustrate the real data for daily growth in number of COVID-19 cases in Brazil. The blue vertical dashed line illustrates the date when the mean of daily growth in COVID-19 cases appears, which is May 9. The red vertical dashed line illustrates the date when the median of daily growth in COVID-19 cases appears, which is April 26. The green vertical dashed line illustrates the date when the maximum daily growth in COVID-19 cases appears, which is June 19.

As is shown by the graph, the predicted curve for total number of confirmed COVID-19 cases in Brazil is mostly fits that of the real data. The predicted curve for daily growth in number of COVID-19 cases in Brazil also mostly reflects the distribution of the real data. The residual standard error of the model on the Brazil data is 5352 on 113 degrees of freedom. The values of the coefficients a, b, and c of the model were estimated at 3671076, 9.712, and 15.059, respectively. The raw prediction values for total confirmed cases and the inaccuracy rate of the basic gamma prediction model based on the Brazil data is as follows:

Table 10: Prediction Inaccuracy Rate of Brazil

Type	6-23	6-24	6-25	6-26	6-27	6-28	6-29	6-30	7-01	7-02	7-03	7-04	7-05	7-06
Real	1145906	1188631	1228114	1274974	1313667	1344143	1368195	1402041	1448753	1496858	1539081	1577004	1603055	1623284
Prediction	1101939.0	1135176.3	1168735.0	1202598.5	1236750.2	1271173.2	1305850.6	1340765.4	1375900.4	1411238.4	1446762.4	1482455.0	1518299.2	1554277.9
Inaccuracy Rate (%)	3.83	4.49	4.83	5.66	5.85	5.42	4.55	4.37	5.02	5.72	5.99	5.99	5.28	4.25

As is shown by table, the inaccuracy between the prediction value and the real value of the first day is only 3.8369%. With the range of prediction improved, the inaccuracy rate of the basic gamma prediction model remains around 5% with regards to the Brazil data. This implies that the short-term prediction accuracy of the basic gamma prediction model is satisfactory for the Brazil data, as is the long-term prediction accuracy. The basic gamma prediction model performed stably for the data of Brazil.

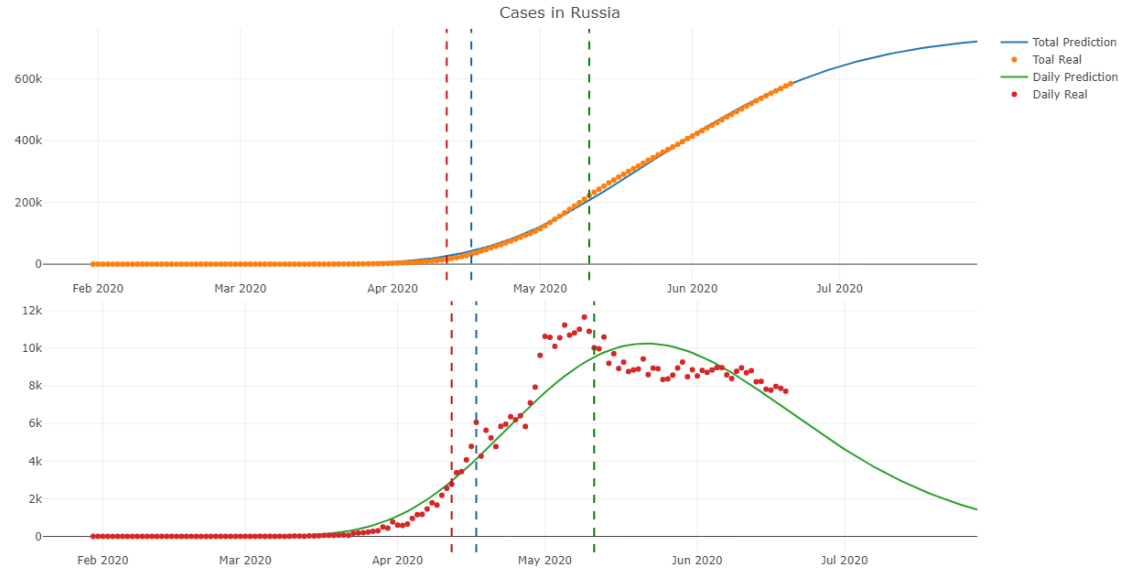


Figure 27: Prediction Result of Basic Gamma Model of Russia

Figure 27 illustrates the prediction results of our basic gamma prediction model for Russia. The blue curve in the top graph illustrates the prediction results for total number of confirmed COVID-19 cases in Russia, by our basic gamma prediction model. The green curve illustrates the prediction results of daily growth in number of COVID-19 cases in Russia. The orange dots illustrate the real data of total confirmed COVID-19 cases in Russia, and the red dots illustrate the real data for daily growth in number of COVID-19 cases in Russia. The blue vertical dashed line illustrates the date when the mean daily growth in number of COVID-19 cases appears, which is April 17. The red vertical dashed line illustrates the date when the median daily growth in COVID-19 cases appears, which is April 12. The green vertical dashed line illustrates the date when the maximum daily growth in number of COVID-19 cases appears, which is May 11.

As is shown by graph, the predicted curve for total number of confirmed COVID-19 case in Russia mostly fits the distribution of the real data. The predicted curve for daily growth in number of COVID-19 cases in Russia also mostly reflects the distribution of the real data. The residual standard error for model on the Russian data is 826.6 on 139 degrees of freedom. The values of the coefficients a, b, and c of the model were estimated at 745054.4, 16.262, and 7.381, respectively. The raw prediction values for total number of confirmed cases, and the inaccuracy rate of the basic gamma prediction model based on the Russia data, is as follows:

Table 11: Prediction Inaccuracy Rate of Russia

Type	6-23	6-24	6-25	6-26	6-27	6-28	6-29	6-30	7-01	7-02	7-03	7-04	7-05	7-06
Real	598878	606043	613148	619936	626779	633563	640246	646929	653479	660231	666941	673564	680283	686852
Prediction	608642.4	615846.8	622893.3	629781.1	636509.9	643079.5	649490.0	655741.6	661834.9	667770.6	673549.6	679173.1	684642.2	689958.4
Inaccuracy Rate (%)	1.63	1.61	1.58	1.58	1.55	1.50	1.44	1.36	1.27	1.14	0.99	0.83	0.64	0.45

As is shown by the table, the inaccuracy between the prediction value and the real value of the first day is only 1.6304%. With the range of prediction improved, the inaccuracy rate of the basic gamma prediction model stays around 1% with regards to the Russian data. This implies that the short-term prediction accuracy of the basic gamma prediction model is satisfactory on the Russia data of Russia, and the long-term prediction accuracy performed very well. Overall, the basic gamma prediction model performed stably for the data from Russia.

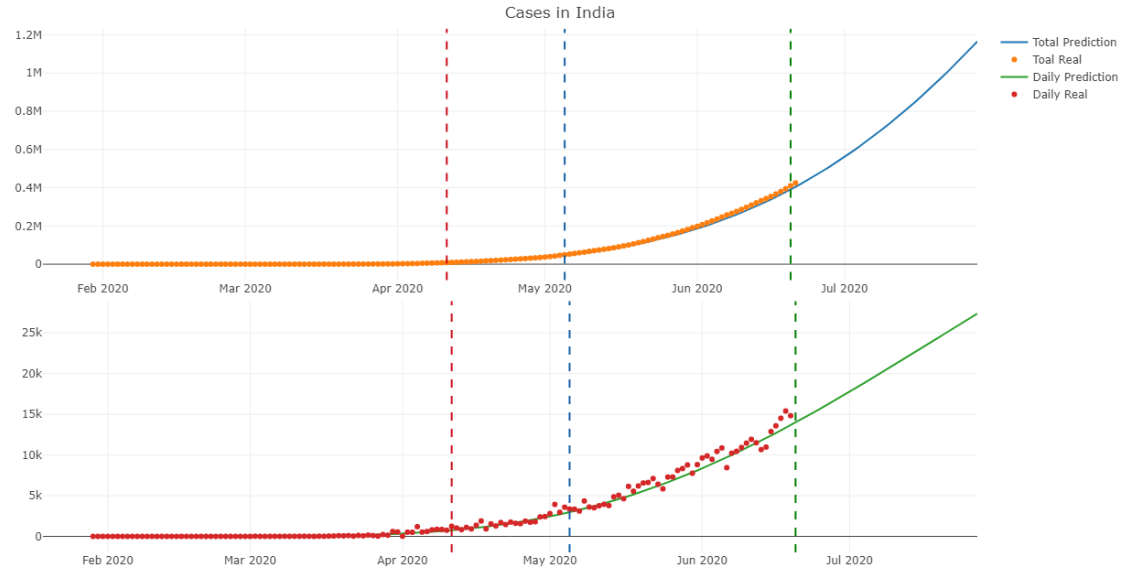


Figure 28: Prediction Result of Basic Gamma Model of India

Figure 28 illustrates the prediction results for our basic gamma prediction model in India. The blue curve on the top graph illustrates the prediction results for total number of confirmed COVID-19 cases in India, by our basic gamma prediction model, and the green curve illustrates the prediction results for daily growth in number of COVID-19 cases in India. The orange dots illustrate the real data distribution for total confirmed COVID-19 cases in India, and the red dots illustrate the real data for daily growth in number of cases. The blue vertical dashed line illustrates the date when the mean daily growth in numbers of COVID-19 cases appears, which is May 5. The red vertical dashed line illustrates the date when the median daily growth in number of COVID-19 cases appears, which is April 11. The green vertical dashed line illustrates the date when the maximum daily growth in number of COVID-19 cases appears, which is June 20.

As is shown by graph, the predicted curve for total number of confirmed COVID-19 case in India fits that of the real data in India. The predicted curve for daily growth in number of COVID-19 cases in India also basically reflects the distribution of the real data. The residual standard error of the model on the data from India is 433.3 on 140 degrees of freedom. The values of the coefficients a, b, and c of the model were estimated at 1247920, 8.018, and 39.175, respectively. The raw prediction values for total number of confirmed cases, and the inaccuracy rate of the basic gamma prediction model for the India data, is as follows:

Table 12: Prediction Inaccuracy Rate of India

Type	6-23	6-24	6-25	6-26	6-27	6-28	6-29	6-30	7-01	7-02	7-03	7-04	7-05	7-06
Real	456183	473105	490401	508953	528859	548318	566840	585481	604641	625544	648315	673165	697413	719664
Prediction	418935.2	433429.9	448259.9	463428.1	478937	494789.2	510987.1	527533.2	544429.5	561678.3	579281.6	597241.4	615559.4	634237.5
Inaccuracy Rate (%)	8.16	8.38	8.59	8.94	9.43	9.76	9.85	9.89	9.95	10.20	10.64	11.27	11.73	11.87

As is shown by table, the inaccuracy between the prediction value and the real value for the first day is only 8.1651%. With the range of prediction improved, the inaccuracy rate of the basic gamma prediction model increases to 11.8703% for this data. Thus, the prediction results are unsatisfactory. This implies that our basic gamma prediction model should be changed to improve the performance for the data from India.

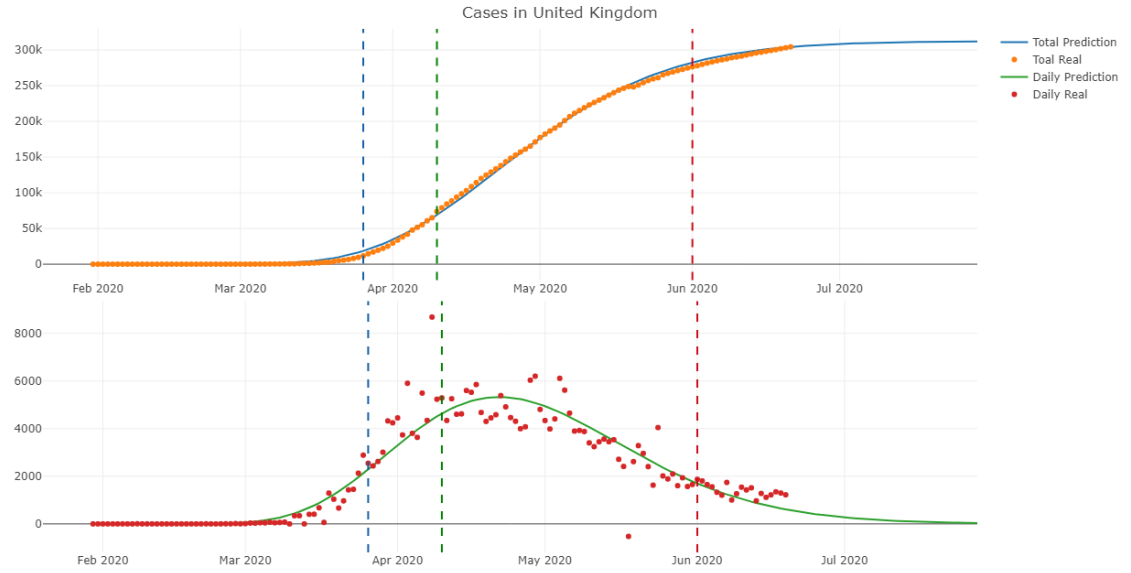


Figure 29: Prediction Result of Basic Gamma Model of UK

Figure 29 illustrates the prediction results of our basic gamma prediction model for the United Kingdom. The blue curve of the top graph illustrates the prediction results for total number of confirmed COVID-19 cases in the United Kingdom by our gamma prediction model. The green curve illustrates the prediction results for daily growth in number of COVID-19 cases in the United Kingdom. The orange dots illustrate the real data for total confirmed COVID-19 cases in the United Kingdom, while the red dots illustrate the real data for daily growth in number of cases. The blue vertical dashed line illustrates the date when the mean daily growth in number of COVID-19 cases appears, which is March 26. The red vertical dashed line illustrates the date when the median daily growth in number of COVID-19 cases appears, which is June 1. The green vertical dashed line illustrates the date when the maximum daily growth in COVID-19 cases appears, which is April 10.

As is shown by graph, the predicted curve for total number of confirmed COVID-19 cases in the United Kingdom generally fits the distribution of the real data for the United Kingdom. The predicted curve for daily growth in number of COVID-19 cases in the United Kingdom also basically reflects the distribution of the real data. The residual standard error of the model on the data from the United Kingdom is 703.5 on 139 degrees of freedom. The values of the coefficients a, b, and c of the model were estimated at 312342.6, 13.64, and 6.541, respectively. The raw prediction values for total confirmed cases and the inaccuracy rate of the basic gamma prediction model based on the United Kingdom data is as follows:

Table 13: Prediction Inaccuracy Rate of The United Kingdom

Type	6-23	6-24	6-25	6-26	6-27	6-28	6-29	6-30	7-01	7-02	7-03	7-04	7-05	7-06
Real	279566	280340	281037	281675	282308	282703	283307	283710	283770	283774	284276	284900	285416	285768
Prediction	279849.4	280195.3	280518.8	280821.2	281103.7	281367.5	281613.7	281843.3	282057.4	282257.0	282442.8	282615.8	282776.8	282926.5
Inaccuracy Rate (%)	0.10	0.05	0.18	0.30	0.42	0.47	0.59	0.65	0.60	0.53	0.64	0.80	0.92	0.99

As is shown by table, the inaccuracy between the prediction value and the real value for the first day is only 0.1014%. With the range of prediction improved, the inaccuracy rate of the basic gamma prediction model increases to 0.9943% with regards to the United Kingdom data. The inaccuracy remains below 1% for fourteen days which implies that the short-term prediction accuracy, and the long-term prediction accuracy of the basic gamma prediction model are both satisfactory using the United Kingdom data.

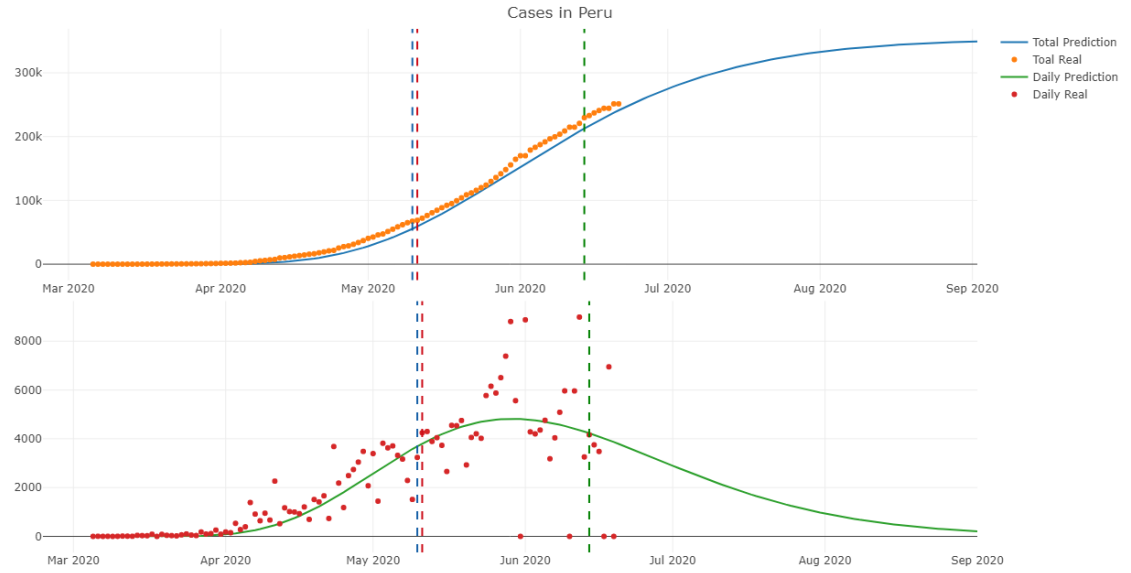


Figure 30: Prediction Result of Basic Gamma Model of Peru

Figure 30 illustrates the prediction results of our basic gamma prediction model for Peru. The blue curve of the top graph illustrates the prediction results for total number of confirmed COVID-19 cases in Peru, and the green curve illustrates the prediction results for daily growth in number of COVID-19 cases in Peru. The orange dots illustrate the real data for the total number of confirmed COVID-19 cases in Peru, and the red dots illustrate the real data for daily growth in number of COVID-19 cases. The blue vertical dashed line illustrates the date when the mean daily growth in number of COVID-19 cases appears, which is May 10. The red vertical dashed line illustrates the date when the median daily growth in COVID-19 cases appears, which is May 11. The green vertical dashed line illustrates the date when the maximum daily growth in number of COVID-19 cases appears, which is June 14.

As is shown by graph, the predicted curve for total number of confirmed COVID-19 cases in Peru generally fits the distribution of the real data for Peru. The predicted curve for daily growth in number of COVID-19 cases in Peru also reflects the distribution of the real data. The residual standard error of the model on the data for Peru is 1356 on 104 degrees of freedom. The values of the coefficients a, b, and c of the model were estimated at 352777, 9.7, and 9.8, respectively. The raw prediction values for total confirmed cases and the inaccuracy rate of the basic gamma prediction model based on the Peru data is as follows:

Table 14: Prediction Inaccuracy Rate of Peru

Type	6-23	6-24	6-25	6-26	6-27	6-28	6-29	6-30	7-01	7-02	7-03	7-04	7-05	7-06
Real	260810	264689	268602	272364	275989	279419	282365	285213	288477	292004	295599	299080	302718	305703
Prediction	252829.0	256735.9	260573.3	264340.0	268035.1	271657.6	275207.0	278682.6	282084.0	285410.9	288663.2	291840.7	294943.5	297971.8
Inaccuracy Rate (%)	3.06	3.00	2.98	2.94	2.88	2.77	2.53	2.28	2.21	2.25	2.34	2.42	2.56	2.52

As is shown by table, the inaccuracy between the prediction value and the real value for the first day is only 3.0601%. With the range of prediction improved, the inaccuracy rate of the basic gamma prediction model decreases to 2.529% with regards to the Peru data. The overall inaccuracy remains below 3.1% for fourteen days of prediction. This implies that the short-term prediction accuracy of the basic gamma prediction model is satisfactory for Peru, and the long-term prediction accuracy also performed well.

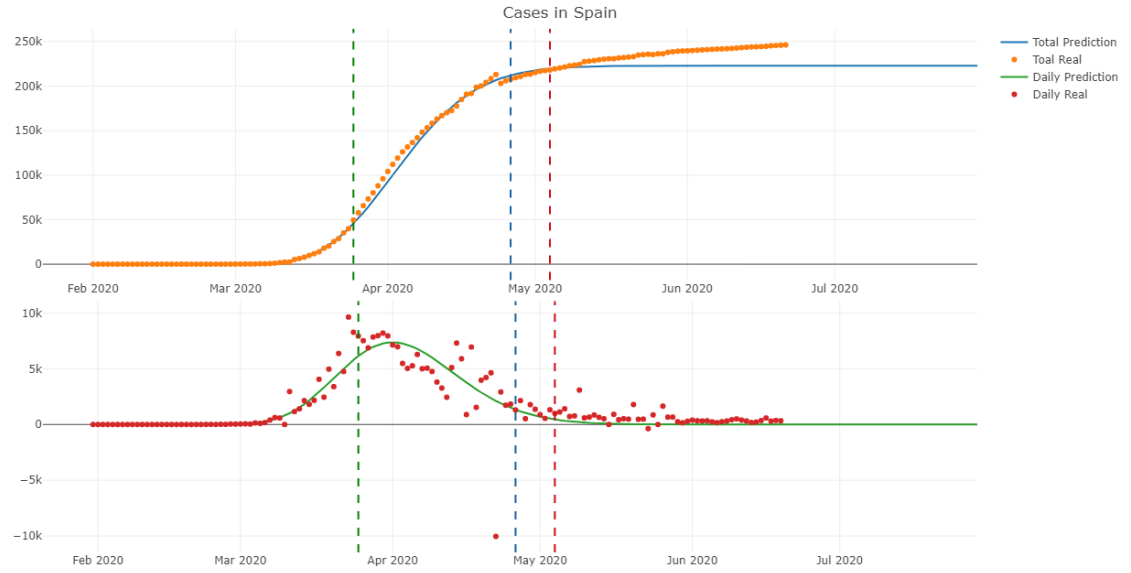


Figure 31: Prediction Result of Basic Gamma Model of Spain

Figure 31 illustrates the prediction results of our basic gamma prediction model for Spain. The blue curve on the top graph illustrates the prediction results for total number of confirmed COVID-19 cases in Spain by basic gamma prediction model. The green curve illustrates the prediction results for Spain's daily growth in number of COVID-19 cases. The orange dots illustrate the real data for total confirmed COVID-19 cases in Spain, and the red dots illustrate the real data for daily growth in number of COVID-19 cases. The blue vertical dashed line illustrates the date when the mean daily growth in COVID-19 cases appears, which is April 26. The red vertical dashed line illustrates the date when the median daily growth in COVID-19 cases appears, which is May 4. The green vertical dashed line illustrates the date when the maximum daily growth in COVID-19 cases appears, which is March 25.

As is shown by graph, the predicted curve for the total number of confirmed COVID-19 cases in Spain generally fits the distribution of the real data. The predicted curve for daily growth in number of COVID-19 cases in Spain also generally reflects the distribution of the real data. The residual standard error of the model on the data from Spain is 1418 on 138 degrees of freedom. The values of the coefficients a, b, and c of the model were estimated at 222835.5, 26.671, and 2.375, respectively. The raw prediction values for total confirmed cases, and the inaccuracy rate of the basic gamma prediction model based on the Spain data, is as follows:

Table 15: Prediction Inaccuracy Rate of Spain

Type	6-23	6-24	6-25	6-26	6-27	6-28	6-29	6-30	7-01	7-02	7-03	7-04	7-05	7-06
Real	246752	247086	247486	247905	248469	248770	248970	249271	249659	250103	250545	250545	250545	251789
Prediction	222835.6	222835.6	222835.6	222835.6	222835.7	222835.7	222835.7	222835.7	222835.7	222835.7	222835.7	222835.7	222835.7	222835.7
Inaccuracy Rate (%)	9.69	9.81	9.96	10.11	10.31	10.42	10.49	10.60	10.74	10.90	11.05	11.05	11.05	11.49

As is shown by table, the inaccuracy between the prediction value and the real value for the first day is 9.6925%. With the range of prediction improved, the inaccuracy rate of the basic gamma prediction model increases to 11.499% with regards to the data from Spain. The inaccuracy stays around 10% for 14 days of predictions. This implies that both the short-term prediction accuracy and the long-term prediction accuracy of the basic gamma prediction model is not very good for the data from Spain. However, the performance of the model stays stable, so we can manually estimate the error and correct it.

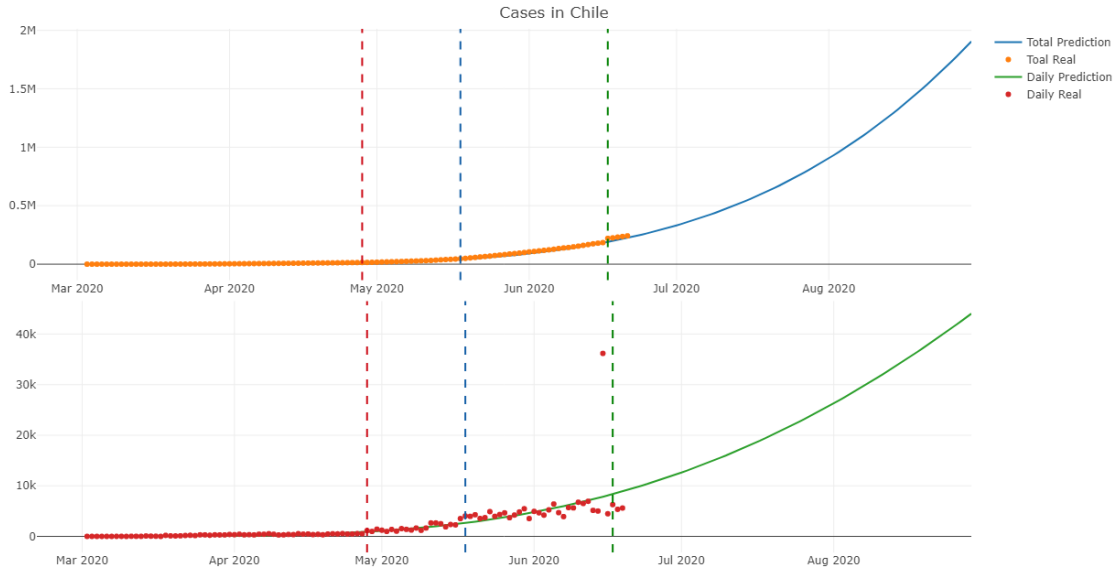


Figure 32: Prediction Result of Basic Gamma Model of Chile

Figure 32 illustrates the prediction results of our basic gamma prediction model for Chile. The blue curve on the top graph illustrates the prediction results for the total number of confirmed COVID-19 cases in Chile, by our basic gamma prediction model. The green curve illustrates the prediction results for daily growth in number COVID-19 cases in Chile. The orange dots illustrate the distribution of the real data for total confirmed COVID-19 cases in Chile, and the red dots illustrate the real data for daily growth in number of COVID-19 cases. The blue vertical dashed line illustrates the date when the mean daily growth in number of COVID-19 cases appears, which is May 18. The red vertical dashed line illustrates the date when the median daily growth in number of COVID-19 cases appears, which is April 28. The green vertical dashed line illustrates the date when the maximum daily growth in number of COVID-19 cases appears, which is June 17.

As is shown by graph, the predicted curve for the total number of confirmed COVID-19 case in Chile generally fits the distribution of the real data. The predicted curve for daily growth in number of COVID-19 cases in Chile also generally reflects the distribution of the real data. The residual standard error of the model for the data from Chile is 2855 on 107 degrees of freedom. The values of the coefficients a, b, and c of the model were estimated at 423589400, 5.02, and 169.9, respectively. The raw prediction values for total confirmed cases and the inaccuracy rate of the basic gamma prediction model based on the data from Chile is as follows:

Table 16: Prediction Inaccuracy Rate of Chile

Type	6-23	6-24	6-25	6-26	6-27	6-28	6-29	6-30	7-01	7-02	7-03	7-04	7-05	7-06
Real	250767	254416	259064	263360	267766	271982	275999	279393	282043	284541	288089	291847	295532	298557
Prediction	211682.9	217440.1	223050.7	228507.1	233802.7	238932.3	243891.2	248675.9	253283.8	257713.2	261963.3	266034.0	269925.9	273640.5
Inaccuracy Rate (%)	15.58	14.53	13.90	13.23	12.68	12.15	11.63	10.99	10.19	9.42	9.06	8.84	8.66	8.34

As is shown by table, the inaccuracy between the prediction value and the real value for the first day is 15.5858%. With the range of prediction improved, the inaccuracy rate of the basic gamma prediction model decreases to 8.3456%. This implies that the prediction accuracy of the basic gamma model increases as the range in prediction improves for the data from Chile.

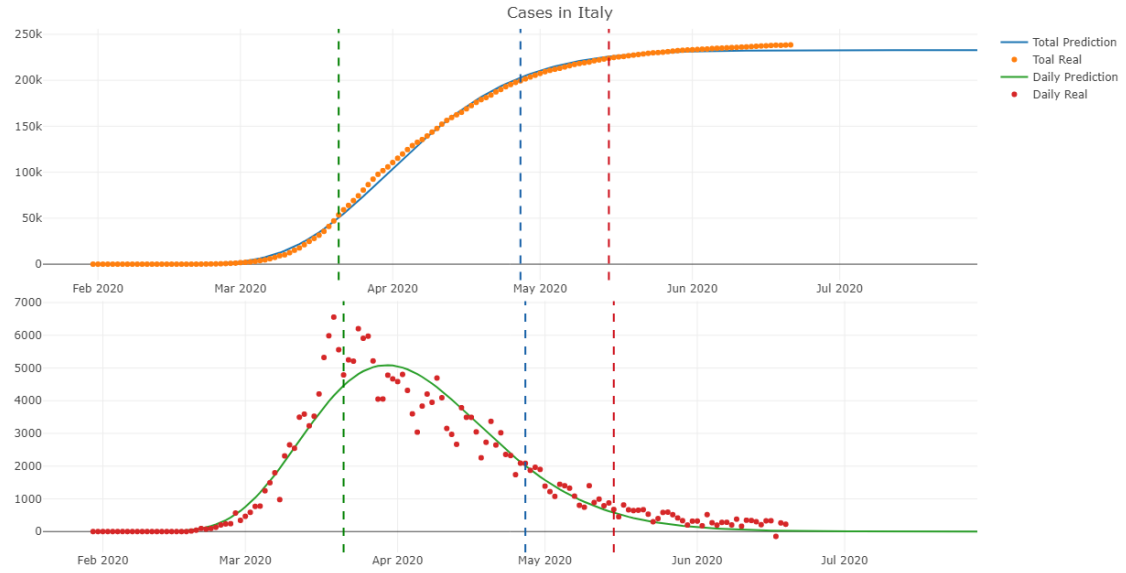


Figure 33: Prediction Result of Basic Gamma Model of Italy

Figure 33 illustrates the prediction results of our basic gamma prediciton model for Italy. The blue curve of the top graph illustrates the prediction results for the total number of confirmed COVID-19 cases in Italy, by our basic gamma prediction model. The green curve illustrates the prediction results for the daily growth in number of COVID-19 cases in Italy. The orange dots illustrate the distribution of the real data for total confirmed number of COVID-19 cases in Italy, and the red dots illustrate the real data for daily growth in number of COVID-19 cases. The blue vertical dashed line illustrates the date when the mean daily growth in number of COVID-19 cases appears, which is April 27. The red vertical dashed line illustrates the date when the median daily growth in number of COVID-19 cases appears, which is May 15. The green vertical dashed line illustrates the date when the maximum daily growth in COVID-19 cases appears, which is March 21.

As is shown by graph, the predicted curve for total number of confirmed COVID-19 cases in Italy generally fits the distribution of the real data. The predicted curve for daily growth in number of COVID-19 cases in Italy also generally reflects the distribution of the real data. The residual standard error of the model for the data from Italy is 429.5 on 139 degrees of freedom. The values of the coefficients a, b, and c of the model were estimated at 232799.6, 11.899, and 5.487, respectively. The raw prediction values for the total number of confirmed cases, and the inaccuracy rate of the basic gamma prediction model for the Italy data, is as follows:

Table 17: Prediction Inaccuracy Rate of Italy

Type	6-23	6-24	6-25	6-26	6-27	6-28	6-29	6-30	7-01	7-02	7-03	7-04	7-05	7-06
Real	238833	239410	239706	239961	240136	240310	240436	240578	240760	240961	241184	241419	241611	241819
Prediction	232771.0	232786.3	232800.0	232812.3	232823.3	232833.2	232842.1	232850.1	232857.2	232863.6	232869.3	232874.4	232879.0	232883.1
Inaccuracy Rate (%)	2.53	2.76	2.88	2.97	3.04	3.11	3.15	3.21	3.28	3.36	3.44	3.53	3.61	3.69

As is shown by table, the inaccuracy between the prediction value and the real value for the first day is only 2.5382%. With the range of prediction improved, the inaccuracy rate of the basic gamma prediction model increases to 3.6953% with regards to the data from Italy. Even though the inaccuracy increases slightly, the overall accuracy of the basic gamma prediction model performs well in the short-term and the long-term based on the data from Italy.

Lastly, we evaluated our application gamma prediction model for the data from Iran, the prediction curve for which is shown below:

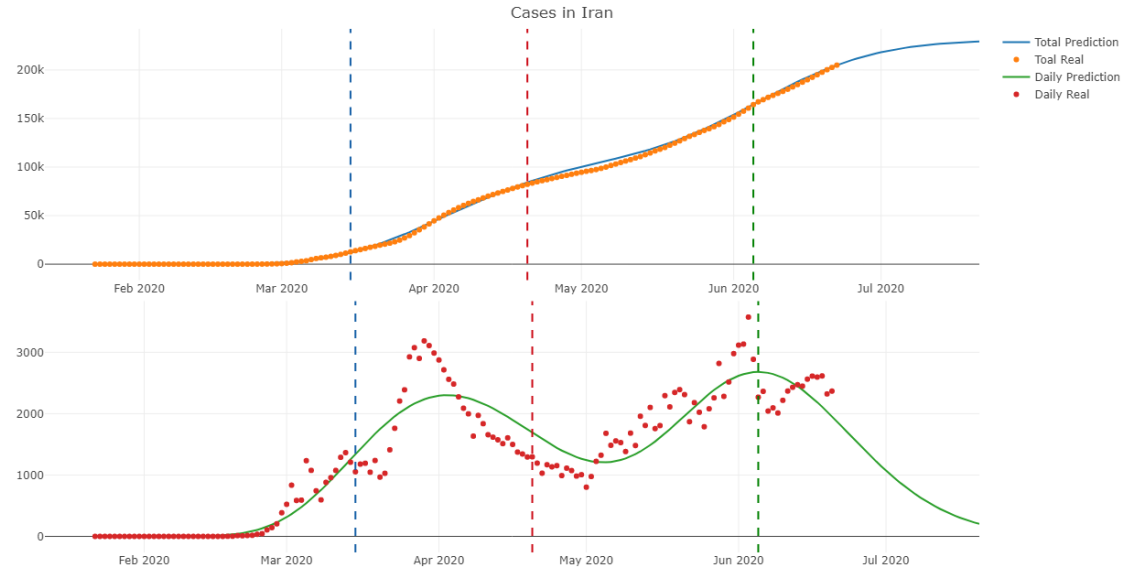


Figure 34: Prediction Result of Application Gamma for Iran

Figure 34 illustrates the prediction results of our application gamma prediction model for Iran. The blue curve on the top graph illustrates the prediction results for the total number of confirmed COVID-19 cases in Iran, by our application gamma prediction model. The green curve illustrates the prediction results for daily growth in number COVID-19 cases in Iran. The orange dots illustrate the distribution of the real data for total confirmed COVID-19 cases in Iran, and the red dots illustrate the real data for daily growth in number of COVID-19 cases. The blue vertical dashed line illustrates the date when the mean daily growth in number of COVID-19 cases appears, which is March 15. The red vertical dashed line illustrates the date when the median daily growth in number of COVID-19 cases appears, which is April 20. The green vertical dashed line illustrates the date when the maximum daily growth in number of COVID-19 cases appears, which is June 5.

As is shown by graph, the predicted curve for total number of confirmed COVID-19 cases in Iran generally fits the distribution of the real data for Iran. The predicted curve for daily growth in number of COVID-19 cases in Iran also generally reflects the trend of the real data. The residual standard error of the model for the data from Iran is 352.3 on 147 degrees of freedom. The values of the coefficients a, b, c, d, and e of the model were estimated at 230971.042, 59.748, 2.314, 14.011, and 5.514, respectively. The raw prediction values for total confirmed cases and the inaccuracy rate of the application gamma prediction model based on the data for Iran is as follows:

Table 18: Prediction Inaccuracy Rate of Iran

Type	6-23	6-24	6-25	6-26	6-27	6-28	6-29	6-30	7-01	7-02	7-03	7-04	7-05	7-06
Real	209970	212501	215096	217724	220180	222669	225205	227662	230211	232863	235429	237878	240438	243051
Prediction	206784.9	208494	210119.9	211662.8	213123.5	214503	215802.9	217024.8	218170.8	219243.2	220244.5	221177.2	222044.2	222848.4
Inaccuracy Rate (%)	1.51	1.88	2.31	2.78	3.20	3.66	4.17	4.67	5.23	5.84	6.44	7.02	7.65	8.31

As is shown by table, the inaccuracy between the prediction value and the real value for the first day is only 1.5169%. With the range of prediction improved, the inaccuracy rate of the basic gamma prediction model increases to 8.3121% with regards to the Iran data. However, the inaccuracy stays below 5% until the eight day. This implies that the short-term prediction accuracy of the application gamma prediction model is satisfactory using the data from Iran, and the long-term prediction accuracy of the model is also basically acceptable. Therefore, the application gamma prediction model is an effective forecasting tool for data that is not similar to the

standard gamma distribution.

According to our evaluation of the performance of our prediction models, using the data from the 10 countries with the highest number of total confirmed COVID-19 cases as of June 22, we made some conclusions. First, the predicted curves for the total number of confirmed COVID-19 cases produced by our models, generally fits the distributions of the real data from the ten countries. Second, the predicted curves for the daily growth in number of COVID-19 cases produced by our models also basically reflects the trends shown by the real data from the different countries. Third, the prediction model generally has high accuracy for short-term predictions, while its accuracy for long-term predictions depend more on the distribution of the real data. Fourth, the performance of the application gamma prediction model on the data from Iran showed significant improvement from the basic gamma prediction model using the same data. To further verify the predictive effect of our model using data from other countries, we calculated the inaccuracies of our prediction models using the data from the 19 countries with the highest number of total confirmed COVID-19 cases as of June 22. The results are as follows:

Table 19: Prediction Inaccuracy Rate of Top 19 Countries

Country	Type	2020-06-23	2020-06-24	2020-06-25	2020-06-26	2020-06-27	2020-06-28	2020-06-29	2020-06-30	2020-07-01	2020-07-02	2020-07-03	2020-07-04	2020-07-05	2020-07-06
US	Real	2347491.00	2382426.00	2422299.00	2467554.00	2510259.00	2549294.00	2590668.00	2636414.00	2687588.00	2742049.00	2795361.00	2841241.00	2891124.00	2936077.00
	Prediction	2323456.00	2338671.00	2353538.00	2368058.00	2382237.00	2396078.00	2409585.00	2422763.00	2435616.00	2448147.00	2460363.00	2472268.00	2483865.00	2495162.00
	Inaccuracy Rate (%)	1.02	1.84	2.84	4.03	5.10	6.01	6.99	8.10	9.38	10.72	11.98	12.99	14.09	15.02
Brazil	Real	1145906.00	1188631.00	1228114.00	1274974.00	1313667.00	1344143.00	1368195.00	1402041.00	1448753.00	1496858.00	1539081.00	1577004.00	1603055.00	1623284.00
	Prediction	1101939.05	1135176.34	1168734.99	1202598.49	1236750.18	1271173.20	1305850.62	1340765.39	1375900.38	1411238.44	1446762.38	1482455.02	1518299.22	1554277.88
	Inaccuracy Rate (%)	3.84	4.50	4.83	5.68	5.86	5.43	4.56	4.37	5.03	5.72	6.00	6.00	5.29	4.25
Russia	Real	598878.00	606043.00	613148.00	619936.00	626779.00	633563.00	640246.00	646929.00	653479.00	660231.00	666941.00	673564.00	680283.00	686852.00
	Prediction	608642.40	615846.81	622893.28	629781.13	636509.94	643079.54	649490.00	655741.63	661834.93	667770.63	673549.65	679173.08	684642.20	689958.44
	Inaccuracy Rate (%)	1.63	1.62	1.59	1.59	1.55	1.50	1.44	1.36	1.28	1.14	0.99	0.83	0.64	0.45
India	Real	456183.00	473105.00	490401.00	508953.00	528859.00	548318.00	566840.00	585481.00	604641.00	625544.00	648315.00	673165.00	697413.00	719664.00
	Prediction	418935.20	433429.90	448259.90	463428.10	478937.00	494789.20	510987.10	527533.20	544429.50	561678.30	579281.60	597241.40	615559.40	634237.50
	Inaccuracy Rate (%)	8.17	8.39	8.59	8.94	9.44	9.76	9.85	9.90	9.96	10.21	10.65	11.28	11.74	11.87
United Kingdom	Real	279566.00	280340.00	281037.00	281675.00	282308.00	282703.00	283307.00	283710.00	283770.00	283774.00	284276.00	284900.00	285416.00	285768.00
	Prediction	279849.43	280195.31	280518.80	280821.18	281103.68	281367.47	281613.67	281843.32	282057.43	282256.96	282442.80	282615.81	282776.79	282926.52
	Inaccuracy Rate (%)	0.10	0.05	0.18	0.30	0.43	0.47	0.60	0.66	0.60	0.53	0.64	0.80	0.92	0.99
Peru	Real	260810.00	264689.00	268602.00	272364.00	275989.00	279419.00	282365.00	285213.00	288477.00	292004.00	295599.00	299080.00	302718.00	305703.00
	Prediction	252829.01	256735.94	260573.34	264340.05	268035.10	271657.64	275206.99	278682.58	282083.99	285410.90	288663.15	291840.67	294943.50	297971.80
	Inaccuracy Rate (%)	3.06	3.00	2.99	2.95	2.88	2.78	2.54	2.29	2.22	2.26	2.35	2.42	2.57	2.53
Spain	Real	246752.00	247086.00	247486.00	247905.00	248469.00	248770.00	248970.00	249271.00	249659.00	250103.00	250545.00	250545.00	250545.00	251789.00
	Prediction	222835.59	222835.61	222835.63	222835.64	222835.65	222835.66	222835.66	222835.67	222835.67	222835.68	222835.68	222835.68	222835.68	222835.68
	Inaccuracy Rate (%)	9.69	9.81	9.96	10.11	10.32	10.43	10.50	10.61	10.74	10.90	11.06	11.06	11.06	11.50
Chile	Real	250767.00	254416.00	259064.00	263360.00	267766.00	271982.00	275999.00	279393.00	282043.00	284541.00	288089.00	291847.00	295532.00	298557.00
	Prediction	211682.94	217440.14	223050.69	228507.06	233802.75	238932.28	243891.17	248675.87	253283.80	257713.23	261963.32	266033.99	269925.93	273640.52
	Inaccuracy Rate (%)	15.59	14.53	13.90	13.23	12.68	12.15	11.63	10.99	10.20	9.43	9.07	8.84	8.66	8.35
Italy	Real	238833.00	239410.00	239706.00	239961.00	240136.00	240310.00	240436.00	240578.00	240760.00	240961.00	241184.00	241419.00	241611.00	241819.00
	Prediction	232771.05	232786.28	232799.97	232812.27	232823.31	232833.21	232842.09	232850.05	232857.19	232863.57	232869.29	232874.41	232878.98	232883.07
	Inaccuracy Rate (%)	2.54	2.77	2.88	2.98	3.05	3.11	3.16	3.21	3.28	3.36	3.45	3.54	3.61	3.70
Iran	Real	209970.00	212501.00	215096.00	217724.00	220180.00	222669.00	225205.00	227662.00	230211.00	232863.00	235429.00	237878.00	240438.00	243051.00
	Prediction	206784.90	208494.00	210119.90	211662.80	213123.50	214503.00	215802.90	217024.80	218170.80	219243.20	220244.50	221177.20	222044.20	222848.40
	Inaccuracy Rate (%)	1.52	1.89	2.31	2.78	3.20	3.67	4.17	4.67	5.23	5.85	6.45	7.02	7.65	8.31
France	Real	192452.00	192265.00	192010.00	193346.00	193152.00	192429.00	194109.00	194373.00	194985.00	195458.00	195904.00	195546.00	195535.00	196748.00
	Prediction	172949.87	172949.89	172949.91	172949.93	172949.94	172949.94	172949.95	172949.95	172949.96	172949.96	172949.96	172949.96	172949.97	172949.97
	Inaccuracy Rate (%)	10.13	10.05	9.93	10.55	10.46	10.12	10.90	11.02	11.30	11.52	11.72	11.56	11.55	12.10
Germany	Real	192480.00	192871.00	193371.00	194036.00	194458.00	194693.00	195042.00	195418.00	195893.00	196370.00	196780.00	197198.00	197523.00	198064.00
	Prediction	169215.89	169215.91	169215.93	169215.94	169215.96	169215.97	169215.97	169215.98	169215.98	169215.99	169215.99	169215.99	169215.99	169216.00
	Inaccuracy Rate (%)	12.09	12.26	12.49	12.79	12.98	13.09	13.24	13.41	13.62	13.83	14.01	14.19	14.33	14.56
Turkey	Real	190165.00	191657.00	193115.00	194511.00	195883.00	197239.00	198613.00	199906.00	201098.00	202284.00	203456.00	204610.00	205758.00	206844.00
	Prediction	185347.02	185582.54	185804.36	186013.23	186209.85	186394.88	186568.96	186732.69	186886.65	187031.37	187167.39	187295.19	187415.23	187527.97
	Inaccuracy Rate (%)	2.53	3.17	3.79	4.37	4.94	5.50	6.06	6.59	7.07	7.54	8.01	8.46	8.91	9.34
Pakistan	Real	188926.00	192970.00	195745.00	198883.00	202955.00	206512.00	209337.00	213470.00	217809.00	221896.00	221896.00	225283.00	231818.00	234509.00
	Prediction	155805.28	160424.21	164965.04	169420.81	173785.14	178052.24	182216.91	186274.55	190221.15	194053.29	197768.13	201363.39	204837.33	208188.73
	Inaccuracy Rate (%)	17.53	16.87	15.72	14.81	14.37	13.78	12.96	12.74	12.67	12.55	10.87	10.62	11.64	11.22
Mexico	Real	191410.00	196847.00	202951.00	208392.00	212802.00	216852.00	220657.00	226089.00	231770.00	238511.00	245251.00	252165.00	256848.00	261750.00
	Prediction	182437.07	187488.10	192600.65	197773.65	203006.00	208296.55	213644.16	219047.66	224505.85	230017.53	235581.45	241196.38	246861.05	252574.18
	Inaccuracy Rate (%)	4.69	4.75	5.10	5.10	4.60	3.95	3.18	3.11	3.13	3.56	3.94	4.35	3.89	3.51
Bangladesh	Real	119198.00	122660.00	126606.00	130474.00	133978.00	137787.00	141801.00	145483.00	149258.00	153277.00	156391.00	159679.00	162417.00	165618.00
	Prediction	110477.75	113985.89	117514.77	121061.63	124623.75	128198.39	131782.86	135374.48	138970.61	142568.64	146165.99	149760.16	153348.66	156929.08
	Inaccuracy Rate (%)	7.32	7.07	7.18	7.21	6.98	6.96	7.06	6.95	6.89	6.99	6.54	6.21	5.58	5.25
Canada	Real	103767.00	104087.00	104463.00	104629.00	104878.00	105193.00	105830.00	106097.00	106288.00	106643.00	106962.00	107185.00	107394.00	107815.00
	Prediction	105327.03	105589.90	105838.66	106073.93	106296.33	106506.43	106704.82	106892.04	107068.64	107235.12	107391.99	107539.73	107678.79	107809.63
	Inaccuracy Rate (%)	1.50	1.44	1.32	1.38	1.35	1.25	0.83	0.75	0.73	0.56	0.40	0.33	0.27	0.00
Qatar	Real	89579.00	90778.00	91838.00	92784.00	93663.00	94413.00	95106.00	96088.00	97003.00	97897.00	98653.00	99183.00	99799.00	100345.00
	Prediction	85119.89	86222.26	87290.40	88324.46	89324.62	90291.15	91224.39	92124.72	92992.58	93828.46	94632.88	95406.43	96149.70	96863.34
	Inaccuracy Rate (%)	4.98	5.02	4.95	4.81	4.63	4.37	4.08	4.12	4.13	4.16	4.08	3.81	3.66	3.47
China	Real	84653.00	84673.00	84701.00	84725.00	84743.00	84757.00	84780.00	84785.00	84816.00	84830.00	84838.00	84857.00	84871.00	84889.00
	Prediction	81146.16	81146.16	81146.16	81146.16	81146.16	81146.16	81146.16	81146.16	81146.16	81146.16	81146.16	81146.16	81146.16	81146.16
	Inaccuracy Rate (%)	4.14	4.17	4.20	4.22	4.24	4.26	4.29	4.29	4.33	4.34	4.35	4.37	4.39	4.41

According to the table, the inaccuracy of our prediction models for the first day were over 10% based on using the data from the following four countries: Chile, France, Germany, and Pakistan. For the other fourteen countries, the inaccuracies of the first-day predictions were all lower than 10%. In these fourteen countries, there were twelve countries for which the inaccuracies of the model were lower than 5% for the first-day prediction. Furthermore, with countries for which the model predicted with an inaccuracy rate lower than 5% for the first day, the inaccuracy rate for the other thirteen days mostly remained below 5%. These results imply that, if the prediction model performs well using the data from the first-day of a given country, it will probably continue to perform well for the following thirteen days.

5 COVID-19 Pandemic Forecasting Web Page

We developed a web page designed to provide the following information to users: the graph of real daily growth in confirmed COVID-19 cases, the graph of real total confirmed COVID-19 cases, the graph of predicted daily growth in confirmed cases, the graph of predicted total confirmed cases, and the corresponding predicted data over 14 days for the 19 countries with the highest number of confirmed cases. Since all of the data processing and analysis in this thesis are implemented by R, and the data visualization is implemented by the R library Plotly, a framework that is integrated with the R library Plotly would be optimal for the information presented in this thesis. Thus, we chose the Dash as the framework to build our web page. Dash for R is an open-source framework for building analytical applications, with no javascript required, and it is tightly integrated with the Plotly graphing library [44].

Dash is a productive framework for building web applications in both R and Python. Written on top of Fiery/Flask, Plotly.js, and React.js, Dash is ideal for building data visualization apps with highly customizable user interfaces in pure R or Python. It's especially well-suited for anyone who works with data. Through a couple of simple patterns, Dash simplifies all of the technologies and protocols that are required to build an interactive web-based application. Dash apps are rendered in the web browser, and the developer can deploy apps to servers and then share them through URLs. Since Dash apps are viewed in web browsers, Dash is inherently across-platforms and is mobile ready.

Our COVID-19 pandemic forecasting web page is now online at [45]. The COVID-19 forecasting web page is composed of two subpages. The front end of the home page is shown below:

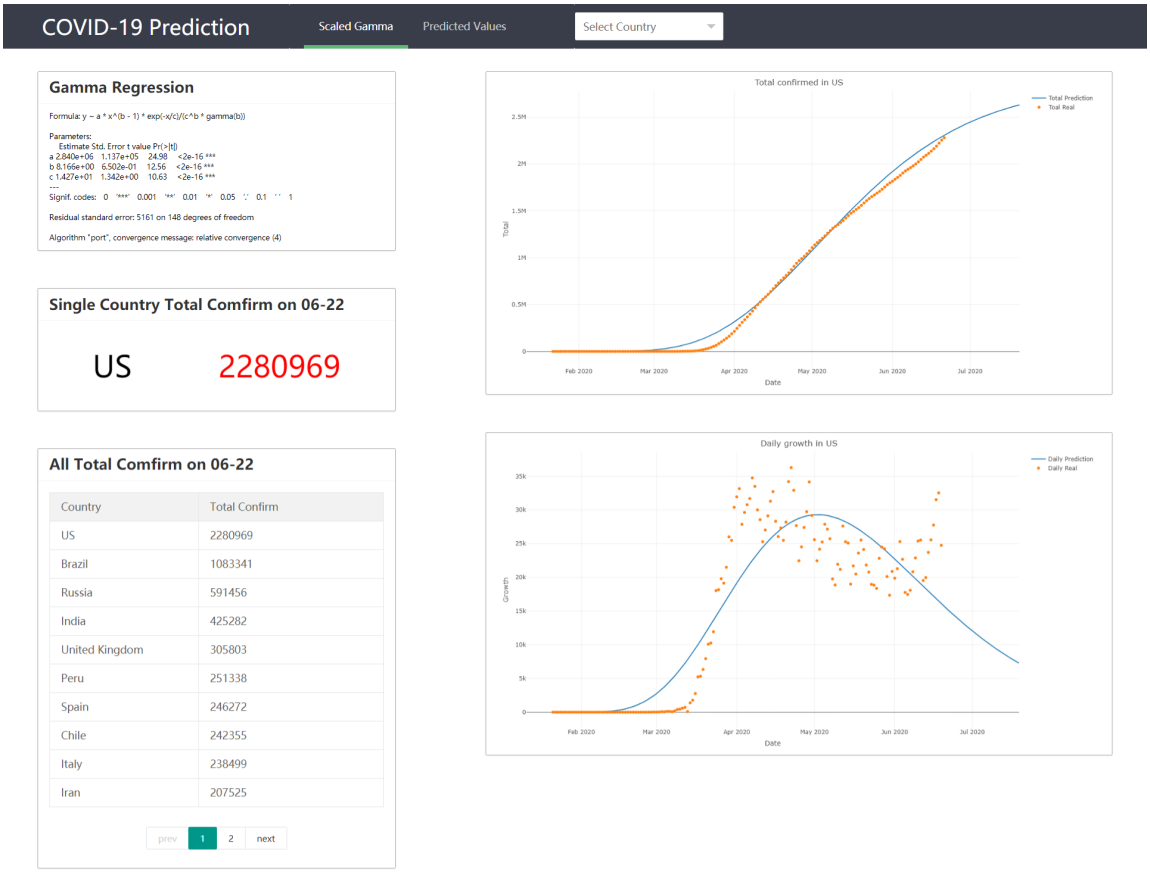
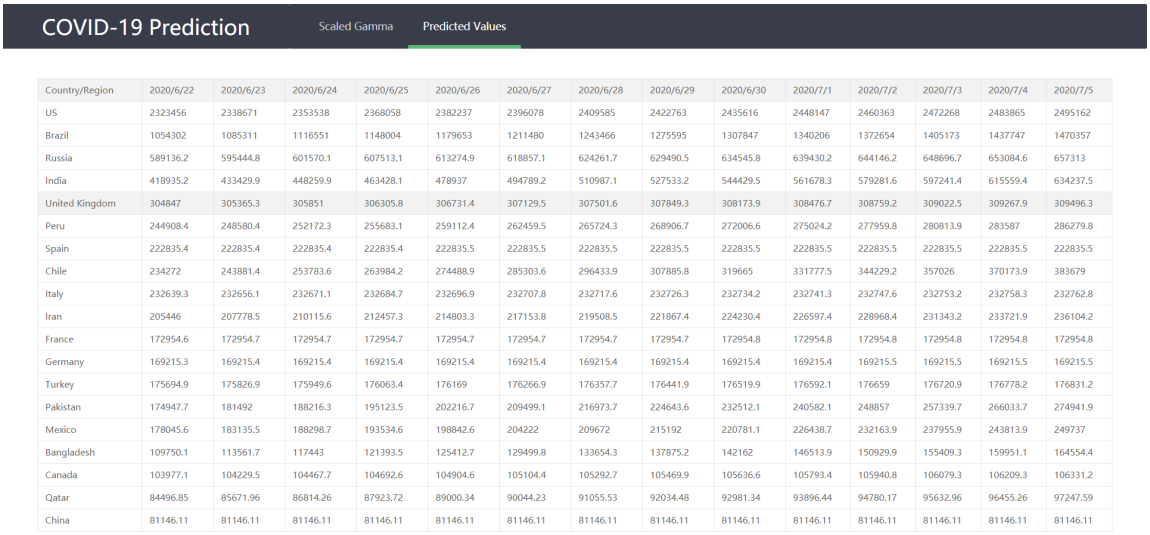


Figure 35: Front End of the Home Page

The home page of the COVID-19 pandemic forecasting web page is mainly composed of five divisions, two buttons, and one drop-down box. The upper left division was developed to summarize the information from the model trained by the data of the selected country. The middle division on the left side was developed to illustrate

the total number of confirmed COVID-19 cases for the selected country as of June 22. The bottom left division was developed to illustrate a list of the 19 countries with the highest number of confirmed cases as of June 22. The upper right division was developed to display the data for the real total number of confirmed COVID-19 cases in the selected country as of 22 June, as well as the predicted curve for the total number of confirmed COVID-19 cases generated by the prediction model of this thesis. The bottom right division was developed to display the data for the real daily growth in number of COVID-19 cases in the selected country as of June 22, as well as the predicted curve for the daily growth in cases as generated by the prediction model of this thesis. The drop-down box at the top was designed for selecting which country's data is to be displayed. The second button on the top was developed to jump to the second subpage of the COVID-19 pandemic forecasting website. The front end of the second sub-page is shown below:



Country/Region	2020/6/22	2020/6/23	2020/6/24	2020/6/25	2020/6/26	2020/6/27	2020/6/28	2020/6/29	2020/6/30	2020/7/1	2020/7/2	2020/7/3	2020/7/4	2020/7/5
US	2323456	2338671	2353538	2368058	2382237	2396078	2409585	2422763	2435616	2448147	2460363	2472268	2483865	2495162
Brazil	1054302	1085311	1116551	1148004	1179653	1211480	1243466	1275595	1307847	1340206	1372654	1405173	1437747	1470357
Russia	589136.2	595444.8	601570.1	607513.1	613274.9	618857.1	624261.7	629490.5	634545.8	639430.2	644146.2	648696.7	653084.6	657313
India	418935.2	433429.9	448259.9	463428.1	478937	494789.2	510987.1	527533.2	544429.5	561678.3	579281.6	597241.4	615559.4	634237.5
United Kingdom	304847	305365.3	305851	306305.8	306731.4	307129.5	307501.6	307849.3	308173.9	308476.7	308759.2	309022.5	309267.9	309496.3
Peru	244908.4	248580.4	252172.3	255683.1	259112.4	262459.5	265724.3	268906.7	272006.6	275024.2	277959.8	280813.9	283587	286279.8
Spain	222835.4	222835.4	222835.4	222835.4	222835.5	222835.5	222835.5	222835.5	222835.5	222835.5	222835.5	222835.5	222835.5	222835.5
Chile	234272	243881.4	253783.6	263984.2	274488.9	285303.6	296433.9	307885.8	319665	331777.5	344229.2	357026	370173.9	383679
Italy	232639.3	232656.1	232671.1	232684.7	232696.9	232707.8	232717.6	232726.3	232734.2	232741.3	232747.6	232753.2	232758.3	232762.8
Iran	205446	207778.5	210115.6	212457.3	214803.3	217153.8	219508.5	221867.4	224230.4	226597.4	228968.4	231343.2	233721.9	236104.2
France	172954.6	172954.7	172954.7	172954.7	172954.7	172954.7	172954.7	172954.7	172954.8	172954.8	172954.8	172954.8	172954.8	172954.8
Germany	169215.3	169215.4	169215.4	169215.4	169215.4	169215.4	169215.4	169215.4	169215.4	169215.4	169215.5	169215.5	169215.5	169215.5
Turkey	175694.9	175826.9	175949.6	176063.4	176169	176266.9	176357.7	176441.9	176519.9	176592.1	176659	176720.9	176778.2	176831.2
Pakistan	174947.7	181492	188216.3	195123.5	202216.7	209499.1	216973.7	224643.6	232512.1	240582.1	248857	257339.7	266033.7	274941.9
Mexico	178045.6	183135.5	188298.7	193534.6	198842.6	204222	209672	215192	220781.1	226438.7	232163.9	237955.9	243813.9	249737
Bangladesh	109750.1	113561.7	117443	121393.5	125412.7	129499.8	133654.3	137875.2	142162	146513.9	150929.9	155409.3	159951.1	164554.4
Canada	103977.1	104229.5	104467.7	104692.6	104904.6	105104.4	105292.7	105469.9	105636.6	105793.4	105940.8	106079.3	106209.3	106331.2
Qatar	84496.85	85671.96	86814.26	87923.72	89000.34	90044.23	91055.53	92034.48	92981.34	93896.44	94780.17	95632.96	96455.26	97247.59
China	81146.11	81146.11	81146.11	81146.11	81146.11	81146.11	81146.11	81146.11	81146.11	81146.11	81146.11	81146.11	81146.11	81146.11

Figure 36: Front End of the Predict Page

The prediction page was developed to display the table showing the raw prediction values for the following 14 days. This shows the predictions for the total number of confirmed COVID-19 cases for the 19 aforementioned countries. Depending on the country name selected by users, the content of the home page would update to reflect that country's data. The process used for the back end of the COVID-19 pandemic forecasting web page is as follows:

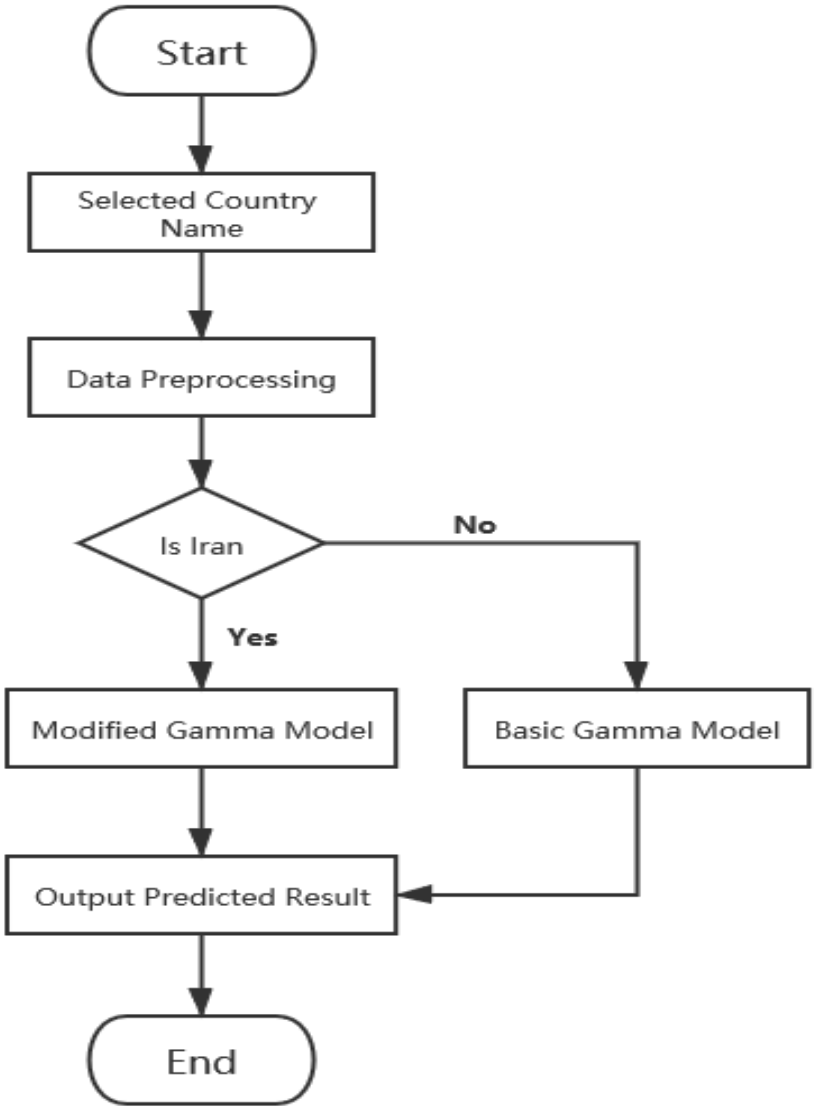


Figure 37: Process of the Back End of the COVID-19 Pandemic Forecasting Web Page

After users select a country, the name of the country will be sent to the back- end

program. The specific process for the data preprocessing is discussed in chapter 3. Then, the back-end program decides which prediction model will be utilized to generate the prediction data. If the selected country is Iran, the back-end program will choose the application gamma prediction model. If the selected country is not Iran, the back-end program will choose the basic gamma prediction model. The specific processes for the basic and application gamma prediction models, respectively, are discussed in chapter 4. After the model has been trained to generate the prediction values, the prediction value will be used to generate the prediction curves for total confirmed COVID-19 cases, and daily growth in COVID-19 cases, for the selected countries.

The information provided by the COVID-19 pandemic forecasting web page can be used by individuals as well as public organizations. Public organizations can use the information to more efficiently allocate medical resources, as well as establish policy to prevent the spread of COVID-19. Individuals can use the information to help them make decisions regarding their own life plan during the COVID-19 pandemic. For example, whether or not to participate in or cancel large gatherings, whether to wear a mask, and whether to reduce unnecessary travel.

6 Conclusion and Future Work

6.1 Conclusion

In this project, we built two prediction models to forecast the total number of confirmed cases of COVID-19, and the daily growth in the number of cases of COVID-19, for different countries. COVID-19 is a highly contagious atypical pneumonia attributed to a novel coronavirus. The global economy and people's lives have been tremendously affected by COVID-19 pandemic. WHO made the announcement that the event constitutes a Public Health Emergency of International Concern on January 30. The core of our prediction algorithms is composed of a gamma distribution with shape K and scale θ as parameters. The model has two versions. The simpler version is referred to as the basic gamma prediction model, and lower consumption of computing power is implemented for prediction in most countries. The advanced version is referred to as the application gamma prediction model, and a higher consumption of computing power is implemented for the prediction of countries with more complex data.

The data is extracted from the original data of the interactive web-based dashboard developed by the Center for System Science and Engineering (CSSE) at Johns Hopkins University, Baltimore, MD, USA. The corresponding data preprocessing procedure used to construct the data for regression, and to construct the data for visualization, has been introduced in the thesis. The basic analysis for the top 10 countries from the original data source has been performed in the thesis in order to

estimate the mathematical core of the prediction model.

The evaluation of the performance of the prediction models in this thesis has been done using the data from the ten countries with the highest number of total confirmed COVID-19 cases as of June 22. The predicted curves for the total number of confirmed COVID-19 cases produced by our models have been shown to generally fit the real data for the different countries. The predicted curves for the daily growth in the number of COVID-19 cases produced by our models have also been shown to generally reflect the trends displayed by the real data for the different countries. The prediction model generally has a high level of accuracy for short-term predictions, while the accuracy of its long-term predictions depends on the distribution of the real data. The inaccuracies of our prediction models using the data from the 19 countries with the highest total number of confirmed COVID-19 cases as of June 22 have been calculated. If the prediction model shows high accuracy after the first day using data from a given country, it is like that it will continue performing well for the following thirteen days.

A web page has been developed as a part of this thesis meant to provide information to users. This included the graphs of the real daily growth in the number of confirmed cases, the graphs showing the real total number of confirmed cases, the graphs showing our predictions for both of these values, and the corresponding predicted data over 14 days for the 19 countries that have the high number of total confirmed cases. Dash for R, an open-source framework, has been chosen for the COVID-19

pandemic forecasting web page in this thesis.

6.2 Future Work

As for future work, it will be interesting to expand the prediction model for other data metrics since we only used the total number of confirmed cases and the daily growth in the number of cases of COVID-19 in different countries for this model. For example, we could use the mortality and recovery rate data for the different countries as well. Furthermore, researching to learn how to decrease the inaccuracies of our prediction models for some countries would be worthwhile. Since the data about COVID-19 is going to increase only time, some prediction models could require a large amount of data to train them. For example, deep learning will be worth attempting in the future.

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A Sample of Original Data

Province/State	Country/Region	Lat	Long	1/22/20	1/23/20	1/24/20	1/25/20	1/26/20	1/27/20	1/28/20	1/29/20	1/30/20	1/31/20	5/27/20	5/28/20	5/29/20	5/30/20	5/31/20	6/13/20	6/14/20	6/15/20	6/16/20	6/17/20	6/18/20	6/19/20	6/20/20	6/21/20
Australian Capital Territory	Australia	-35.4735	149.0124	0	0	0	0	0	0	0	0	0	0	107	107	107	107	107	108	108	108	108	108	108	108	108
New South Wales	Australia	-33.8688	151.2093	0	0	0	0	0	3	4	4	4	4	3000	3002	3002	3005	3008	3128	3131	3134	3135	3137	3143	3144	3149	3151
Northern Territory	Australia	-12.4634	130.8456	0	0	0	0	0	0	0	0	0	0	29	29	29	29	29	29	29	29	29	29	29	29	29	29
Queensland	Australia	-28.0167	153.4	0	0	0	0	0	0	0	1	3	2	1058	1058	1058	1058	1058	1065	1065	1065	1066	1066	1066	1066	1066	1066
South Australia	Australia	-34.9285	138.6007	0	0	0	0	0	0	0	0	0	0	440	440	440	440	440	440	440	440	440	440	440	440	440	440
Tasmania	Australia	-41.4545	145.9707	0	0	0	0	0	0	0	0	0	0	228	228	228	228	228	228	228	228	228	228	228	228	228	228
Victoria	Australia	-37.8136	144.9631	0	0	0	0	0	1	1	1	2	3	1628	1634	1645	1649	1653	1720	1732	1741	1780	1792	1792	1836	1847	
Western Australia	Australia	-31.9505	115.8605	0	0	0	0	0	0	0	0	0	0	570	577	585	586	589	602	602	602	603	603	604	605	605	
	Brunei	4.5353	114.7277	0	0	0	0	0	0	0	0	0	0	141	141	141	141	141	141	141	141	141	141	141	141	141	141
	Bulgaria	42.7339	25.4858	0	0	0	0	0	0	0	0	0	0	2400	2477	2485	2499	2513	3206	3220	3341	3453	3542	3674	3755	3872	3995
	Burkina Faso	12.2383	-1.5616	0	0	0	0	0	0	0	0	0	0	845	847	847	847	847	847	892	894	895	899	900	901	903	
	Cabo Verde	16.5388	-23.0418	0	0	0	0	0	0	0	0	0	0	300	300	405	421	435	726	750	760	781	792	823	848	863	890
	Cambodia	11.55	104.9167	0	0	0	0	0	1	1	1	1	1	124	124	124	125	125	128	128	128	128	128	129	129	129	129
	Cameroon	3.848	11.5021	0	0	0	0	0	0	0	0	0	0	5436	5436	5436	5436	5004	5004	8681	8681	9864	9864	10638	11650	11882	
Nova Scotia	Canada	44.682	-63.7443	0	0	0	0	0	0	0	0	0	0	1053	1055	1055	1056	1056	1061	1061	1061	1061	1061	1061	1061	1061	1061
Ontario	Canada	51.2538	-85.3232	0	0	0	0	0	1	1	1	1	3	27043	28320	28700	29023	29300	33025	33806	34174	34582	34674	34789	35044	35217	
Prince Edward Island	Canada	46.5107	-63.4168	0	0	0	0	0	0	0	0	0	0	27	27	27	27	27	27	27	27	27	27	27	27	27	27
Quebec	Canada	52.9399	-73.5491	0	0	0	0	0	0	0	0	0	0	49148	49711	50222	50651	51039	53824	53062	54054	54146	54263	54383	54550	54674	54706
Saskatchewan	Canada	52.9399	-106.4509	0	0	0	0	0	0	0	0	0	0	637	639	641	645	646	664	665	683	684	693	708	716	726	746
	Chad	15.4542	18.7322	0	0	0	0	0	0	0	0	0	0	715	726	759	759	778	848	850	850	853	854	854	858	858	858
	Chile	-35.6751	-71.543	0	0	0	0	0	0	0	0	0	0	82269	86945	90038	94858	99088	167355	174233	179496	184449	239028	225103	231393	236748	242355
Anhui	China	31.8257	117.2264	1	9	15	39	60	70	106	152	200	237	991	991	991	991	991	991	991	991	991	991	991	991	991	991
Beijing	China	40.1824	116.4142	14	22	36	41	68	80	91	111	114	139	503	503	503	503	503	637	673	700	731	772	777	777	821	830
Chongqing	China	30.0572	107.874	6	9	27	57	75	110	132	147	182	211	579	579	579	579	579	580	582	582	582	582	582	582	582	582
Fujian	China	26.0789	117.9874	1	5	10	18	35	59	80	84	101	120	358	358	358	358	358	361	362	362	362	362	362	362	363	363
Gansu	China	37.8099	101.0583	0	2	2	4	7	14	19	24	26	29	139	139	139	139	139	139	139	139	149	150	151	151	151	151
Bermuda	United Kingdom	32.3078	-64.7505	0	0	0	0	0	0	0	0	0	0	139	140	140	140	140	142	142	144	144	144	144	146	146	146
Cayman Islands	United Kingdom	19.3133	-81.2546	0	0	0	0	0	0	0	0	0	0	140	140	141	141	141	187	187	187	193	193	193	195	195	195
Channel Islands	United Kingdom	49.3723	-2.3644	0	0	0	0	0	0	0	0	0	0	560	560	560	560	560	565	565	568	568	570	570	570	570	570
Gibraltar	United Kingdom	36.1408	-5.3536	0	0	0	0	0	0	0	0	0	0	157	158	161	169	170	176	176	176	176	176	176	176	176	176
Isle of Man	United Kingdom	54.2361	-4.5481	0	0	0	0	0	0	0	0	0	0	336	336	336	336	336	336	336	336	336	336	336	336	336	336
Montserrat	United Kingdom	16.7425	-62.1874	0	0	0	0	0	0	0	0	0	0	11	11	11	11	11	11	11	11	11	11	11	11	11	11
	United Kingdom	55.3781	-3.436	0	0	0	0	0	0	0	0	2	2	267240	269127	271222	272836	274702	294575	295680	296857	298106	299251	300469	301815	303110	304351
	Uruguay	-32.5228	-55.7658	0	0	0	0	0	0	0	0	0	0	803	811	816	821	823	847	848	848	849	849	850	853	859	876
	US	37.0902	-95.7129	1	1	2	2	5	5	5	5	7	7	1707423	1730359	1754747	1776993	1799922	2074326	2094058	2119026	2157731	2196290	2191062	2222579	2255119	2278679
	Uzbekistan	41.3775	64.5853	0	0	0	0	0	0	0	0	0	0	3089	3444	3468	3546	3623	4066	4080	5263	5493	5682	5767	5946	6153	6315
	Venezuela	6.4238	-66.5897	0	0	0	0	0	0	0	0	0	0	1245	1325	1370	1459	1510	2004	2078	3062	3150	3386	3483	3501	3769	3917
	Vietnam	16	108	0	2	2	2	2	2	2	2	2	2	327	327	328	328	328	334	334	334	334	335	342	349	349	349
	Zambia	-15.4167	28.2833	0	0	0	0	0	0	0	0	0	0	1057	1057	1057	1057	1057	1357	1358	1382	1405	1412	1416	1430	1430	1430

B Basic Gamma Prediction Model R Program

```
rm(list=ls())
library(openxlsx)
library(dplyr)
library(pracma)
library(grid)
library(gridExtra)
library(ggplot2)
library("cowplot")
library(plotly)
library(RCurl)
options(scipen = 200)


# Load original data from GitHub to data frame
raw_data<-getURL("https://raw.githubusercontent.com/
    CSSEGISandData/COVID-19/master/csse_covid_19_data/csse_covid
    _19_time_series/time_series_covid19_confirmed_global.csv")
df<-read.csv(text=raw_data)




# Load data of initial coefficient
coef_gam<-as.matrix(read.csv("coef_gam.csv"))
```

```
Basic_Gamma(df,1, country_names)[[1]]


country_names <- c("US", "Brazil","Russia","India", "United␣
   Kingdom", "Peru", "Spain", "Chile", "Italy", "Iran", "France
   ", "Germany",
"Turkey","Pakistan","Mexico","Bangladesh","Canada","Qatar","
   China")



# Main fuction to finish all work
Basic_Gamma <- function(df, j, country_names){
country <- df %>% filter(Country.Region == country_names[j])
# Construct data for regression
if (country_names[j]%in% c("China", "Canada")) {
country <- country[,-c(1:4)]
country <- apply(country, 2, sum)} else if (country_names[j] %
   in% c("United␣Kingdom", "France")){
country <- country %>% filter(Province.State=="")
country <- country[,-c(1:4)]} else country <- country[,-c(1:4)]
country <- as.vector( t(country))
ind <- which(country != 0)
country <- country[ind]
x <- 1:length(country)
train_data<-data.frame(x=x[-1],y=diff(country))
```

```
# Load initial coefficient of correspond country
wsp_gam <- coef_gam[j,]

# Estimate the coefficients in basic gamma distribution by nls
    ()
mod_gam <- nls(y ~ a * x^(b-1)*exp(-x/c) / (c^b*gamma(b)), data
     = data.frame(y=diff(country), x=x[-1]),
start = list(a = wsp_gam[1], b = wsp_gam[2], c=wsp_gam[3]),
control=nls.control(maxiter = 10000, tol = 1e-05, minFactor = 1
    /1024, printEval = FALSE, warnOnly = FALSE),
algorithm = "port")

# Construct data frame to save the real data
data_diff <- data_cum <- NULL
day <- as.Date("22-01-2020", format="%d-%m-%Y")+0:(length(
    country)-1)+min(ind)-1
total <- as.vector(country)
data_cum <- data.frame(day, total, type=rep("real", length(
    country)))
data_diff <- data.frame(day=day[-1], growth=diff(total), type=
    rep("real", length(country)-1))

# Construct data frame to save the prediction result
```

```
t<-0:200


s <- min(ind)-1


wsp_gam <- coef(mod_gam)
wsp_list<-c(wsp_gam[1],wsp_gam[2],wsp_gam[3])


data_cum <- rbind(data_cum, data.frame(day = as.Date("
   22-01-2020", format="%d-%m-%Y")+t+s,
total=wsp_gam[1]*pgamma(t, shape = wsp_gam[2],
scale =  wsp_gam[3]),
type=rep("scaled Gamma", length(t))))



data_diff <- rbind(data_diff, data.frame(day = as.Date("
   22-01-2020", format="%d-%m-%Y")+t-1+s,
growth=wsp_gam[1]*dgamma(t, shape = wsp_gam[2],
scale =  wsp_gam[3]),
type=rep("scaled Gamma", length(t))))



library(scales)
library(plotly)
```

```
#Construct data frame for ploting the prediction curve and real
    value data of total confirmed cases
real_cum<-data_cum[which(data_cum$type=='real'),]
gamma_cum<-data_cum[which(data_cum$type=='scaled␣Gamma'),]
c<-NA
for (i in 1:(length(gamma_cum$day)-length(real_cum$day)-1)) {
c<-c(c,NA)
}
real_cum_data<-c(real_cum$total,c)
plot_cum_data<-data.frame(day=gamma_cum$day,scaled=gamma_cum$
   total,real=real_cum_data)

#Construct data frame for ploting the prediction curve and real
    value data of daily growth cases
real_diff<-data_diff[which(data_diff$type=='real'),]
gamma_diff<-data_diff[which(data_diff$type=='scaled␣Gamma'),]
d<-NA
for (i in 1:(length(gamma_diff$day)-length(real_diff$day)-1)) {
d<-c(d,NA)
}
real_diff_data<-c(real_diff$growth,d)
plot_diff_data<-data.frame(day=gamma_diff$day,scaled=gamma_diff
   $growth,real=real_diff_data)
```

```
# Date of Mean
mean_value<-mean(real_diff$growth)
index1<-max(c(which(abs(mean_value-real_diff$growth)==min(abs(
   mean_value-real_diff$growth)))))
mean_date<-real_diff$day[index1]

# Date of Median
median_value<-median(real_diff$growth)
index2<-max(c(which(abs(median_value-real_diff$growth)==min(abs
   (median_value-real_diff$growth)))))
median_date<-real_diff$day[index2]

# Date of Maximum
max_value<-max(real_diff$growth)
max_date<-real_diff[which(real_diff$growth==max_value),1]

#Save all dates in a list
all_dates<-list(mean_date,median_date,max_date)

# Draw Vertical Line
vline <- function(x = 0, color = "red") {
list(
type = "line",
y0 = 0,
```

```
y1 = 1,
yref = "paper",
x0 = x,
x1 = x,
line = list(color = color,dash="dash")
)
}

#Plot the prediction curve and real value data of total
   confirmed cases
fig_cum<-plot_ly(plot_cum_data,x=~day,y=~scaled,name = 'Total␣
   Prediction',type = 'scatter',mode='lines')
fig_cum<-fig_cum%>%add_trace(y=~real,name='Toal␣Real',mode='
   markers')
fig_cum<-fig_cum%>%layout(title=paste("Total␣confirmed␣in",
   country_names[j]),
xaxis=list(title="Date"),
yaxis=list(title="Total"))
#fig_cum<-fig_cum%>%layout(shapes=list(vline(mean_date,color =
   'rgb(22, 96, 167)'),vline(median_date,color = 'rgb(205, 12,
   24)'),vline(max_date,"Green")))

#Plot the curve of real value of total confirmed cases
```

```
fig_real_cum<-plot_ly(real_cum,x=~real_cum$day,y=~real_cum$
    total,name = 'Real Total Confirmed',type = 'scatter',mode='
    lines+markers')
fig_real_cum<-fig_real_cum%>%layout(title=paste("Real Total
    Confirmed in", country_names[j]),
xaxis=list(title="Date"),
yaxis=list(title="Total"))



#Plot the prediction curve and real value data of daily growth
    cases
fig_diff<-plot_ly(plot_diff_data,x=~day,y=~scaled,name = 'Daily
    Prediction',type = 'scatter',mode='lines')
fig_diff<-fig_diff%>%add_trace(y=~real,name='Daily Real',mode='
    markers')
fig_diff<-fig_diff%>%layout(title=paste("Daily growth in",
    country_names[j]),
xaxis=list(title="Date"),
yaxis=list(title="Growth"))
#fig_diff<-fig_diff%>%layout(shapes=list(vline(mean_date,color
    = 'rgb(22, 96, 167)'),vline(median_date,color = 'rgb(205,
    12, 24)'),vline(max_date,"Green")))
```

```
#Plot the curve of real value of daily growth cases
fig_real_diff<-plot_ly(real_diff,x=~real_diff$day,y=~real_diff$
    growth,name = 'Real Daily Grwoth',type = 'scatter',mode='
    lines+markers')
fig_real_diff<-fig_real_diff%>%layout(title=paste("Real Daily 
    Growth in", country_names[j]),
xaxis=list(title="Date"),
yaxis=list(title="Growth"))

#Stitch the plot of total confirmed prediction with plot of
    daily growth prediction
fig_nail<-subplot(fig_cum,fig_diff,nrows = 2)
fig_nail<-fig_nail%>%layout(title=paste("Cases in",country_
    names[j]))

#Stitch the plot of real total confirmed with plot of real
    daily growth
fig_nail_real<-subplot(fig_real_cum,fig_real_diff,nrows = 2)
fig_nail_real<-fig_nail_real%>%layout(title=paste("Real Cases 
    in",country_names[j]))

return(list(fig_nail,all_dates,summary(mod_gam)))}
```

C Application Gamma Prediction Model R Program

```
rm(list=ls())
library(openxlsx)
library(dplyr)
library(pracma)
library(plotly)
library(RCurl)
options(scipen = 200)

# Load original data from GitHub to data frame
raw_data<-getURL("https://raw.githubusercontent.com/
   CSSEGISandData/COVID-19/master/csse_covid_19_data/csse_covid
   _19_time_series/time_series_covid19_confirmed_global.csv")
df <- read.csv(text=raw_data)

# Construct data for regression
iran <- df %>% filter(Country.Region == "Iran")
iran <- as.vector(t(iran[,-c(1:4)]))
ind<-iran[which(iran!=0)]
x<-1:length(iran)

# Load data of initial coefficient
wsp_gam <- c(186855, 63.6, 2.01, 23, 3.2)
```

```
# Estimate the coefficients in gamma distribution by nls()
mod_gam2 <- nls(y ~ a/2 * (x^(b-1)*exp(-x/c) / (c^b*gamma(b)) +
    x^(d-1)*exp(-x/e) / (e^d*gamma(d))),
data = data.frame(y=c( iran[1],diff(iran)), x=x),
start = list(a = wsp_gam[1], b = wsp_gam[2], c=wsp_gam[3], d=
   wsp_gam[4], e=wsp_gam[5]),
control=nls.control(maxiter = 10000, tol = 1e-05, minFactor = 1
   /1024, printEval = FALSE, warnOnly = FALSE),
algorithm = "port")

# Construct data frame to save the real data
data_diff<-data_cum<-NULL
day<-as.Date("22-01-2020", format="%d-%m-%Y")+0:(length(iran)
   -1)+min(ind)-1
total <- as.vector(iran)
data_cum <- data.frame(day, total, type=rep("real", length(iran
   )))
data_diff <- data.frame(day=day[-1], growth=diff(total), type=
   rep("real", length(iran)-1))

#Construct data frame to save the prediction result
t <- 0:180
s <- min(ind)-1
wsp_gam2 <- coef(mod_gam2)
```

```
a <- wsp_gam2[1]
b <- wsp_gam2[2]
c <- wsp_gam2[3]
d <- wsp_gam2[4]
e <- wsp_gam2[5]
daily_prediction<-a /2 * (t^(b-1)*exp(-t/c) / (c^b*gamma(b)) +
   t^(d-1)*exp(-t/e) / (e^d*gamma(d)))

data_cum <- rbind(data_cum, data.frame(day = as.Date("
   22-01-2020", format="%d-%m-%Y")+t+s,
total=cumsum(daily_prediction),
type=rep("scaled Gamma", length(t))))

data_diff <- rbind(data_diff, data.frame(day = as.Date("
   22-01-2020", format="%d-%m-%Y")+t-1+s,
growth=daily_prediction,
type=rep("scaled Gamma", length(t))))
library(scales)

#Construct data frame for ploting the prediction curve and real
    value data of total confirmed cases
real_cum<-data_cum[which(data_cum$type=='real'),]
gamma_cum<-data_cum[which(data_cum$type=='scaled Gamma'),]
blank<-NA
```

```
for (i in 1:(length(gamma_cum$day)-length(real_cum$day)-1)) {
blank<-c(blank,NA)
}
real_cum_data<-c(real_cum$total,blank)
plot_cum_data<-data.frame(day=gamma_cum$day,scaled=gamma_cum$
    total,real=real_cum_data)



#Plot the prediction curve and real value data of total
    confirmed cases
fig_cum<-plot_ly(plot_cum_data,x=~day,y=~scaled,name = 'Total␣
    Prediction',type = 'scatter',mode='lines')
fig_cum<-fig_cum%>%add_trace(y=~real,name='Toal␣Real',mode='
    markers')
fig_cum<-fig_cum%>%layout(title=paste("Total␣confirmed␣in␣Iran"
    ),
xaxis=list(title="Date"),
yaxis=list(title="Total"))
fig_cum<-fig_cum%>%layout(shapes=list(vline(mean_date,color = '
    rgb(22,␣96,␣167)'),vline(median_date,color = 'rgb(205,␣12,␣
    24)'),vline(max_date,"Green")))
```

```
#Construct data frame for ploting the prediction curve and real
    value data of daily growth cases
real_diff<-data_diff[which(data_diff$type=='real'),]
gamma_diff<-data_diff[which(data_diff$type=='scaled Gamma'),]
blank2<-NA
for (i in 1:(length(gamma_diff$day)-length(real_diff$day)-1)) {
blank2<-c(blank2,NA)
}
real_diff_data<-c(real_diff$growth,blank2)
plot_diff_data<-data.frame(day=gamma_diff$day,scaled=gamma_diff
   $growth,real=real_diff_data)

#Plot the prediction curve and real value data of daily growth
   cases
fig_diff<-plot_ly(plot_diff_data,x=~day,y=~scaled,name = 'Daily
   Prediction',type = 'scatter',mode='lines')
fig_diff<-fig_diff%>%add_trace(y=~real,name='Daily Real',mode='
   markers')
fig_diff<-fig_diff%>%layout(title=paste("Daily growth in Iran")
   ,
xaxis=list(title="Date"),
yaxis=list(title="Growth"))
fig_diff<-fig_diff%>%layout(shapes=list(vline(mean_date,color =
    'rgb(22, 96, 167)'),vline(median_date,color = 'rgb(205, 12,
```

```
   ␣24)'),vline(max_date,"Green")))



#Stitch the plot of total confirmed prediction with plot of
   daily growth prediction
fig_nail<-subplot(fig_cum,fig_diff,nrows = 2)
fig_nail<-fig_nail%>%layout(title=paste("Cases␣in␣Iran"))

fig_nail



# Date of Mean
mean_value<-mean(real_diff$growth)
index1<-max(c(which(abs(mean_value-real_diff$growth)==min(abs(
   mean_value-real_diff$growth)))))
mean_date<-real_diff$day[index1]

# Date of Median
median_value<-median(real_diff$growth)
index2<-max(c(which(abs(median_value-real_diff$growth)==min(abs
   (median_value-real_diff$growth)))))
median_date<-real_diff$day[index2]

# Date of Maximum
```

```
max_value<-max(real_diff$growth)
max_date<-real_diff[which(real_diff$growth==max_value),1]


# Draw Vertical Line
vline <- function(x = 0, color = "red") {
list(
type = "line",
y0 = 0,
y1 = 1,
yref = "paper",
x0 = x,
x1 = x,
line = list(color = color,dash="dash")
)
}
```