Application of Occam's inversion to airborne time-domain electromagnetics

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Airborne time-domain electromagnetics (ATDEM) methods are regularly used for mining, hydrocarbon, and groundwater exploration. A large quantity of data is collected along survey lines from an aircraft, and there is an incentive to interpret these data in a systematic way. When the geology is appropriate, the use of 1D inversion methods is justified. Among these methods are: conductivity-depth transform (CDT) (Wolfgram and Karlik, 1995), layered-earth inversion (Sattel, 1998), Zohdy's method (Sattel, 2005), and Occam's inversion (Constable et al., 1987; Sattel, 2005). These methods either require considerable tuning to get realistic results, are limited to step response data, or require considerable experimentation with the initial guess to ensure a reasonable result. The advantage of the Occam's algorithm is that it can be easily adapted to different ATDEM methods and is not strongly dependent on the initial guess. Furthermore, there are not a lot of parameters to tune in order to get a reasonable result. The weakness of the Occam's inversion is that for ATDEM data, the process requires a great deal of computer time. In this paper, we review details of the application of Occam's method to ATDEM data and we present the results of some of our experiments.

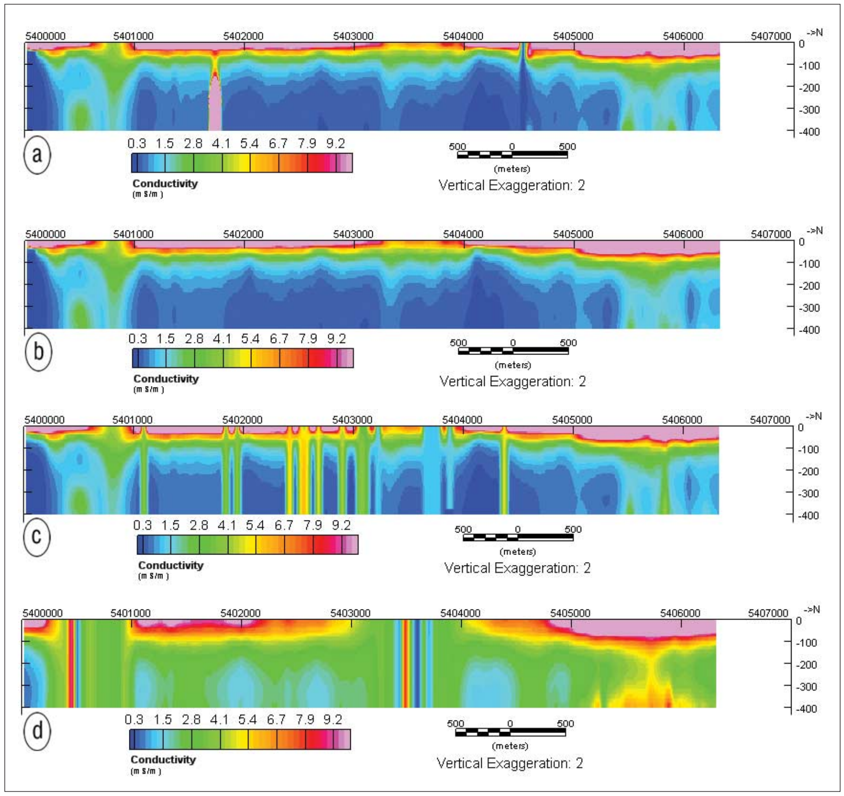

Figure 1. Occam's inversion over Line 15 of Reid-Mahaffy test site, with (a) noise level = 100 pT/s and requested misfit = 50; (b) noise level = 1000 pT/s and requested misfit = 5; (c) noise level = 10 000 pT/s and requested misfit = 0.5; and (d) noise level estimated from the data and requested misfit = 1.

Methodology

Occam's inversion algorithm is a smooth inversion method. The usual weighted least-square criterion can be written as

$$X^2 = \sum_{j=1}^{M} \left(d_j - F_j[m]\right)^2 \Big/ \sigma_j \qquad (1)$$

where d_j is the j observation, F_j[m] is the j functional relating the model to the observations, and σ_j is the uncertainty in the jth datum. The inversion process involves finding a model m that minimizes the functional X^2. A constrained functional U is formed by means of a Lagrange multiplier μ^{-1}.

$$U = \left\|\partial m\right\|^2 + \mu^{-1}\{\left\|Wd - WF[m]\right\|^2 - X_*^2\} \qquad (2)$$

where $\|\partial m\|^2$ is the roughness of the model, W is the diagonal $M \times M$ matrix, $W = diag\{1/\sigma_1, 1/\sigma_2 ..., 1/\sigma_M\}$ and X_*^2 is the requested misfit. The multiplier μ can be interpreted as a kind of smoothing parameter: when it is large, the solution is not influenced by the data misfit. Alternatively, when it tends to zero, the roughness term is of little significance.

Constable et al. propose to minimize the functional U in a least squares sense, while systematically varying the smoothness (by changing the Lagrange multiplier). The result is the model of smallest roughness with a data misfit specified by the user. If the misfit requested is not achieved after a maximum number of iterations, the program stops. In the application of this technique, the results obtained can vary significantly depending on the relative size of the uncertainty σ_j and the selection of the required misfit.

Reid-Mahaffy results

To illustrate the impact of the selection of the required misfit and noise level, we examine the results from Occam's inversion of ATDEM data collected over a test site. The ATDEM data are 90-Hz GEOTEM *dB/dt* data (Annan and Lockwood, 1991) collected in 2006 as part of a test survey over the Reid-Mahaffy test site, which has been used regularly since 1999 to calibrate and compare geophysical instruments (Witherly et al., 2004). The geology at this site consists of a conductive overburden of variable thickness between 17 and 60 m (ascertained from drill information). The bedrock is resistive volcanic rock and a number of generally vertical conductive structures. Figure 1 shows the results for different combinations of noise level (in units of measurement) and required misfit (dimensionless) for Reid-Mahaffy line 15. In all cases, the section image shows a conductive overburden of irregular thickness over a resistive basement. The thickness of this layer (less than 100 m) is consistent with the drill information. However, all sections except Figures 1b and 1d are affected by artifacts. In Figure 1a, the noise level is too low (100 pT/s) and the misfit too high (50). On the contrary, in Figure 1c, the noise level is too high (10 000 pT/s) and the requested misfit is too low (0.5). Finally, noise levels were selected by looking at the standard deviation of the raw data compared with the processed data. The results (Figure 1d) look very similar to those in Figure 1b. Our experience from this data set leads us to recommend a misfit between 1 and 10, with appropriate values of the noise level, either a fixed value (in this case 1000) or a value estimated from the data. One feature of the Occam's method is that it is generally easy to recognize when the two tuning parameters (noise level and requested misfit) are not appropriately selected, as the model is extremely smooth (conductivity constant with depth). In the fitting of these data, we only used the EM response measured during the offtime, and we set the maximum of iterations to 20.

As the next step, in order to evaluate the results from Occam's inversion, we compare these results to the results from two different methods applied to the inversion of ATDEM data also collected over the same line at the Reid-Mahaffy test site.

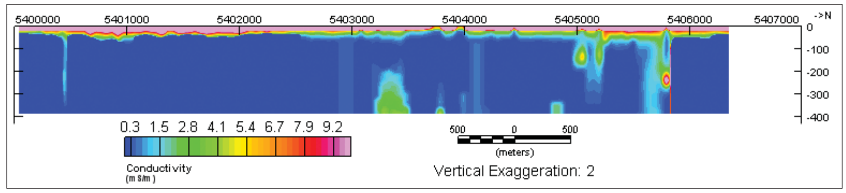

Figure 2. *CDT inversion of B-field over Reid-Mahaffy line 15.*

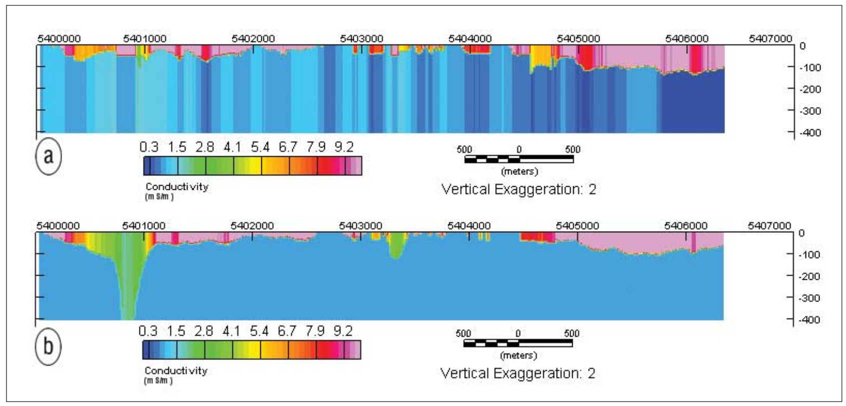

Figure 3. *Layered-earth inversion of Reid-Mahaffy line 15 with a two-layer model: (a) with half-space conductivity that can vary and (b) with half-space conductivity fixed at 1 mS/m.*

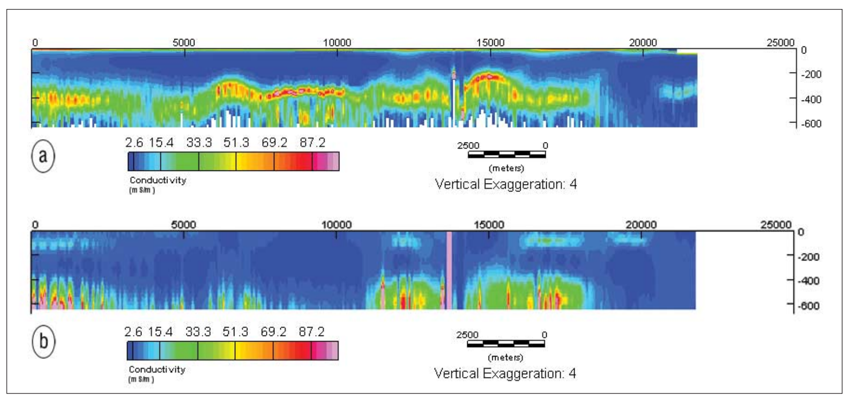

Figure 4. *Inversion of Golden Valley Mines deep conductor example: (a) CDT, (b) Occam's inversion.*

CDT

The conductivity-depth transform (CDT) is a technique developed by Wolfgram and Karlik (1995) to image GEOTEM time-domain data using a 1D model. Figure 2 shows the application of the technique on the B-field data collected on line 15. The technique images a conductive superficial layer over a resistive basement. The thickness of the overburden is variable, but generally less than 50 m. A local conductor is imaged at Northing 5403300 at a depth greater than 200 m. This conductor has been intersected by a drill hole at a depth of 120 m below 50 m of overburden and interpreted to be a vertical plate-like structure (Smith and Lee, 2002). This conductor was not clearly imaged on the Occam's sections (Figure 1) derived from the same B-field data.

Layered-earth inversion with AIRBEO

The AIRBEO program from CSIRO (Raiche, 1998; Chen and Raiche, 1998) allows the inversion of GEOTEM data based on a layered-earth model with a limited number of layers. The strength of the AIRBEO program is that various constraints can be applied to the model parameters. The inversion for a two-layer model was explored; the results are presented in Figure 3a. Based on the expected highly resistive volcanic rocks, we further simplified the two-layer inversion by fixing the conductivity of the basement at 1 mS/m.

The results for the second case are shown in Figure 3b. Both cases show a conductive overburden with a thickness consistent with the drill information. Like the Occam's inversion, there is no strong or obvious indication of the bedrock conductor. The two AIRBEO results differ. The second shows a more smoothly varying overburden. Whether or not this is more realistic requires more geological control. Other techniques were tried on this data and these are reported in Vallée and Smith (2007).

Deep conductive layer (Golden Valley Mines)

In this application of the GEOTEM system, the background conductivity is larger than in the first case. Figure 4 presents the interpreted section over one line. The CDT section (Figure 4a) is clearly affected by the "bulge" effect discussed by Hunter and Macnae (2001). In this effect, there will be a conductive unit in the ground which is roughly uniform below a certain depth. However, the CDT section will show a peak or bulge in the conductivity close to the top of this unit, but the interpreted conductivity on the section will fall away below this. This manifests itself on the CDT sections (Figure 4a) as a red feature at some depth near the top of the conductive feature, but with yellow green and blue (resistive) below. Similar resistive features at depth were also seen on the CDT sections of Smith et al. (2004). The Occam's section (Figure 4b) was generated using noise levels derived from the standard deviation of the data and a misfit of 1. The section does not show the bulge effect as strongly, as there is never a blue feature below the red conductive features on Figure 4b. Otherwise, the main features on the two sections in Figure 4 agree very closely.

Conclusions

Occam's inversion is an alternative tool for assisting in the interpretation of airborne time-domain electromagnetic data in quasi layered-earth environments. Results are consistent with the CDT sections and with AIRBEO inversion using a two-layer model. The Occam's method is therefore a useful alternative that can be used to help resolve the 1D conductivity structure. The advantage of the Occam's method is that far fewer tuning parameters (2) are required compared with other techniques like CDT (9 parameters). If the two tuning parameters required by Occam are not selected optimally, then some artifacts can appear. These are normally easy to identify. However, for the examples we considered, it was possible to select a set of parameters where no or few artifacts are present. The automatic method of selecting the noise estimates normally gave good results.

Compared with the CDT algorithm, the Occam's algorithm showed more lateral coherence of features. Also, the Occam's sections did not show the bulge artifact as strongly—the CDT sections frequently show a resistive feature at depth.

One disadvantage of the Occam's results is that vertical conductors, like the one on line 15 at Reid-Mahaffy, might not be imaged. Also, the computer time required to invert the sections is an hour or so per line, compared with a minute or so for the CDT sections. This means that experimenting with the two tuning parameters can take a very long time.

Suggested reading. "An application of airborne GEOTEM in Australian conditions" by Annan and Lockwood (*Exploration Geophysics*, 1991). "Inverting AEM data using a damped eigenparameter method" by Chen and Raiche (*Exploration Geophysics,* 1998). "Approximate 2D inversion of airborne TEM data" by Christensen and Wolfgram (ASEG *Expanded Abstract*, 2006). "Occam's inversion: A practical algorithm for generating smooth models from electromagnetic sounding data" by Constable et al. (Geophysics, 1987). "Inversion of airborne electromagnetic data" by Ellis (*Exploration Geophysics,* 1998). "Subsurface conductivity structure as approximated by conductivity-depth transforms" by Hunter and Macnae (ASEG 2001 *Expanded Abstract*). "An example of 3D conductivity mapping using the TEMPEST airborne electromagnetic system" by Lane et al. (*Exploration Geophysics*, 2000). "Modeling the time-domain response of AEM systems" by Raiche (*Exploration Geophysics*, 1998). "Conductivity information in three dimensions" by Sattel (*Exploration Geophysics*, 1998). "Inverting airborne electromagnetic (AEM) data with Zohdy's method" by Sattel (Geophysics, 2005). "The moments of the impulse response: a new paradigm for the interpretation of electromagnetic data" by Smith and Lee (Geophysics, 2002). "Using airborne electromagnetic surveys to map the hydrogeology of an area near Nyborg, Denmark" by Smith et al. (*Near Surface Geophysics*, 2004). "Comparison of fixed-wing airborne electromagnetic 1D inversion methods" by Vallée and Smith (Exploration '07 *Expanded Abstract*). "Reid-Mahaffy Test Site, Ontario Canada: an example of benchmarking in airborne geophysics" by Witherly et al. (SEG 2004 *Expanded Abstracts*). "Conductivity-depth transform of GEOTEM data" by Wolfgram and Karlik (*Exploration Geophysics*, 1995). TLE

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