Snow rubbing squeal vibrations, and roaring cold snow

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Submitted to; Can. J. Phys. April 10, 2013.

Abstract: The objective of this study is the investigation of the mechanism responsible for the squeal vibrations excited when bodies with fairly rough surfaces are rubbed on a cold dry slightly compacted snow bed. To this end, signals were recorded and analyzed when the snow surface was rubbed by the ends of baseball bats, the ends of circular wood rods, and by the thick sole of a rubber boot. It is argued that the vibration modes are confined in the rubbing bodies and that the role of the snow bed is limited to providing the right conditions for the stick-slip effect to be applicable at the rubbing interface. An attempt is made to account for the reported very intense sound emission from a sheared very cold snow bed in terms of coherent snow granule column vibrations around the shearing body, as in the case of a sheared singing sand bed.

PACS Nos: 68.35.Ja, *4328.Hr, 62.20.de, 81.40.Jj

1. Introduction

A squealing, squeaking, sound, with dominant frequency, f_d in the range of 1000 Hz, is usually emitted when one walks on a dry cold snow band, several mm thick, on top of a well compacted snow bed below. It is best heard when the cold dry snow band is fairly recent and the snow had fallen during a relatively cold temperature, about -20 C^o. The sound is more intense and more musical when the thick shoe sole is made of rubber and, it seems, when the sole bottom is divided into sections.

The question arises naturally as to the mechanism responsible for such a sound; is it due to vibrations in the shoe sole, or in the sheared snow band under the sole, or in both? A similar squeal sound can be emitted when the same shoe sole is rubbed on the wet surface of an irregularly shaped glass piece, a few cm in overall diameter, implying that the sound originates with vibrations in the shoe sole. However, no sound can be emitted when the same sole is rubbed on a clear ice surface. A discussion of similar acoustic emissions can be found in Patitsas [1].

A clearer musical sound can be realized when the handle cap end of a regular wood baseball bat is rubbed on a fresh cold dry slightly compacted snow bed surface, usually by turning the bat about its axis with a slight downward push. It is noteworthy that the paint has to be removed with a medium grade sand paper from the surface of the bat in contact with the snow surface. When the temperature and snow conditions are more ideal, wood rods with diameter about 2.5 cm and with the ends somewhat rounded can be used in the same way to produce similar sounds. Furthermore, regular bottle corks held by regular pliers can evoke similar sounds, but with higher frequencies in the range of 1500 to 3500 Hz when rubbed on a slightly compacted snow bed surface or on a wet glass surface.

The following signals were microphone recorded at the bottom of a five step staircase on the west side of the house at 158 Harry Crescent, Sudbury, ON, Canada. The snow rubbing was effected at the side of a 40 cm wide walkway where the snow bed was slightly compacted. For reason of relatively not well defined geometry, the discussion of the acoustic emissions from shoe sole rubbing on a snow bed is presented after that of the rods. Additionally, the discussion of the reported intense sound emission during snowshoeing in a very cold snow bed is placed in the Appendix at the end of the paper, since it is based on coherent vibrations in the snow granule columns extended a few cm into the snow bed, as opposed to vibrations in the shearing body.

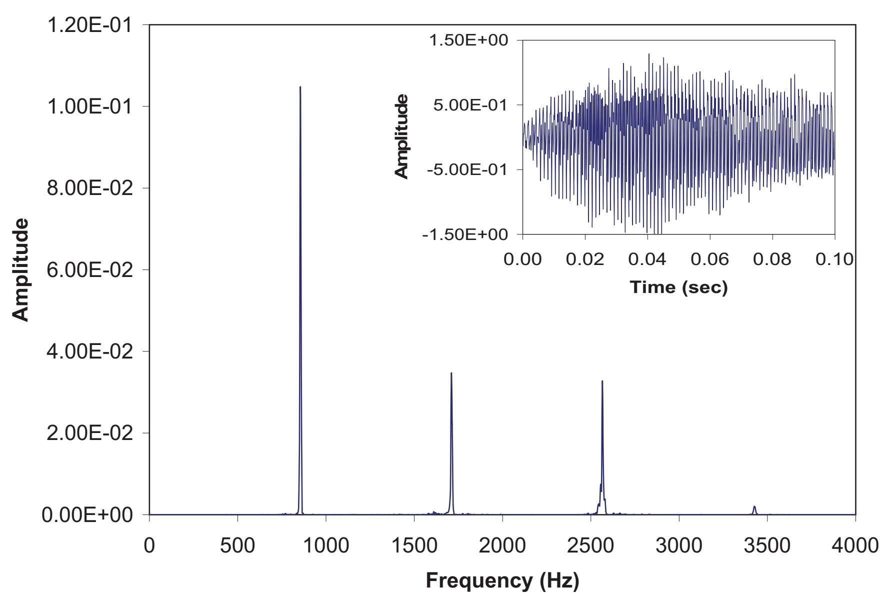
2. Preliminary results using baseball bats

During the winter of 2011-2012, several objects with smooth surfaces, such as, glass or plastic bottles and metallic or painted bodies, were rubbed on a slightly compacted snow bed surface with negative results. Evidently, the friction between the rubbing surfaces was not sufficient for the stick-slip effect to become applicable. Alternatively, all such bodies would evoke sonorous sounds when rubbed on a bed of singing sand, since the sound emission is due to grain layers sliding and rubbing on one another some distance under and ahead of the shearing body, Patitsas [2, 3], Bagnold [4].

Then, an old unpainted adult wood baseball bat produced a musical sound with $f_d \approx$764 Hz and weak harmonics of f_d, when the handle cap end was rubbed on the snow surface by twisting it back and forth with a slight push downwards. About one quarter of the cap had been brocken off, suggesting that the origin of the sound is not to be found in the bat cap. No sound could be evoked when the bat was inverted and the thick strike end was similarly rubbed on the snow surface.

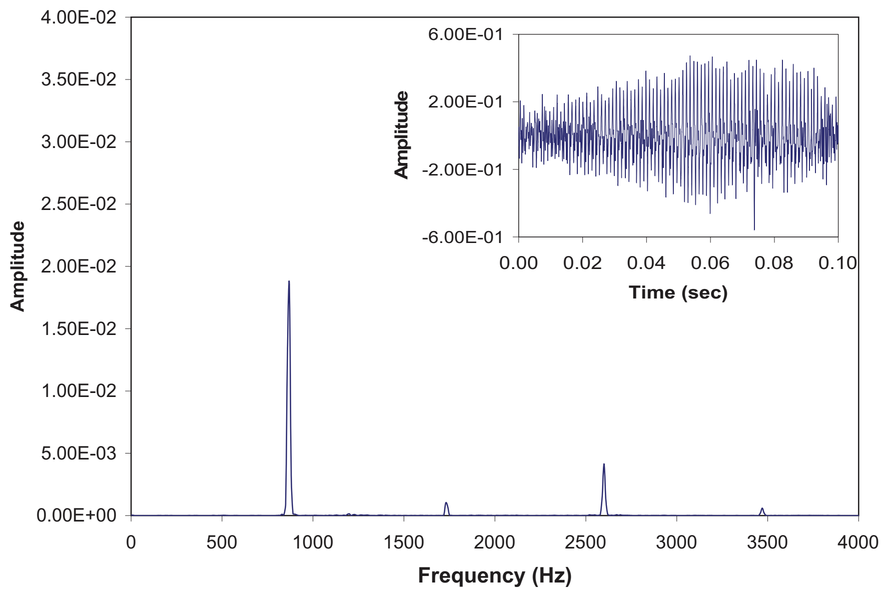
On January 20, 2012, at about midnight, the cap end of a smaller bat was rubbed similarly on a slightly compacted snow bed surface at temperature, $T \approx -20\,C^o$, resulting in the signal and its frequency spectrum shown in Fig. 1. The bat dimensions were as follows; 63 cm in length, 2.5 cm in diameter at the handle end, below the 4.8 cm diameter cap, and 5.0 cm in diameter at the strike end. The paint at the cap end had to be removed with a medium grade sand paper. The main peak lies at $f_d = f_1 \approx$853 Hz and the others at multiples f_1.

Fig. 1. Frequency spectrum of the microphone recorded signal when the handle cap end of a baseball bat was rubbed on a slightly compacted snow bed surface, January 2012. $f_1 \approx 853$ Hz.



Nearly one year later, on January 17, 2013 at 11:00 AM, with plenty of sunlight at $T \approx -18\ C^o$, the same bat was similarly rubbed resulting in the plots shown in Fig. 2, with $f_1 \approx$863 Hz and harmonics of f_1. The proximity of the values of f_i in the two plots suggests that the modes of vibration are defined in the bat. However, the surface texture of the snow granules could play a role in the degree of excitation of a given mode. The sound was produced more readily by twisting the bat about its axis, but forcing the cap end to slide sidewise would also result in sound emission.

Fig. 2. The same as in Fig. 1 but one year later. $f_1 \approx$863 Hz.



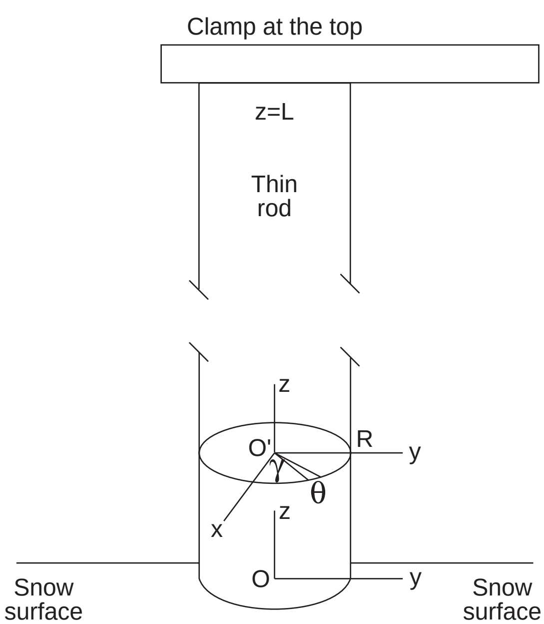
When the thin section, above the cap end, was grasped by the hand, the squeal sound ceased, implying that the vibration was defined primarily in this section as opposed to thick strike end at the top. No sound could be evoked when the bat was similarly rubbed on a clear ice surface. No sound could be evoked when the bat was inverted and similarly twisted by hand at the cap end, but when regular pliers were used to hold and twist the bat just below the cap end, the sound returned with a very weak excitation of the fundamental mode at $f_1 \approx$208 Hz and strong excitations of the modes with frequencies equal to the next three harmonics of f_1.

It could be argued that when the thick end of the bat is rubbed on the snow surface, the thin end tends to vibrate as a free end, resulting in appreciable attenuation

of the vibration amplitude due to absorption in the hand skin. Additionally, when the thick end is on top, it plays the role of a mass load, resulting in nearly zero vibration amplitude and in not significant absorption in the hand skin. In the case of the wood rods, the squeal sound is hardly emitted until a steel clamp is attached at the top end, or the rod is held at the top by a pair of regular pliers, effectively, until the top end becomes nearly a fixed end as opposed to a free end.
In what follows, the simplicity of the rod geometry is utilized in an attempt to elucidate the modes of vibration responsible for such squeal sounds

3. Torsional vibrations in thin rods

Fig. 3. A thin rod, diameter, $D = 2R$, length, L, with a steel clamp attached at the top, in a vertical position, is twisted back and forth on a slightly compacted snow bed surface.



The diagram shown in Fig. 3 depicts a vertical thin rod of length, L, and diameter, $D = 2R$, loaded with a clamp of moment of inertia, I, about the rod axis. In such a system, there could exist torsional as well as longitudinal and flexural modes of vibration. Whereas the twisting of the rod end on the snow surface allows for the applicability of the stick-slip effect in the excitation of the torsional modes, no such effect exists for the excitation of the longitudinal or flexural modes. However, in cases where the snow surface is not very smooth and the top load is appreciable, excitation of the latter modes could not be ruled out totally.

The axes, xyz, are assumed to be fixed on the rod which is assumed to be thin enough so that the particle displacement due to wave propagation does not depend on the distance, r, from the rod axis, or the angle, γ, *i.e.*, the vibration displacement angle , θ, can be expressed as, $\theta = \theta(z)\mathrm{e}^{\mathrm{j}\omega \mathrm{t}}$. Then, the wave equation can be written as, Graff [5], page 127,

$$\frac{\partial^2\theta}{\partial z^2} = \frac{1}{c_s^2}\frac{\partial^2\theta}{\partial t^2} \tag{1}$$

where $c_s = \sqrt{G/\rho}$ is the shear velocity, and G, ρ are the shear modulus and the mass density of the rod respectively. Then, the expression for θ becomes,

$$\theta = [A\cos\alpha z + B\sin\alpha z]\mathrm{e}^{\mathrm{j}\omega \mathrm{t}} \tag{2}$$

where $\omega = \alpha c_s$.

While the time span of a back and forth twist of the rod amounts to about one second, the angle θ changes sign hundreds and even thousands of times per second.

The torque exerted by the snow surface on the twisted rod end is not zero, implying that the rod end is not free and $\frac{\partial\theta}{\partial z} \neq 0$ at $z = 0$. With rod diameter equal to 3.0 cm, rod weight equal to 0.75 kg and as much downward push, and friction coefficient between the rubbed surfaces equal to 0.5, the torque exerted by the snow surface on the rod end amounts to 0.08 Nm. It could be argued that during the stick phase, the relative velocity between the rubbing surfaces is nearly zero, implying that during the same phase, when energy from the hand is converted into vibration energy, the rod end surface is nearly fixed relative to the snow surface. The following experimental results support such an assumption. Thus, (2) becomes,

$$\theta = B\sin\alpha z\mathrm{e}^{\mathrm{j}\omega \mathrm{t}} \tag{3}$$

At $z = L$, the torque exerted by the rod on the clamp is,

$$K = -C\frac{\partial\theta}{\partial z} = -CB\alpha\cos\alpha L\mathrm{e}^{\mathrm{j}\omega \mathrm{t}} \tag{4}$$

Where, the torsional rigidity, C, is equal to JG, where, $J = (1/2)\pi R^4$, [5], p. 126. Additionally,

$$K = I\frac{\partial^2\theta}{\partial t^2} = -I\sin\alpha L\omega^2 \tag{5}$$

resulting in the transcendental equation,

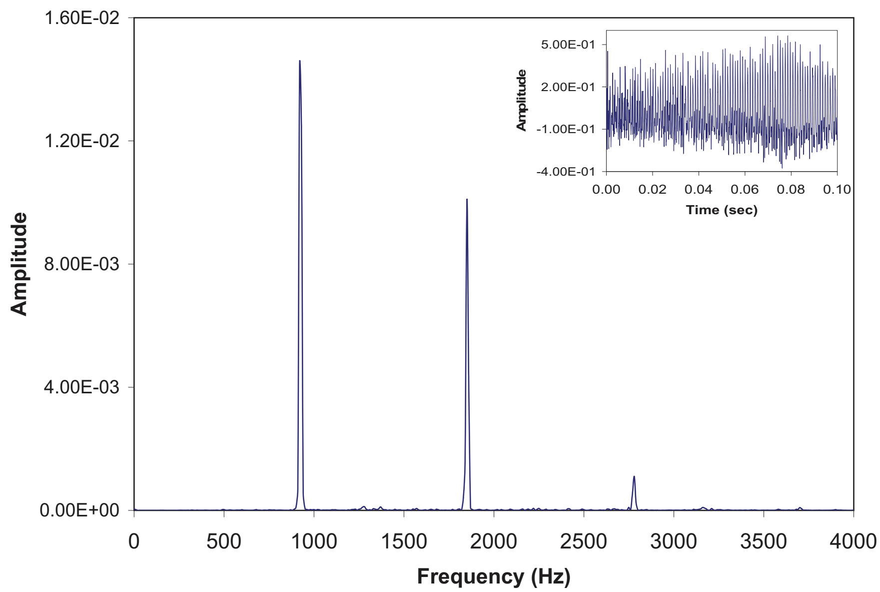
$$cot\alpha L = \frac{I}{J\rho L}\alpha L \tag{6}$$

Even with the smallest steel clamp, with mass equal to 120 gr and center of mass approximately 4 cm from the rod axis, and rod diameter and length equal to 2.2 cm and 120 cm respectively, the slope, $\zeta = I/(J\rho L)$ in (6), acquires the relatively large value of 14. With such a large value of the slope ζ, the straight line, $\zeta\alpha L$ intersects the curves, $cot\alpha L$ at nearly $\alpha L = n\pi, n = 1, 2, 3..$ in agreement with the frequency plots that follow. The mode corresponding to the intersection below $\pi/4$ is absent in the following plots. It could be argued that for such a mode, the rod would oscillate about its axis like a short spring loaded with the clamp mass, and excitation of such a mode would require appreciably more energy than that provided by the present experimental arrangement. Additionally, the oscillation would be absorbed by the hand holding the clamp or the pliers at the top of the rod.

4. Results using thin wood rods

Figure 4 depicts the signal emitted, and its frequency spectrum, when a thin short wood rod, likely from a whitewood tree, diameter, D=2.2 cm, length, L=30 cm, with a relatively small 120 gr steel clamp attached at the top, was twisted back and forth manually on a somewhat compacted snow bed surface, as seen in Fig. 3. The recording took place on February 16, 2013, at about 1:00 AM after a light snow had fallen a few hours earlier at $T \approx -15\,C^o$. The air temperature was, $T \approx -20\,C^o$ and the depth of penetration into the snow surface was, $h \approx$5 mm. The fundamental mode, at $f_1 \approx$920 Hz, was mostly excited and then the modes at harmonics of f_1.

Fig. 4. Frequency spectrum of the microphone recorded signal when a short thin wood rod, D=2.2 cm, L=30 cm, was twisted back and forth on a slightly compacted snow bed surface. $f_1 \approx$920 Hz, $f_2 = 2f_1$ etc.



Approximately 24 hours later, a similar recording, not shown, with a heavier, 492 gr clamp at the top, resulted in the excitation of only the fundamental mode at $f_1 \approx$938 Hz. Occasionally, such changes in the spectrum structure could take place in the span of only a few seconds, suggesting that mode excitation depends not only on snow conditions but also in the manner the rod is handled, the angle from the vertical direction, for example.

Figure 5 depicts the spectrum of the signal, recorded several seconds after that in Fig 4, using a longer thin rod, *i.e.*, D=2.2 cm, L=120 cm, with the 492 gr clamp attached at the top. The fundamental mode at $f_1 \approx$265 Hz was hardly excited and the 5th mode at $f_5 \approx$1384 Hz was mostly excited. The ratio of the fundamental frequencies in Figs. 4 and 5 is, 920/265=3.47, not quite equal to 4.0, which is the length ratio. It is possible that the two rods did not have exactly the same elastic properties, even though they were purchased from the same store. Additionally, it could be argued that, since the rod ends were not totally fixed, the wavelength, λ_1, corresponding to the fundamental mode, was somewhat larger than $2L$, and that such inequality was more pronounced in the case of the appreciably shorter rod. It is possible to obtain a rough estimate of the shear velocity, c_s, *i.e.*, $c_s = f_1\lambda_1 = f_1 2L$ =636 m/s, that leads to the shear modulus, $G = c_s^2\rho = 0.3 \times 10^9 Pa$, with $\rho = 720\, Kg/m^3$, as for oak wood.

Fig. 5. Same as in Fig. 4 but with a longer wood rod, D=2.2 mm, L=120 cm, twisted on a slightly compacted snow bed surface. f_1 ≈265 Hz and f_5 ≈1384 Hz.

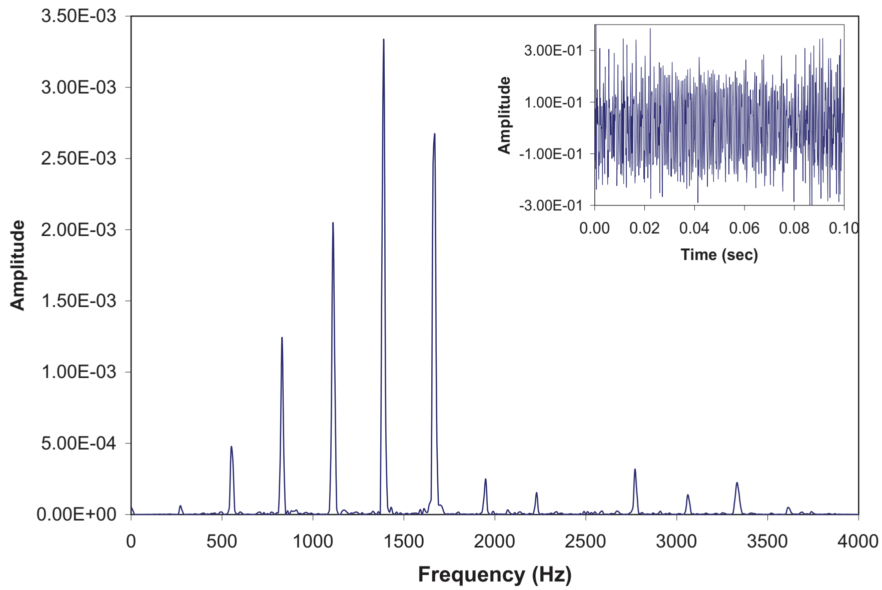
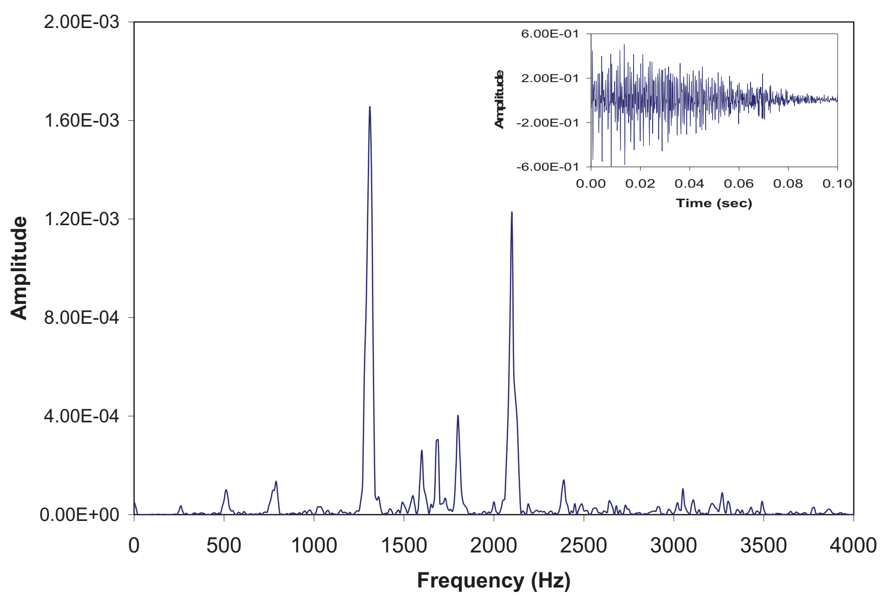
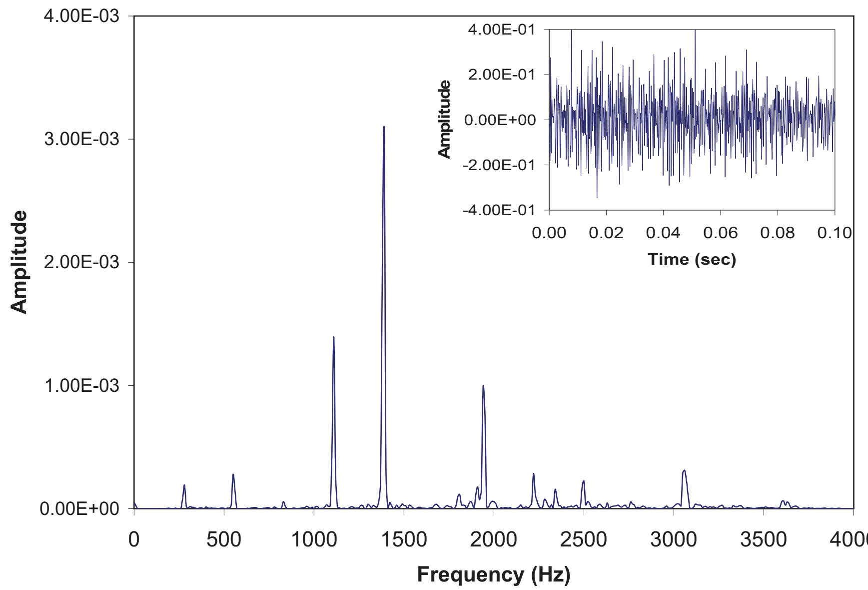


Fig. 6. The same as in Fig. 5 but 24 hours later. The cluster of peaks at around 1700 Hz implies mode excitation other than torsional. f_5 ≈1308 Hz.



The signal shown in Fig. 6 was recorded approximately twenty four hours after the signal in Fig. 5 was recorded using the same thin rod, D=2.2 cm, L= 120 cm, as in Fig. 5, but with a heavier 664 gr clamp at the top. The frequencies are slightly lower than those in Fig. 5, *i.e.*, $f_1 \approx 256$ instead of 265 Hz, and $f_5 \approx 1308$ Hz instead of 1384 Hz. It is included here in order to see the cluster of peaks between 1500 and 1800 Hz suggesting mode excitation other than the torsional modes.

Fig. 7. same as in Fig. 5 but at an earlier date with a medium diameter rod, D=2.5 cm, same L=120 cm. $f_5 \approx 1384$ Hz.



The effect of the rod diameter on the frequency spectrum is exhibited in Fig. 7 where D=2.5 instead of 2.2 cm and L=120 cm. The time and temperature were about the same as in the Figures above, but on February 4, 2013 instead of February 16, 2013. The fundamental frequency is, $f_1 \approx 275$ Hz instead of 265 Hz in Fig. 5 and $f_5 \approx$ 1384 Hz. Several seconds after the signal in Fig. 7 was recorded, a thicker wood rod was used similarly, D=3.2 cm, L=120 cm. The resulting spectrum, not shown, is characterized by a major peak at $f_5 \approx 1327$ Hz and a barely excited fundamental mode at 265 Hz, as in Fig. 5. These results tend to validate the assumption that such rods can be treated as thin rods, *i.e.*, the frequency spectra are independent of the rod diameter.

On February 6, 2013 at about 1:00 AM, with $T \approx -14\,C^o$, it was difficult to obtain the squeal sound when the rods were twisted on the snow surface in the usual place where the snow was fairly compacted. The temperature had risen to about -10 C^o during the previous day and a light wet snow had fallen. However, about 20 cm further away from the walkway, where the snow had not been stepped on, the sound could be evoked when the rods were pushed downwards to a depth of 5 to 10 cm where the snow bed was sufficiently compacted. The spectrum in Fig. 8 was obtained by using the medium thickness rod, D=2.5 cm, L=120 cm, with the 664 gr clamp at the top. The fundamental mode at $f_1 \approx 284$ Hz was mostly excited followed by that with $f_6 \approx 1744$ Hz. The value of f_1 compares well with that in Fig. 7. The signal from the thinner rod, D=2.2 cm, L=120 cm, not shown, resulted in the spectrum with exactly the same peak at 284 Hz but no other peaks.

Fig. 8. Same as in Fig. 7, D=2.5 cm, L=120 cm, but two days later, at lower temperature, $T \approx -14\,C^o$. f_1 ≈284 Hz, f_6 ≈1744 Hz.

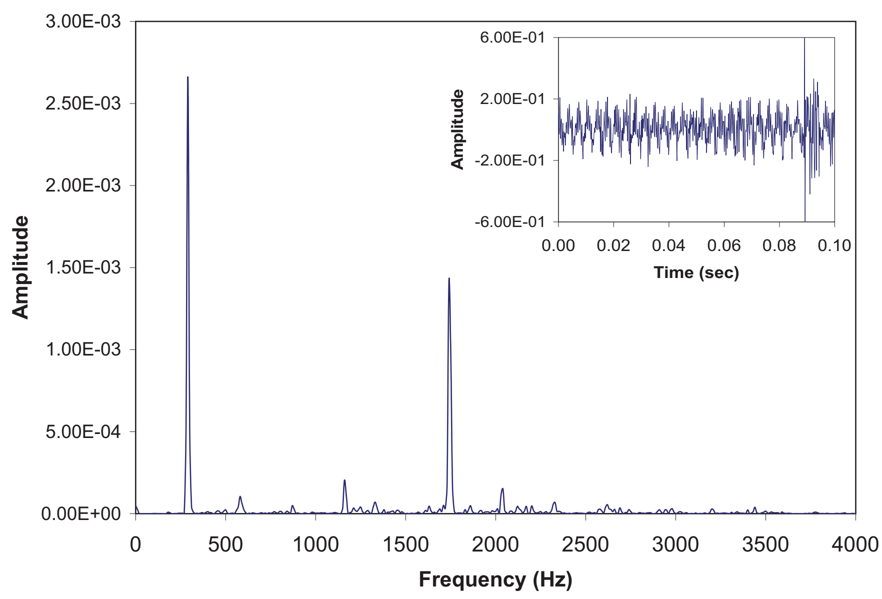
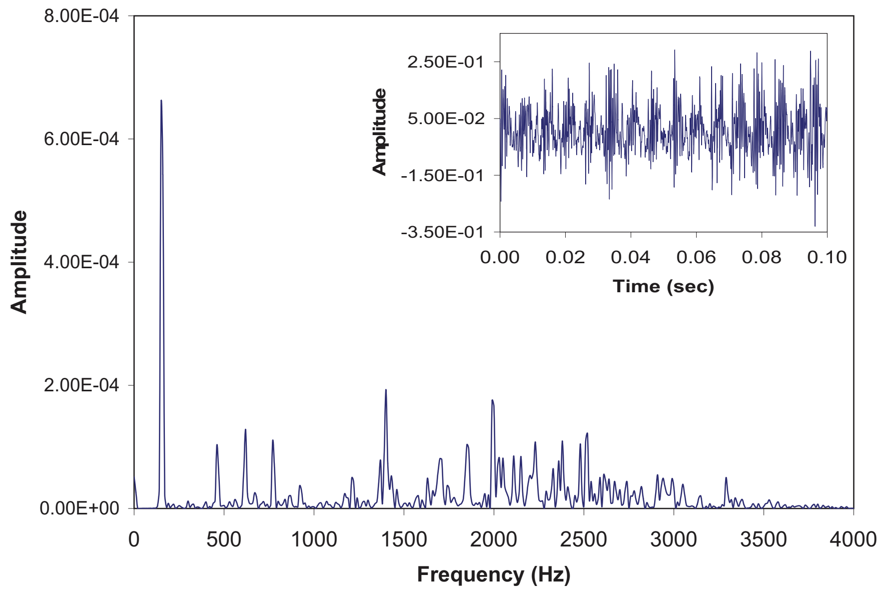


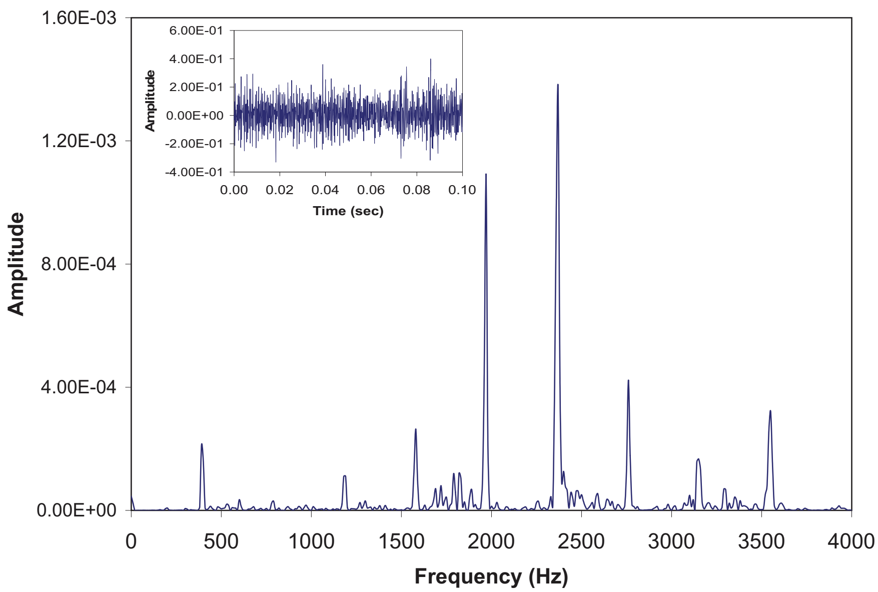
Fig. 9. Spectrum of the recorded signal during the same session as in Fig. 8, using a thicker and longer rod, D=3.2 cm, L=240 cm. The fundamental mode was primarily excited with $f_1 \approx$144 Hz, as in Fig. 8.



When the thick long rod, D=3.2 cm, L=240 cm, with the heavy 1250 gr clamp on top was similarly twisted, the signal resulted in the spectrum shown in Fig. 9. Similarly to the spectrum in Fig. 8, the fundamental mode was primarily excited at $f_1 \approx$144 Hz, *i.e.*, at half the fundamental frequency for the rod with, D=2.5 cm, L=120 cm. When the same long rod was hand held and twisted at its center, the odd modes, with an antinode at the rod center, were not excited, namely, the major peak occured at 2×144 Hz, the next at 4×144 Hz etc. Such results are consistent with the assumption adopted earlier in that the rod ends are nearly fixed.

One day earlier, on February 5, 2013, at about 1:00 AM, the conditions were more suitable with $T \approx -22\,C^o$, and when the thin shorter rod, D=2.2 cm, L=90 cm, with the 492 gr clamp on top, was twisted in a penetration depth of only a few mm, the signal resulted in the spectrum shown in Fig. 10. The fundamental mode corresponds to $f_1 \approx$379 Hz, while the mostly excited mode corresponds to $f_6 \approx$2351 Hz, and even the mode with $f_9 \approx$3536 Hz was appreciably excited. Furthermore, the ratio of the fundamental frequencies in this and Fig. 5 is 379/265=1.43, which is close enough to the length ratio, 120/90=1.33.

Fig. 10. The same as in Fig. 9, but 24 hours earlier at $T \approx -22\,C^o$ using the thin rod, D=2.2 cm and L=90 cm. Only the modes with higher frequencies were primarily excited. $f_6 \approx$2351 Hz.



5. Results using a rubber boot sole

Figure 11 depicts the microphone recorded signal and its frequency spectrum when the heel of the sole of a rubber boot was twisted about, by the corresponding leg, on a fairly well compacted snow surface at the temperature, $T \approx -20\,C^o$, on January 14, 2012, in the walkway described earlier. The sole bottom was divided into 14×22 mm sections separated by a distance of about 3 mm with average depth of separation of about 4 mm. The three dominant components at 863, 920 and 948 Hz could correspond to rubbing by three adjacent sole sections. There is considerable rubbing noise as there is in Fig. 12 that was generated by rubbing nearly similarly the same sole heel on a glass pot cover, 24 cm in diameter by about 6 mm in thickness, covered with a thin water layer. The boot was left outside for several hours prior to recording the sound inside the house.

Fig. 11. Spectrum of the microphone recorded signal when the heel of a rubber boot was twisted and rubbed on a somewhat compacted snow bed surface.

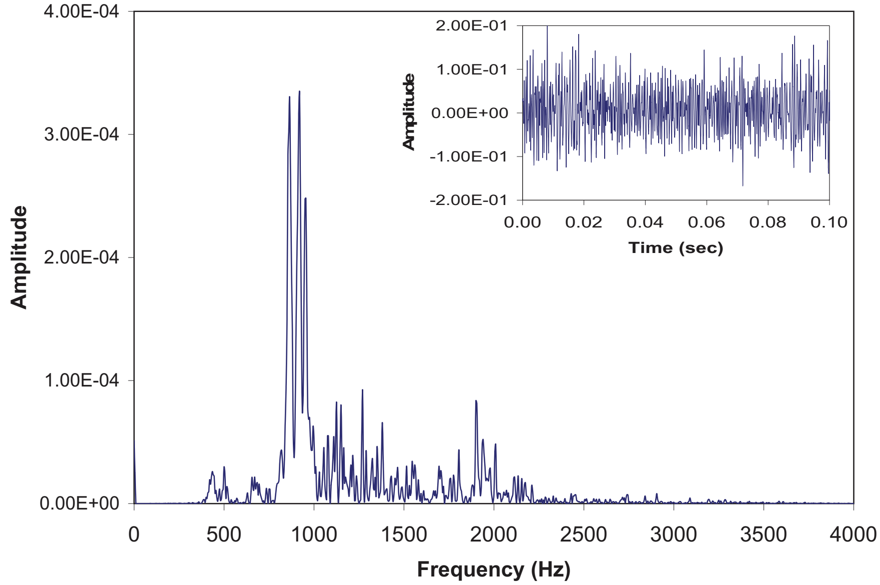
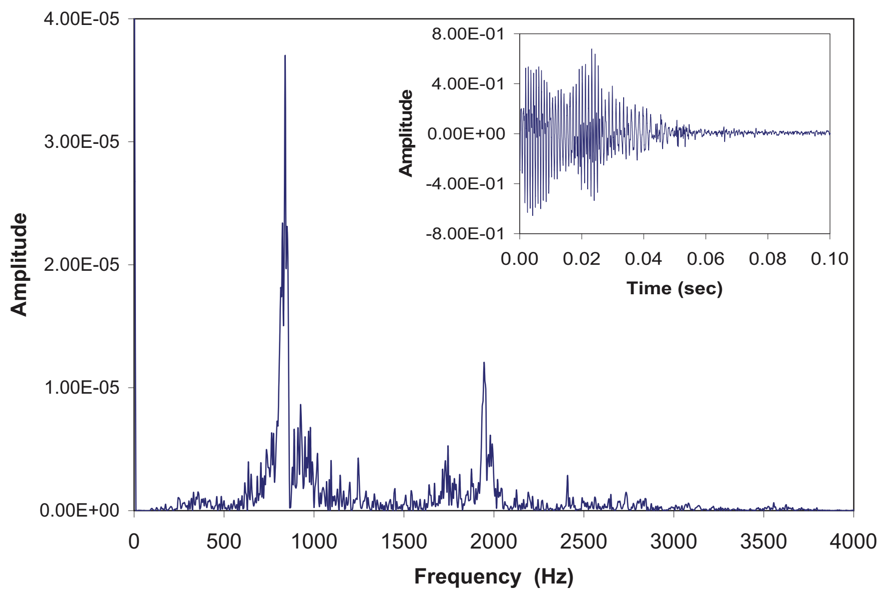
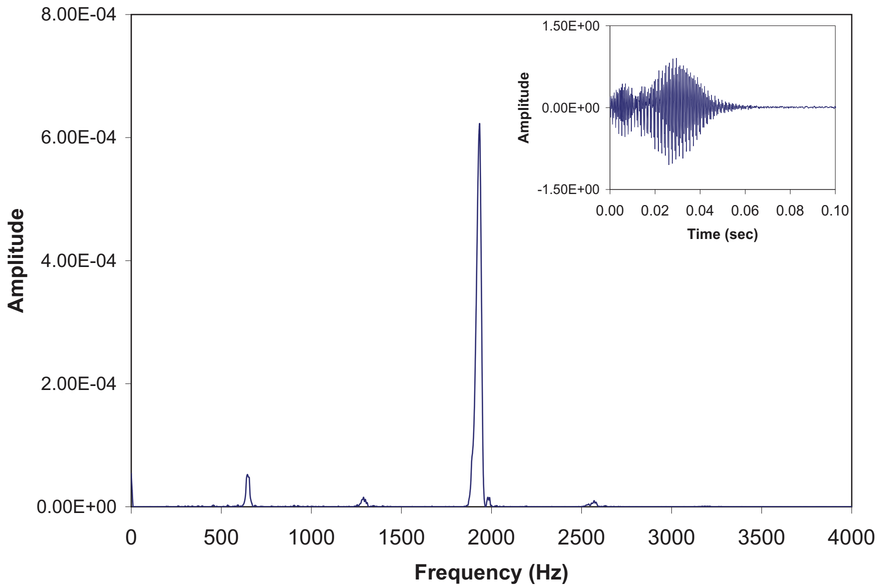


Fig. 12. Same as in Fig. 11 but the heel of the rubber boot was rubbed on a wet glass surface. $f_d \approx 834$ Hz.



The main peak in Fig. 12 lies at 834 Hz, reasonably close to the main peaks in Fig.11, suggesting that the role of the cold snow surface is the same as that of the wet glass surface. Then, about 4 seconds later in the rubbing process, another recording resulted in Fig. 13 with frequency, f_1 ≈645 Hz and harmonics of f_1. The large deviation of the peak frequency in Fig. 12 at 834 Hz from f_1=645 Hz in Fig. 13 could be due to different shearing geometries and different sole sections responsible for the corresponding emissions.

Fig. 13. Same as in Fig. 12 but during another rub a few seconds later. $f_1 \approx 645$ Hz, $f_3 \approx 1935$ Hz.



As in the case of the thin rods, there is no obvious explanation as to why the modes with frequencies f_1, f_2 in Fig 13 were hardly excited . For some reason the stick-slip effect responsible for the conversion of leg to mode vibration energy favored the mode with frequency f_3. It is safe to assume that the geometry of the sole section and that of the shearing process plays a major role in such mode excitation selection.

The presence of the noise in Figs. 11 and 12 merits a brief comment. In absence of any vibration in the boot sole, when it slides over the snow surface, there are collisions between the rubber asperities and the surface snow granules that result in elastic waves in the two surfaces, and then in the air, that are totally incoherent, resulting in the usual noisy hissing rubbing sound. But, in the presence of an intense vibration, there could be little contact between the two surfaces during the slip phase, and very little relative motion between the two surfaces during the stick phase, when the leg energy is converted into vibration energy, resulting in no rubbing noisy sound, as seen in Fig. 13. In the case of the sounds from bats and rods, discussed earlier, a tactile vibration can be felt when the spectrum is free of noise.

6. Conclusions

The experimental results from rubbing the ends of baseball bats, the ends of thin circular wood rods with a mass load on top, and a rubber boot sole on the surface of a slightly compacted dry cold snow bed reveal that the frequency spectra of the squeal vibrations consist of the fundamental frequency, f_1, and harmonics of f_1. In the case of the thin wood rods, such spectra can be attributed to shear wave propagation in the rods, with the ends nearly fixed.

In all cases, there is evidence that the vibration modes are confined in the rubbing bodies, and that the role of the slightly compacted snow bed surface is to provide the right conditions for the applicability of the stick-slip effect, which is responsible for the conversion of hand to mode vibration energy. Effectively, the friction coefficient between the rubbed surfaces has to be sufficiently high and it must decrease with increased relative velocity between the same surfaces.

There is some evidence that the snow conditions and the manner the rods are held and twisted on the snow surface influence the mode excitation selection. However, the mechanism responsible for such excitation selection remains undetermined.

Acknowledgment

Considerations are due to the Research, Development, and Creativity Office of Laurentian University for their financial and moral support towards the pursuit of this study, and to: S. N. Patitsas, G. G. Clark, D. I. Chortis an J. N. Noel for their assistance in the graph preparation.

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Appendix. Coherent vibrations in a sheared cold snow bed

During a Canadian Broadcasting Corporation radio program, CBC Living Out Loud, January 27, 2012, it was remarked that while snowshoeing at a temperature, $T \approx -45\,C^o$, an intense sound could be generated that could be heard as far as two to three km away. It is unlikely that such an intense sound emission could originate with vibrations in the wood frame of the snowshoe laced with straps. Additionally, there is no mechanism for the transfer of leg energy into snowshoe vibration energy. It could be suggested that, in such a setting, the very cold snow granules behave like singing sand grains, when sheared and forced to slide one over another by the snowshoe. Then, there are collective snow granule vibrations around the snowshoe, where transfer of leg energy is effected by the stick-slip effect as outlined below.

If the horizontal velocity of the snowshoe is considerable when impacting the snow surface, then, in the context of the treatment of acoustic emissions from singing sands, Patitsas [1], it could be argued that in front and under the snowshoe, a band of snow granules is formed, roughly 40×20 cm in area by a few cm in depth, comprising a large number of granule columns of the same depth. The contact areas of the granules in a given column behave like short springs resulting in well defined modes of vibration in the columns, and since neighboring columns are closely interlocked, they all vibrate in phase and the entire band becomes a dynamic vibrator. The leg muscle energy is converted into elastic vibration energy via the stick-slip effect, applicable at the granule contact areas, as the granules slide over one another. Then, the radiated energy can propagate very efficiently in the cold air.

In the paper by Lewis [2], it is stated that a roaring sound, dominant frequency $f_d \approx$ 200 Hz, was emitted when the sand on the slip face of a dune, in the Kalahari desert, South Africa, longitude $22^o28'$, latitude $28^o34'$, was pushed downhill by a wood board in a heaped up manner. In this sense, the very cold dry snow could be characterized as, *roaring snow*.

However, if the horizontal velocity of the snowshoe is nearly zero when impacting the snow surface, then, the geometry resembles better that of a pestle impacting a bed of beach singing sand. Thus, when the thick strike end of a baseball bat, diameter $D=$ 5 cm, impacted vertically a bed of singing sand collected at the mouth of the Brevort River flowing into the North shore of Lake Michigan, about 25 km West of the city of St. Ignace, MI, USA, a sonorous sound was emitted with $f_d \approx 315$ Hz.

In this case, it could be argued that the granule columns are formed somewhat under and around the snowshoe where the snow granules are forced to slide over one another. The vibrations are in phase, even when the columns lie on opposite sides of the snowshoe, due to the close interlocking between neighboring columns and the close proximity to the snowshoe, which also partakes in the snow vibration as a nearly rigid body.

The wavelength of the fundamental mode in a given granule column is, $\lambda_1 \approx 2L$, where L is the column length, resulting in fundamental frequency, $f_1 = V_p/(2L)$, where V_p is the longitudinal wave propagation velocity in the granule column. Whereas, V_p is defined by the elastic properties of the granule contact shear bands, L remains undefined as does the frequency f_1. On first thought, it could be argued that L is determined by the granule flow dynamics defined by the friction at the granule contact shear bands and by the elastic properties of such bands. There are no conditions on the relative velocity between the sliding granule layers.

However, there is another train of thought worthy of presentation. In 1966, Bagnold [3], argued that when a sand grain layer is forced to slide over another below with relative velocity, Δu, it tends to oscillate as it moves from the configuration of closest contact to that of least contact, when sliding takes place, and then back to that of closest contact and so on. Thus, the dominant frequency, $f_d = f_1$, would be equal to $\Delta u/d$, where d is the average grain dimeter, or equivalently, $f_1 = f_c$, where f_c is the average grain-grain collision frequency.

With average snow granule diameter, d=1 mm, and f_d=200 Hz, Δu=20 cm/s, and if the granule column length is, $L = 30d$, then, the layer velocity adjacent to the snowshoe surface would amount to the unrealistically high value of 6 m/s. However, as in the case of freely avalanching sand grains, [1], the value Δu=20 cm/s could apply only to a few layers adjacent to the snowshoe surface, while the layers farther away could slide at appreciably lower values of Δu. Thus, again, the value of the collision frequency, f_c, is determined by the flow dynamics of the granules adjacent to the snowshoe surface and of those farther away. Effectively, the system becomes self-organized in that the vibration extends to distance L so that $f_1 = f_c$.

References

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