Singing sands, booming dune sands, and the stick-slip effect

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Abstract: The origin of the acoustic and seismic emissions from impacted singing grains and from avalanching dune sand grains is sought in modes of vibration in discreet grain columns. It is postulated that when the grains in a column are pressed together, elastic shear bands are formed at the contact areas with distinct elastic properties. The central part of such contact shear bands, where the stress level is maximum, is more in a liquid-like rather than in a solid-like state, resulting in very low elastic moduli. In a given column, the elastic moduli would assume the lowest values just below the impacting pestle and higher values further below. The transfer of energy from the pestle to the modes of vibration of such columns is effected by the stick-slip effect. The concept of grain flowability is used to justify the great disparity between the acoustic emissions from impacted singing grains and from avalanching dune sand grains. The concept of grain columns is assumed to apply in the avalanching sand band, but with larger length to justify the lower frequencies. The concept of contact shear bands can be used to justify the variation of the emission frequency with blade speed and pile height when a grain pile is pushed by a blade. Finally, this approach can provide explanations as to why ordinary sands do not sing, and why singing sands do not boom and booming sands do not sing.

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1. Introduction

The mechanism responsible for the seismic and acoustic emissions, when a bed of singing sand or silica gel grains is impacted by a pestle, was the subject of a recent paper by Patitsas [1]. The mechanism was sought in shear modes of vibration in a well defined shear band (slip channel) comprising several grain layers ahead of the impacting pestle. It was argued that due to some not yet understood physico-chemical effect on the grain surface, a bed of singing (musical) grains is characterized by a relatively high level of rigidity. Thus, the grains just ahead of the pestle are subjected to a relatively high stress level resulting in the partial fluidization of the tips of the grain asperities and any grain coating at the contact areas, resulting in turn in drastically reduced elastic moduli in the slip channel.

However, the assumption in [1] that the length of the slip channel, in directions nearly normal to the direction of the pestle motion, is also well defined needs reconsideration. Such an assumption was necessary in order to view the slip channel as a cavity with well defined walls, with well defined shear standing wave patterns and with well defined eigenfrequencies. It is highly likely that the walls at the channel ends are not well defined and that there are propagating (traveling) waves along the slip channel as opposed to standing wave patterns. Thus, the determination of the frequencies of the seismic and acoustic emissions remains an open question. Whereas, it could be argued that the boundary of the slip channel adjacent to the pestle is well defined, *i.e.*, the height of the bumps is small compared with the wavelength of any waves present, it is difficult to argue that this is also the case on the lower boundary. A more gradual transition from very low elastic moduli below the pestle to nominal values in the grain bed, well below the pestle, is a more realistic assumption. Effectively, the stress level generated by the pestle decreases with depth resulting in lower fluidization at the grain contact areas. Furthermore, it is unreasonable to expect that continuum mechanics alone can provide a satisfactory solution when the width of the channel is only about ten times the particle (grain) size.

Similarly, there are serious questions that need be considered regarding the recent approaches in accounting for the seismic and acoustic emissions from avalanching dune sands. Four such approaches have been published over the past seven years: In the mainly experimental report by Andreotti [2], it is shown that during a booming dune avalanche, there are elastic waves propagating along the dune surface extending several cm below the surface. It is then argued that the grains would oscillate according to the particle displacement dictated by such waves. Furthermore, such waves would synchronize the grain-grain collisions and would become excited by such collisions. It is known that when grains of any kind are induced to avalanche down an inclined plane, the average time required by one grain to overtake another is given by the expression, $T_c = 1/0.4\sqrt{(\bar{d}/g)}$, where \bar{d} is the average grain diameter and $g = 9.8m/s^2$ (Andreotti [2], MiDi [3]). Thus, the dominant frequency of the propagating waves and that of the seismic and acoustic emissions can be defined as $f_d = 1/T_c$. However, in the paper by Vriend at al. [4], it is reported that dune vibrations were detected even when there was no apparent avalanche in progress and moreover, during avalanches the dominant frequency f_d was accompanied by several harmonics.

Subsequent reports by Bonneau et al. [5, 6] go to great lengths to elucidate the properties of waves propagating along a dune surface where the elastic moduli increase with sand depth. However, there is no clear identification of the modes corresponding to frequencies equal to multiples of f_d . Even in the latest report by Andreotti and Bonneau [7], the question of harmonics of f_d is not addressed. In this latter report, it is assumed that a thin shear band is formed between the avalanching sand band and the static sand below, and that leads to the excitation of the surface waves. Whereas, such a shear band is evident when a sand plate breaks off and begins to slide downhill, there is no such evidence after the plate breaks up and a free avalanche ensues. On page 253 in Bagnold [8], it is stated that the velocity of a grain layer at depth z decreases linearly with z until it is zero at the depth z = H. Furthermore, from video recordings, it can be argued that the avalanche thickness decreases with distance from the avalanche front. Therefore, for both these reasons the argument in [5] that for booming to occur the avalanche thickness must exceed a certain threshold is rather tenuous. Effectively, it is argued that the wave frequency defined by the overtake time T_c would be lower than the cutoff frequency of propagation in the avalanching sand band. The argument is more tenuous when applied to the case of a sand pile pushed by a blade in the study by Douady et al. [9], where the geometry is even more ill-defined.

When about 0.5 kg of booming sand grains from Sand Mountain, Nevada, USA, were placed in a glass jar, 7 cm in diameter by 16 cm in length, and shaken horizontally along the jar axis, the dominant frequency of the acoustic emission was 280 Hz (Leach and Rubin [10]). It can be argued that there is grain layer rollover and a high stress level, when the grain mass collides with the jar wall, as there is layer rollover and high stress level when the avalanche front collides with the static sand ahead. The dominant frequency, f_d , is about four times as large in the former than in the latter case. According to Nori et al. [11], $f_d \approx 65$ Hz in the latter case. According to Leach et al. [12], f_d tends to increase with decreased grain mass in the jar. It appears that the mechanism responsible for the emissions from highly localized events could also be responsible for the emissions from large scale events during the sliding of sand plates and during large surface avalanches.

In the report by Patitsas [1], the concept of a shear band (slip channel) several mm thick, under a sliding sand plate or under a freely avalanching sand band, was used to explain the relevant emissions as originating with shear modes of vibration in the channel with shear phase velocity about 1 m/s such that $\lambda \approx$ twice the channel thickness. But, even if such a channel existed in the case of free avalanche, it would not be well defined at the lateral ends, as in the case of the slip channel under an impacting pestle. However, this approach could explain the harmonics of f_d . The paper has also been archived at the Laurentian University Library.

In the experimental report by Douady et al. [9], it is also recognized that the frequency, f_d , is defined by the overtake time, T_c , but the synchronization of the collisions is effected by some sort of coupling between adjacent grain layers due to some wave that propagates up-down between the static sand and the surface of the avalanche. There is no attempt to account for overtone frequencies, but such an approach would lead to overtones in the sequence of $3f_d$, $5f_d$ etc. Then, there is the question as to how would such a wave become initially excited. This approach could not explain the sound emission just when sand plates begin to slide. However, the notion of up-down motion of grain layers is in agreement with the notion of up-down grain column oscillations proposed in this study.

In the mainly experimental report by Vriend et al. [4], the booming emission is sought in compression wave propagation along a surficial grain layer about 2 m in height. The frequency is defined by the condition that for specific phase velocities in the substrate, the grain layer and in the air, there is total reflection at the boundaries. This approach was criticized by Andreotti et al. [13] especially regarding the assumption that the phase velocity does not change with depth, and that there is experimental evidence that vibrations do not extend more than about 10 cm below the surface [2]. It is highly unlikely that the energy generated when a few kg of sand are pushed downhill or uphill, inside a hole dug on the face of a dune, would be sufficient to excite a wave in such a large layer in thickness and length. The absence of boomability in certain dunes in a given area is not a strong indicator that the booming mechanism has to lie well beneath the dune surface, since on the surface all dunes appear to be the same. Equivalently, only certain sections of the Eastern and Northern shores of Lake Michigan USA, visited on August 2009, exhibited singability.

2. The grain column approach

Figure 1 depicts an assumed grain configuration inside a slip channel where the five grain layers slide over one another along \hat{x} . For reasons to become clear later, the slip channel can also be referred to as, the vibration shear band. Ultimately the source of all vibrations are the elastic shear bands at the grain contact areas. For the first column on the left hand side, they are labeled as: shear band # 1 at the bottom to shear band #6 at the top. It is understood that the lifetime of a given column is roughly equal to the average time required for a grain to overtake another and that the lifetime of the five column configuration is about five times shorter and that the lifetimes would decrease with increased grain number, N, in the columns. These shear bands are characterized by thickness, b, and by compression and shear moduli that result in the corresponding phase velocities, c_p, c_s . In the context of the analysis that follows, b is assumed to be of the order of 300 nm, higher values can be compensated by higher values of the elastic moduli. In the computations that follow, c_p, c_s are assigned values in the order of a few m/s, implying that the physical composition of the shear bands is more that of a liquid rather than that of a solid. It could be characterized as viscoelastic.

Fig.1. An assumed grain column configuration in a slip channel (vibration shear band). The shaded areas correspond to the elastic contact shear bands with physical properties of their own when the grains are sheared together. They are characterized by compression and shear phase velocities, c_p, c_s and particle displacements ξ_z, ξ_x .



Inside the contact shear bands lie the tips of the surface asperities where transient temperatures can rise to several hundred C^{o} in time intervals up to a few microseconds (Bhushan [14]). It is widely assumed that the space between the asperities is filled with some sort of coating that plays a central role in the production of the musical sound. Several references to this effect can be found in [1, 9]. In the report by Lewis [15], the negative effect of moist conditions on the boomability of sand grains is stressed and in the report by Miwa et al. [16], it is suggested that certain beach sands lost their singability due to water pollutants resulting from nearby construction projects. In the report by Brunet et al. [17], it is shown that in a glass bead packing subjected to ultrasound waves, the dissipation increased by a factor of five when the beads were covered with silicon oil. However, the loss of musicality of polluted grains may not be as much due to viscous absorption as to changes in the friction coefficient that results

in the non-applicability of the stick-slip effect [18].

In a recent report by Patitsas [18], it is demonstrated that the water layer on the epidermis of a finger rubbed on a glass surface acts as the interfacial band that facilitates slipping and also results in the decrease of the friction coefficient with relative velocity resulting in the stick-slip effect. Furthermore, it is argued that the shear modes of vibration responsible for the acoustic emission are to be found in the finger skin. However, there is no reason why the modes of vibration in the skin with thickness $b_s \approx 5$ mm, shear phase velocity $c_s \approx 10$ m/s and wavelength $\lambda \approx 2b_s$ could not also exist in the interfacial band with $c_s \ll 10$ m/s and thickness $b \ll \lambda$. In this sense, the shear modes of vibration with $b \ll \lambda$. The attempt to write, $\lambda = 2b$ results in $c_s \approx 6 \times 10^{-4}$ m/s, for $b = \bar{d}/1000$ and $f_d=1000$ Hz, where the average grain diameter is, $\bar{d}=0.3$ mm. Such unrealistic low value of c_s leads to the conclusion that the shear modes of vibration in the shear bands must be characterized by the conditions, $b \ll \lambda$ and $L \ll \lambda$ along \hat{z} and \hat{x} respectively in Fig. 1, where L is the length of a given band along \hat{x} .

If the motion of the grains in a given column were along \hat{z} only, the shear bands could be replaced by equivalent short weightless springs. It is a straightforward exercise to compute the eigenfrequencies and describe the corresponding modes of vibration for such a system. For N blocks and N + 1 springs, there are N modes of vibration with frequencies, $f_1, f_2...f_N$. For the mode with frequency f_1 , all blocks oscillate in phase while for the mode with frequency f_N , neighboring blocks oscillate out of phase. The frequency f_1 tends to be rather insensitive to permutations of the blocks with different mass.

It appears suitable at this stage to include a short paragraph from the study by Haff [19]. While the author was thinking of the booming dune emissions, the implications for the impacted grains are obvious. "Perhaps the mechanical analogue which most readily comes to mind is the slipstick phenomenon, a nonlinear mechanism by which a steady input of external energy is ultimately released and stored. This is certainly consistent with the oscillatory nature of the system and with its sensitivity to grain surface conditions and hence, presumably, to friction. To ascribe booming to a slipstick mechanism, however, is only to say the words; until we have a clear picture in mind of what the grains are actually doing, we do not really understand the origin of the booming sands".

3. Modes of vibration in a given column

In what follows, the origin, O in Fig. 1, is assumed to coincide with the left side of shear band # 1. The particle displacement, ξ_s , is written as, $\xi_s = \nabla \times \mathbf{A}$, where \mathbf{A} satisfies the vector wave equation with phase velocity c_s . \mathbf{A} is chosen to lie along \hat{y} resulting in, $A_y = [A_1 \cos \alpha z + B_1 \sin \alpha z][A_1' \cos \beta x + B_1' \sin \beta x]e^{j\omega t}$ and this in turn results in, $\xi_z = \beta [A_1 \cos \alpha z + B_1 \sin \alpha z][-A_1' \sin \beta x + B_1' \cos \beta x]$ and, $\xi_x = \alpha [A_1 \sin \alpha z - B_1 \cos \alpha z][A_1' \cos \beta x + B_1' \sin \beta x]$, where the factor $e^{j\omega t}$ is understood to be included. The wave number, $k_s = \omega/c_s$, is given as, $k_s^2 = \alpha^2 + \beta^2$. The question arises as to the nature of the boundary conditions at the ends of a given shear band. If the ends are free, then, $\partial \xi_x / \partial x = 0$ at x = 0, resulting in $B_1' = 0$ and in, $\xi_z = [A_1 \cos \alpha z + B_1 \sin \alpha z]\beta \sin \beta x$ and, $\xi_x = [A_1 \sin \alpha z - B_1 \cos \alpha z]\alpha \cos \beta x$. The problem with the choice of free ends is that ξ_x is greater than ξ_z by several orders of magnitude since $\beta x \to 0$ for the low frequency modes and $\beta \ll \alpha$ since $b \ll L$. If the ends are fixed then,

$$\xi_z = [A_1 \cos\alpha z + B_1 \sin\alpha z]\beta \cos\beta x \tag{1}$$

and

$$\xi_x = [A_1 \sin\alpha z - B_1 \cos\alpha z] \alpha \sin\beta x \tag{2}$$

The latter choice appears reasonable when it is realized that the stress level decreases rapidly as x approaches the ends of the band resulting in a solid-like state at the ends.

In shear band #1, the expression for the particle displacement along \hat{z} simplifies to,

$$\xi_{1z} = [B_1 \sin \alpha z] \beta \cos \beta x \tag{3}$$

assuming that $\xi_{1z}=0$ at z=0, and that in shear band # 2 becomes,

$$\xi_{2z} = [A_2 \cos\alpha z + B_2 \sin\alpha z]\beta \cos\beta x \tag{4}$$

In shear band #3, the coefficient subscripts become 3 and so on for the rest of the bands.

The boundary condition on the top of shear band # 1 at z = b is,

$$\int \sigma_{1zz} dx dy + \int \sigma_{2zz} dx dy = M_1 \partial^2 \xi_{1z} / \partial t^2$$
(5)

where M_1 is the mass of grain # 1 and $\partial^2 \xi_{1z}/\partial t^2$ is evaluated at some point $x \to 0$. The normal stress per unit area along \hat{z} is given as, $\sigma_{1zz} = -(\lambda_e + 2\mu_e)\partial\xi_{1z}/\partial z = -\rho c_p^2 \partial\xi_{1z}/\partial z$, while that along $-\hat{z}$ at the bottom of shear band # 2 is given as, $\sigma_{2zz} = \rho c_p^2 \partial\xi_{2z}/\partial z$. The mass density in the bands was assumed to be equal to that in the grains, *i.e.*, that of quartz equal to 2650 kg/m^3 . Equation (5) is repeated at the top of shear band # 2 until the top of the last band # 6, where the normal shear force σ_{6zz} acts on the equivalent pestle mass M_p , *i.e.*,

$$\int \sigma_{6zz} dx dy = M_p \partial^2 \xi_{6z} / \partial t^2 \tag{6}$$

The result of (5), with $A_1=0$, is the following working equation,

$$[-S_1\rho c_p^2 \alpha \cos(\alpha b) + M_1 \omega^2 \sin(\alpha b)] B_1 - [S_2\rho c_p^2 \alpha \sin(\alpha (b+d_1))] A_2 + [S_2\rho c_p^2 \alpha \cos(\alpha (b+d_1))] B_2 = 0$$

where S_1, S_2 are the areas of bands # 1 and 2.

The grains are assumed to be perfectly rigid, so that $\xi_{1z}(z=b) = \xi_{2z}(z=b+d_1)$, resulting in the working equation,

$$\sin(\alpha b)B_1 - \cos(\alpha(b+d_1))A_2 - \sin(\alpha(b+d_1))B_2 = 0$$
(8)

where d_1 is the overall diameter of grain#1. There are 2N + 1 equations and 2N + 1 coefficients, $B_1, A_2, B_2, \dots A_6, B_6$ for N=5 grains as in Fig. 1.

4. Computations and implications

For a given array of grain diameters in a column and for given values of the compression and shear phase velocities, c_p , c_s , in each of the contact shear bands, the eigenfrequencies, $f_1, f_2, ... f_N$ can be determined by looking for the zeros of the determinant of the coefficients, $B_1, A_2,...$ when $\omega = 2\pi f$ is varied, provided the wavenumber α can be specified in terms of ω . Then, for a given eigenfrequency, the coefficients can be specified relative to the arbitrary value of $B_1=1$, and the nature of the corresponding mode of vibration can be examined. However, the value of the wavenumber β must be specified before α can be specified from the relation, $(\omega/c_s)^2 = \alpha^2 + \beta^2$. It was argued above that for low frequency vibrations, $\lambda_z/b \gg 1$ and also $\lambda_x/L \gg 1$. But, $L/b \approx 100$ and thus it can be argued that $\lambda_x/\lambda_z \approx 100$, resulting in $\alpha/\beta \approx 100$, and $\alpha \approx \omega/c_s$. Thus, it is argued that the relation between α and β for a standing wave pattern in a given shear band must also hold for a low frequency mode of vibration. The attempt to specify β from the condition, $\xi_x=0$ at x = L results in $\beta = \pi/L$ from (2). This in turn results in a cutoff frequency that is not consistent with the experimental results that follow and also with the need for a low frequency corresponding to pestle vibration.

In what follows in the next two paragraphs, it is assumed that the phase velocities, c_s, c_p , are the same in all contact shear bands. The average grain dimeter in a column of 12 grains was assigned the value, $\bar{d}=0.35$ mm to correspond to that of the singing sand collected from the mouth of the Brevort River flowing into the north shore of Lake Michigan, USA, about 25 km west from the city of St. Ignace. The grain diameter, d_j , was varied randomly between 0.25 and 0.55 mm and the band thickness was assigned the value, $b = \bar{d}/1000$. The circular contact areas, S_j , were evaluated by assuming the radius to be equal to the average diameter of the adjacent grains divided by 25. When a rod is hand-held and pushed or tapped into a sand bed, it is impossible to estimate the value of M_p . However, in the case of an 11 mm steel sphere dropped on a Brevort River sand bed, it is possible to evaluate approximately the effective value of M_p on top of a given column. To this end, the sum of the cross-sectional areas of the columns below the sphere was assumed to equal 1/3 of the sphere cross-sectional area.

The value of $c_s = c_p$ was varied until the value of 2.8 m/s resulted in $f_1 = f_d = 802$ Hz corresponding to the dominant peak in Fig. 2. The sequence of, f_p , f_1 , f_2 , f_3 , $..f_{12}$ was as follows: 49, 802, 1362, 2328,... 13888 Hz, with corresponding αb values of: 0.00008, 0.0012, 0.0021, 0.0036, ... 0.0133. Clearly, all 12 modes conform with the condition, $b/\lambda = \alpha b/2\pi \ll 1$. The lowest frequency, $f_p=49$ Hz, corresponds to the pestle vibration and is very sensitive to changes in the equivalent pestle mass, M_p , while the frequencies f_j decrease weakly with increasing M_p . It is worthy of note that $f_2 < 2f_1$ in this case. As in the case of a spring column, there are 12 frequencies with $\alpha b \ll \pi$. The frequencies corresponding to standing wave patterns in the shear bands are quite high, *i.e.*, $\alpha b = \pi$ results in $f = 1/3.5 \times 10^6$ Hz with $b = 3.5 \times 10^{-7}$ m. When b was raised to d/10, the lowest value of αb was 6.23 resulting in f=40000 Hz.

The description of the corresponding modes was effected by computing the coefficients, $B_1, A_2, B_2, ...A_{13}, B_{13}$ and then the particle (grain) displacements, ξ_z, ξ_x at the bottom middle and top of every band. For the lowest frequency, M_p , there was no change in sign in either particle displacement along the column, while for the fundamental, f_1 =802 Hz, the variation of ξ_z was like a sine function, and that of ξ_x nearly that of a negative cosine function, in agreement with (1, 2). For the frequency, f_2 , there was a change of sign in ξ_z and two changes in ξ_x . From (1, 2) it was determined that the ratio of the maximum value of ξ_x divided by that of ξ_z was about 6. From (8), it follows that ξ_z is also the grain displacement along \hat{z} , and since $L \ll \lambda_x, \xi_x$ is a measure of the grain oscillation displacement along \hat{x} .

In analogy with a column of springs, when the number of grains in a given column was increased from 12 to 24 with the same average diameter, the eigenfrequencies were as follows: 36, 378, 881, 1332 Hz for the first four, where in this case, $f_2 > 2f_1$. Thus, the dominant frequency, f_1 =802 Hz for N=12 was reduced by more than a factor of two for N=24. When c_s was increased from 2.8 to 5.9 m/s, f_1 was restored to nearly its value for N=12, *i.e.*, f_1 =797 Hz. Furthermore, when the grains were permutated in several ways, f_1 remained in the range of, 800 ± 40 Hz. Even when the total column mass was increased from 1.62×10^{-5} to 2.03×10^{-5} kg, f_1 remained within this range. It appears that amongst the various columns, f_1 would remain in the range, 800 ± 40 Hz, a spread that lies within the half width of the major frequency envelopes, Fig. 2 for example.

However, the assumption that the elastic moduli of the contact shear bands are the same along a grain column would be more applicable to the case of grain columns in a free avalanching sand band than to the case of compressed grains under a pestle. Since the grains under the pestle can move away laterally, the stress level at the grain contact areas would decrease appreciably with depth below the pestle, resulting in appreciable increase of the elastic moduli with depth. In this sense, the computations described above were repeated with $c_s=10$ m/s for the first shear band at z=0 and then decreasing linearly to a small value at the top of the grain column. The computations were facilitated by assuming again that $\xi_z=0$ at z=0. Then, for $c_s(13)=0.3$

m/s, the eigenfunctions were as follows: 16, 814, 2163, 2971, 4003 Hz etc. After several grain permutations along a given column, it was determined that f_1 assumed values in the range, 815 ± 38 Hz and f_2 assumed values in the wider range, 1375 to 2163 Hz. It will be argued below that the absence of envelopes corresponding to f_2 in Figs. 3 to 5, for example, could be due to such a large variation in f_2 . The relative particle (grain) displacement, ξ_z , increased monotonically with z with $\xi_z=0$ at z=0, while ξ_x increased similarly with $\xi_x=-1$ at z=0 to about $\xi_x=0$ at the top of the column.

5. Experimental results and implications

The frequency spectrum of the microphone recorded signal, when an 11 mm diameter steel sphere was dropped on a Brevort River singing sand bed, is depicted in Fig. 2. The side peak at about 1000 Hz is deemed to be due to a minor vibration shear band with slightly lower effective thickness compared to that corresponding to $f_d=790$ Hz. There is also a trace of low frequency vibration that would correspond to the vibration of the impacting sphere with frequency f_p . The microphone and geophone recorded signals in Figs. 2 and 3 correspond to the same event and the frequency plots correspond quite well except for the absence of the low frequency content in Fig 3 and the content around 1400 Hz in Fig. 2. It is possible that the geophone could not detect very low frequencies, however, this is also the case in the plots in Criswell et al. [20]. It is possible that the up-down oscillations of the impacting sphere are transmitted readily into the air but not so in the grain mass. The frequency envelope at $f \approx 475$ Hz in Fig. 3, that could be due to a thicker vibration band, is hardly present in Fig. 2. Evidently, the corresponding mode of vibration did not result in appreciable surface vibration.

The plots in Fig. 4 imply that when the sphere was dropped from the height, H=25, as opposed to 10 cm in Fig. 3, the vibration bands were considerably thicker, *i.e.*, N was considerably higher and or the elastic moduli had lower values. Moreover, when the sphere was fished out of the sand with the aid of a small spoon, the grain configuration would have changed and so would the character of the vibration shear bands during the following drop. Effectively, the grain configuration was history dependent.

Fig.2. Frequency spectrum and the microphone recorded signal when an 11 mm steel sphere was dropped, height $H \approx 10$ cm, on a Brevort River singing sand bed in a ceramic flower pot, 20 cm rim diameter by 10 cm in depth. The microphone was placed about 10 cm away from the impact point. $f_d \approx 790$ Hz with side peaks at 681, and 886 Hz. The other peaks are at about 1000 and 1381 Hz.



Fig.3. Same as in Fig. 2 but for the geophone recorded signal. The geophone was placed inside the pot near the rim. $f_d \approx 771$ Hz with a side peak at about 695 Hz. There is a pronounced envelope at about 475 Hz and a doublet at about 1000 Hz.



Fig.4. Same as in Fig. 3 but with height $H \approx 25$ cm. $f_d \approx 476$ Hz with minor envelopes at about 305, 571, 666 and 846 Hz.



Figure 5 depicts the frequency spectrum of the microphone recorded signal when a glass sphere was dropped on a silica gel bed. Contrary to expectation for microphone recorded signals, there is no frequency content that would correspond to the frequency $f_2 \approx 1750$ Hz. Figures 6 and 7 are the frequency spectra of the microphone and geophone recorded signals of the same event, namely when a glass rod was pushed into a Brevort River sand bed. The envelope at $2f_d=1180$ Hz in Fig. 6 is absent in Fig.7. A similar envelope at $2f_d$ can be seen in Fig. 4 in [1] when silicagel grains in a small container were impacted by a small pestle. The signal in Fig. 8 was recorded by microphone about 15 months before that in Fig. 6, shortly after the sand was collected in August 2009. The signal was recorded when a 16 mm wood rod was tapped into the sand bed. The major side envelope at about 650 Hz could be attributed to a minor vibration band and the envelopes at about 1000 and 1500 Hz to modes with frequencies f_2 and f_3 . The rest have to be attributed to surface noise effects due to grain collisions with the pestle and other grains. Comparison with Fig. 6 implies that such noise effects depend on such factors as: pestle geometry, pestle speed, angle of impact, confinement of the grain mass and history of the grain bed. In the report by Nori et al. [11], the frequency spectrum of a signal, presumably microphone recorded, from 'squeaking sand' has a major peak at 860 Hz and harmonics of 860 at 1720 and 2580 Hz.

Fig.5. Frequency spectrum and the microphone recorded signal when a 16 mm glass sphere was dropped, $H \approx 10$ cm, on a bed of silica gel grains. $f_d \approx 943$ Hz with a side peak at about 848 Hz.



Fig.6. Frequency spectrum of the microphone recorded signal when a glass rod, length 7.5 cm, diameter 1.5 cm obtained from the Museum of Sand in Nima, Japan, was pushed into a Brevort River singing sand bed. $f_d \approx 590$ Hz with various side peaks. The minor envelope is at about $2f_d=1180$ Hz.





Fig.7. Same as in Fig. 6 but for the geophone recorded signal. $f_d \approx 590$ Hz.

Fig.8. Frequency spectrum and the microphone recorded signal when a 16 mm wood rod was tapped into a Brevort River singing sand bed. $f_d \approx 475$ Hz. There are envelopes at about $2f_d$ and $3f_d$.



The greater part of Section 6 in [1] is devoted to establishing that such harmonics in the frequency spectra are not due to modes of vibration in the slip channel (vibration shear band), viewed then as a continuum, but rather due to grain collisions with the pestle. It is argued that the surface grain collisions are nearly slaved to the vibration of the fundamental mode with frequency f_d and in so doing they emit strings of waves

with time between collisions centered around $T_d = 1/f_d$. Provided the spread around T_d is not too large, the synthesized signal from all grains, is fairly periodic and includes harmonics of f_d . Furthermore, the Fourier spectrum of such a signal can resemble the spectrum plot in Fig. 2 for example. For larger spread around T_d , side peaks appear in the frequency spectrum of the synthesized signal and eventually it resembles that of a noise signal. In the context of the present approach, where the slip channel is composed of grain columns, a similar argument can be made, especially in view of the absence of such frequency envelopes for the signals recorded by the geophone. The plots that follow will further reinforce the argument that, whereas, such synthesized signals from surface grain collisions can propagate readily into the air, they cannot do so in the grain bed. Furthermore, the minor side peaks seen in Fig. 5 to Fig. 7 could be due to variations in the grain number, N, amongst the various columns in the vibration band and also due to incomplete synchronization of the column vibrations.

However, the question remains as to why the modes with frequencies f_2 , f_3 etc are not excited. The geophone ought to detect such modes if excited. It was remarked above that there was a large spread in the values of the frequency f_2 when the grains in a given column were permuted. Thus, such a large spread could render unstable the collective vibration of the grain columns at the frequency f_2 . More likely, according to [18], the stick-slip effect would become inapplicable if the frequency of the grain oscillation along \hat{x} would be high enough so that the period of oscillation would be lower than the relaxation time regarding the grain surface state that defines the change of the friction coefficient with relative velocity.

According to [4], the geophone recorded signals included frequency content at relatively low frequencies, *i.e.* multiples of f_d =85 Hz. Furthermore, according to Lewis [15], the frequencies of acoustic emissions from steady state booming avalanches at the Kalahari Desert, South Africa, ranged from about 132 to 300 Hz. However, these were estimates using tuning pitch pipes. It is possible that the higher end was due to harmonics of the fundamental, about 130 Hz. According to [20], there is a harmonic content at about twice the fundamental, *i.e.*, at about 120 Hz. However, the signals were recorded when the booming sand, at Sand Mountain, NEV, USA, was squeezed under a flat shovel while withdrawn sharply with a downward push.

Figures 9 and 10 depict the frequency spectra of the signals, recorded by microphone and geophone respectively, of a single event when a 13 mm wood rod was tapped lightly into a large bed of a local silent beach sand. The content around 2750 Hz is deemed to be due to grain-grain collisions, while the rest due to grain-pestle collisions. It is surprising that the frequency spectrum in Fig. 10 includes only the lower frequency content similarly to Fig. 7 for the singing sand. Evidently, the signals generated by the grain collisions on or near the surface are readily transmitted into the air but not into the grain bed. However, the similarity between Figs. 7 and 10 suggests that, even in the case of the silent sand, there is a fairly well defined vibration shear band extending well below the pestle. The same effect is more evident in Fig. 11 where the same wood rod was tapped into a large bed of nearly silent sand from the Providence Bay beach in Manitoulin Island, Ontario, Canada. The local natives claim that the sand used to sing in the distant past. Figures 12 and 13 correspond to Figs. 9 and 10 except that the rod was tapped into a large bed of crusher dust, used as road surface in place of pavement. The particles varied in size from about 1 mm in overall diameter to as large and irregular as $10 \times 5 \times 2$ mm. Most of the fine dust had been removed.

Fig.9. Frequency spectrum and the microphone recorded signal when a 13 mm wood rod was tapped into a bed of local silent beach sand.





Fig.10. Same as in Fig. 9 but for the geophone recorded signal. $f_d\approx\!\!457$ Hz.

Fig.11. Frequency spectrum and the geophone recorded signal when a 13 mm wood rod was tapped into a bed of nearly silent beach sand from Providence Bay, Manitoulin, Ontario, CA. $f_d \approx 514$ Hz.



Fig.12. Frequency spectrum of the microphone recorded signal when a 13 mm wood rod was tapped into a bed of crusher dust. The grains were very irregular in shape and some as large as $10 \times 5 \times 2$ mm



Fig.13. Same as in Fig.12 but for the geophone recorded signal. $f_d \approx 362$ Hz.



The implication of Figs. 10, 11 and 13 is that vibration shear bands exist under the pestle in all grain beds. As in the case of the wet finger drawn over a glass surface [18], the key to poor or good singability of the grain bed is sufficient grain-grain slippage and sufficient decrease of the friction coefficient with relative velocity between the grains. Evidently, these conditions were poorly met or not met at all resulting in very small

energy transferred to the column vibration modes in the cases described in Figs. 10, 11 and 13. Additionally, there was not sufficient slippage between the grains that would result in the relief of the stress level between the pestle and the grains that would in turn result in low surface noise level. These arguments are consistent with previous claims that the transition from silent to singing grains is not sudden but gradual [1, 21].

The degree of grain-grain slippage was tested by dropping a 16 mm steel sphere on a grain bed of: (a) Brevort River sand, (b) Providence Bay nearly silent sand and (c) a local beach sand sounding even more silent. In case (a), the sphere was barely visible at the center of the crater, in case (b), nearly 1/3 of the upper hemisphere was visible and in case (c), nearly all of the upper hemisphere was visible. It is worthy of note that the surface texture of the pestle can influence the degree of singability to some extent. In particular, the glass rod described in Fig. 6 tends to produce a low level musical sound even when impacting lightly the sand in case (c) above.

6. The stick-slip effect and the synchronization wave

The role of the stick-slip effect in the realization of the singing sound when a grain bed is impacted by a pestle was outlined in the second last paragraph of the previous section. However, for the stick-slip effect to come into play, the modes of vibration have to be excited somewhat, *i.e.*, the grains have to slide back and forth along \hat{x} in Fig. 1. [18]. To this effect, it can be argued that a minimum (threshold) impact velocity is required for the initial excitation of the column modes of vibration. Such thresholds are always present before a musical event can occur. When a rod, about 15 mm in diameter, was held vertically and forced to move horizontally through a bed of silica gel grains, the immersion depth had to be more than about 2 cm and the velocity had to be more than about 20 cm/s. Moreover, when a plastic bead, 1 cm in diameter, was buried in a flat pile of the Brevort River singing sand and pulled horizontally by a string, the depth had to exceed about 3 cm and the string had to be pulled rather sharply. Similar thresholds are seen in [9] where the booming dune grains were pushed by a blade.

However, the mere excitation of the modes in the grain columns is not sufficient to

result in the intense emissions associated with such phenomena. It could be argued that under the pressure of the pestle, the vibrations in the various columns become self-organized so as to vibrate in phase, since large values of the particle (grain) displacements, ξ_z, ξ_x would facilitate the grain flow away from the pestle. But, such collective vibration of the grain columns could also be realized by the presence of a propagating wave along the vibration shear band. The wave that reaches the geophone and eventually the surface and the microphone has to start propagating somewhere. It is the same concept introduced in [2, 9] regarding the synchronization of the collisions of avalanching sand dune grains. A given column is continuously renewed as grains are replaced by others, resulting in slight changes in the frequency $f_1 = f_d$. However, in view of the role of the synchronization wave, such changes can be smoothed out when averaged over a large number of columns.

In the case of a well defined vibration shear band where the elastic moduli change abruptly from very low to very high values at z=0 in Fig.1, there would be questions regarding the frequency range that would allow propagation of the synchronization wave. However, in view of the more realistic assumption, where the grain columns are allowed to be very long on the condition that the elastic moduli of the contact shear bands increase steadily with depth, there ought to be candidate waves according to the results reported in [5, 6].

7. Grain flowability and grain confinement

It seems fair to argue that what distinguishes primarily a freely avalanching booming sand, from other avalanching sands, is the relatively high avalanche front and the apparent high flowability of the grains. Reference to high flowability, where the sand flow is compared to that of a water stream, can be found in the reports by: Sholtz et al. [22], Bagnold [23] and Humphries [24]. When Brevort River singing sand was placed in a plastic container, 40×35 by 25 cm deep and dumped sharply on the side of a nearby dune ridge with slope over 30° , there was no appreciable avalanche front. The sand flow was sluggish and characterized more by plate-like motion than free surface grain motion. It was more sluggish than that of ordinary silent sand motion when similarly dumped. It could be argued that the relatively low flowability resulted in a very thin vibration shear band, under the sliding plate, and in $f_1 \approx 3000$ Hz for $N \approx =3$ and in wavelength $\lambda_1 < 1.0$ mm. Such a small wavelength would require very well defined vibration band boundaries, and that may not be the case during a plate avalanche. Furthermore, when the frequency is too high, the stick-slip effect could become inapplicable as described above. Finally, values of f_1 in the order of 1000 Hz are not compatible with the grain-grain collision frequency, f_c , as described below. However, when a cupful of sand was tossed with some force at an angle of about 45° on the flat top of a sand pile, the usual sound was evoked.

In [15], it is reported that when booming grains were placed in a glass jar, 17.5 cm in length by 10 cm in diameter, half full, and rapidly tilted, a violent roar could be produced. Similarly, when Brevort River singing sand was placed in a glass jar, 17 cm in length by 8 cm in diameter but only 7 cm at the lip, and then tilted sharply, no sound was produced until most of the sand had flown out and the sand height above the lip was about 15 mm. During the sound emission, the sand appeared to flow out of the jar as in one piece, thus, reinforcing the concept of the slip channel and the vibration shear band adjacent to the jar lip. The low sand height above the jar lip suggests that the slip channel comprised fewer than ten grain layers. Thus, singing sand grains do not boom because of their low flowability.

Booming dune grains do not sing in the sense of emitting a musical or a squeaking sound when placed in a large dish and impacted by a rod with diameter about 2 cm. In the context of this approach, this is so since the relatively high flowability of such grains results in very thick slip channel(s) around the rod and very unstable long grain columns. In other words, the grains can flow away from the rod without the aid of a vibration shear band. However, when the booming dune surface was impacted sharply by the palm of the hand, there was emission with $f_d \approx 73$ Hz [5]. Evidently, the relatively large area of the impacting hand resulted in a sufficiently large degree of confinement that resulted in a vibration shear band a few cm below the hand. Similarly, when booming grains were confined in a 25 cm wide circular channel and pushed by a large blade, they became boomable and or singable [9]. Additionally, they became singable when confined inside a jar [10]. In [1, 21], it is demonstrated that salt, sugar and silent sand grains can exhibit singability when sufficiently confined. In [20], frequencies as low as 53 Hz were detected when a 30×30 cm flat shovel was withdrawn sharply with a downward push from the face of a dune at Sand Mountain. The other peaks at 58, 66, 76 Hz could be attributed to several distinct slip channels under the shovel. Additionally, in Fig. 5 in [20], it is stated that considerable frequency content may be present in the 3 to 30 Hz interval. In the context of this approach, this could be attributed to the pestle frequency f_p . On the other hand, when a flat pile of the Brevort River sand was impacted vertically by the flat end of a wood rod (block) 14 cm in diameter, there was practically no musical sound emission, presumably, due to the relatively low flowability of the grains.

Evidently, the surface state of the booming grains allows for a large degree of slippage and a sufficient rate of decrease of the friction coefficient with relative velocity so that the stick-slip remains applicable. It cannot be argued that the elastic moduli of the contact shear bands are generally lower than those for the singing grains. However, it can be argued that the rate of increase of such moduli with distance from the plane of the forcing agency is considerably lower than for the singing grains, resulting in considerably longer grain columns, *i.e.* considerably thicker effective vibration shear band, resulting in considerably lower frequency f_d .

8. Freely avalanching booming sand

When a sand band, several cm thick, is in a state of avalanche, it is effectively confined by the plane of the static sand below and the thin band of relatively fast moving surface layers above [2, 9]. The effective width of the slip channel (vibration shear band) could be about ten times larger than in the cases of the impacted singing grains, resulting in dominant frequency f_d about ten times lower. It is now argued that the grain layers near the bottom of the avalanche front experience the greatest stress level when they decelerate sharply and are overtaken by the layers above, and that a slip channel could exist in that region. It is possible that the so called 'roar' sound emitted when the sand is pushed downhill in a heaped-up manner is due to grain column vibrations in such a front slip channel. However, the 'hum' that follows the 'roar' represents a steady state acoustic emission that is independent of an avalanche front. It can be maintained by continuously digging a hole where the avalanching sand is deposited [15]. In the context of this approach, it is estimated that the effective grain column number N in the avalanching sand band that results in the observed frequency $f_d \approx 100$ Hz, for average diameter $\bar{d} \approx 0.2$ mm, [2], is about 100.

In the relatively slowly avalanching sand band, the elastic moduli would increase with depth resulting in increased phase velocity with depth. A computation was effected with grain number N=24, shear band thickness, $b = \bar{d}/500$, shear phase velocity, c_s , decreasing linearly from 0.85 at z=0 to 0.25 m/s at the top, and an effective pestle mass M_p equal ten times the average grain mass. Effectively, the top ten surface grain layers would not take part in the column vibrations. The results were as follows: $f_p=39$ Hz, $f_1=101$ Hz, $f_2=217$ Hz, $f_3=369$ Hz etc. If the effective grain column number, N, were 100, then the values of c_s would be about four times larger. These results are reasonable considering the approximation in terminating abruptly the column at z=0. The particle (grain) displacements, ξ_z, ξ_x , were computed for $f_1=101$ Hz and it was determined that ξ_z had a maximum value, while ξ_x had a node, near the column center. This is consistent with the results shown in Fig. 3 in [7], where the "first mode" also has a similar node. Evidently, when the vibration shear band becomes quite large, continuum mechanics is applicable.

The frequency f_p corresponds to the surface mode of vibration. In Fig. 2, parts (b) and (f) in [4], there appears to be evidence of such low frequency content. Minor envelopes centered at 30 Hz, as well as $2f_d$ and $3f_d$, with $f_d \approx 100$ Hz, can be seen in the frequency spectrum of the avalanche signals available at the website in [2]. Permission to this end was obtained from the author some time ago. Within the context of this approach, the low frequency, f_p , would collapse to nearly zero if the vibration shear band were situated two or three cm below the surface as suggested in [1, 6], *i.e.*, if the load mass M_p were that much larger.

The collective grain column vibrations cannot be sustained at frequencies other than the average grain-grain collision rate, $f_c \approx 100$ Hz for $\bar{d} \approx 0.2$ mm. Unlike the case of the singing grains where the energy that excites the column vibrations is derived from the impacting pestle, in this case the energy is derived mainly from the surface grain-grain collisions. Thus, any collective vibration has to be slaved to the frequency f_c . Additionally, the frequency of the surface Rayleigh-Hertz wave [2, 5], that acts to synchronize the column vibrations, as well as the grain-grain collisions, has to be slaved to the same frequency.

However, the frequency $f_1 = f_d$ is defined by the elastic moduli of the contact shear bands, and if f_d is appreciably different from f_c , then, boomability is not possible. Such a conclusion is consistent with the rarity of such a phenomenon. It occurs only during certain periods of the year and not all dunes in a given region can boom. Furthermore, if the thickness of the avalanching sand band becomes too thin, then, f_d is forced to exceed f_c and boomability ceases, as was reported in [5, 9].

The synchronized vibration of a huge number of grain columns extending over areas of several m^2 can justify the huge amounts of seismic and acoustic energy radiated. Not all sections of a given avalanche band need participate in the vibration process, resulting in the non-uniform appearance of a boiling viscous liquid [1]. Furthermore, not all sections would be characterized by the same width of the vibration shear band, *i.e.*, the same column number N, resulting in several side peaks in the frequency spectra, resulting in relative poor sound quality as reported in [8, 11, 15] and Curzon [25].

9. Sliding booming sand plates

On the basis of high flowability of the booming grains, it can be argued that the slip channel between the plate sand and the compacted sand below is thick enough so that the fundamental frequency of the column vibrations, f_1 , is in the general neighborhood of the collision frequency $f_c \approx 100$ Hz for free avalanche. However, there is no reason to assume that f_1 as well as the frequency of the ensuing synchronization wave are slaved to the frequency f_c . As in the case of the impacted grains, the gravitational energy of the plate can be transferred to the column vibrations via the stick-slip effect. At this stage, it is deemed appropriate to include an excerpt from the book by Curzon [25], p. 285, that appears to correspond to the observation by Vriend et al. [4] in that dune vibrations were detected even when there was no apparent avalanche in progress. "By the flowing in of the sand from the sides and the repeated tread [of the traveler] a large part of the whole sand-layer of the slope at last acquires motion, and by its friction against the motionless under-layer produces a noise, which from a humming becomes a murmur, and in the end passes into a roar, and is all the more surprising in that one sees but little of the trickling and general movement of the sand-layer."

10. The pushed booming sand

Similarly to the discussion in the previous section, there is no reason to assume that the column vibrations in the slip channel are slaved to the grain-grain collision frequency f_c in the slip channel. In Fig. 2 in [23], the internal flow (slip channel) is depicted as rising from the bottom of the blade towards the surface at an angle of about 30° from the vertical blade surface. Such an angle, as well as the effective channel width and the elastic moduli of the grain contact shear bands, would vary with the blade velocity, V_b , and the sand height, H, in front of the blade. The first attempt to determine the change in frequency with the manner the sand was pushed on the face of a dune can be found in [15]. It was determined that when the sand was pushed uphill, the frequency increased as opposed to when it was pushed downhill, and it also increased with the speed of push. In [9], it is claimed that frequencies, f_d , as low as 25 Hz were obtained by pushing the booming sand on the face of a dune. Evidently, the means by which the sand was pushed resulted in very low elastic moduli of the contact shear bands in the vibration shear band. Not only the effective grain number, N, in the columns was very large but also the mass load on the columns, M_p , was very large.

Furthermore, in [9], the study of the change of the dominant frequency f_d with blade speed and height of the grain mass in front of the blade was quantified. In Fig. 2 in [9], it can be seen that when the mean shear applied to the grains was, $V_b/H \approx 5.5$, then, $f_d \approx 100$ Hz, suggesting that at this mean shear, the stress level in the slip channel was about equal to that in the vibration shear band in a freely avalanching sand band. The authors claim that larger values of H would result in f_d as low as 25 Hz. In the context of this approach, it could be argued that larger values of V_b for fixed H, would result in larger values of the elastic moduli of the contact shear bands by forcing the grains to slide past one another in less time. Reduced height H for fixed V_b , would have the same effect since it would result in lower stress level and higher elastic moduli of the contact shear bands. When either V_b or H is too low, then the stress level is too low to result in the excitation of the column vibrations in the vibration shear band.

11. Conclusions

The use of discrete, as opposed to continuous, mechanics leads to a better insight into the grain mechanics inside the grain bed that result in the musical sound emissions. The assumed elastic shear bands at the grain contact areas lead to the concept of grain columns that resemble the familiar columns of blocks with light springs interposed. The experimental results lead to the conclusion that when a grain bed, musical or not, is impacted by a pestle, a slip channel (vibration shear band), comprising many grain columns, is formed under the pestle. The excitation of the modes of vibration in such grain columns depends on the degree of slippage between the grains, moving past one another, and on the degree of decrease of the friction coefficient with relative velocity between the grains. Effectively, the degree of singability of the grain bed depends on the degree of applicability of the stick-slip effect.

In this sense, the transition from silency to musicality is gradual, a process consistent with experimental evidence. As the stick-slip effect becomes more applicable, the stress level between the pestle and the grains is reduced, since most of the grain movement away from the pestle is effected in the vibration shear band. Thus, the surface noise level generated by the chaotic grain motion around the pestle is also reduced. Simultaneously, most of the energy delivered by the pestle into the grain mass is transferred into the fundamental mode of the grain column vibrations, resulting in the intense and pleasant sound, provided the dominant frequency is not too high.

The distinction between impacted singing grains and avalanching booming sand grains can be found in the degree of flowability of the grains. The low flowability of the singing grains prevents a free surface avalanche flow and the slip channel (vibration shear band) under the sliding plates would be too thin, resulting in very high frequency, f_d , not supportable by the stick-slip effect. On the other hand, the high flowability of the booming grains does not allow for the formation of a vibration shear band when the grains are impacted by a pestle in a large dish. However, when the grain confinement is large enough, by the use of a very large pestle or a large blade, the grains can become boomable (singable). The large flowability of the booming grains results in low rate of increase of the elastic moduli of the grain contact shear bands with distance from the forcing agency, a pushing blade for example or the surface shearing layers in a dune avalanche. This results in longer grain columns and lower dominant frequency f_d , nearly by a factor of ten, compared to that from the impacted singing grains. It could be argued that two avalanching components contribute to the booming sound: Namely, the relatively fast avalanching band of ten or so surface layers and then, the main band that could be several cm in thickness, which constitutes the vibration shear band. Since the fast avalanching surface layers contribute substantially to the energy that excites the column vibrations in the main band, the frequency of the dominant mode, f_d , and that of the surface band. However, this can happen only if the two frequencies are not far apart. Thus, not all dunes in a given area can boom. In particular, if the thickness of the main band becomes too low, then, f_d is forced to exceed f_c and booming is not possible.

Within the context of this approach, it could be argued that when a sand pile is pushed by a blade, the dominant frequency, f_d , is increased with blade speed since the grain-grain contact time is decreased resulting in higher elastic moduli of the contact shear bands. Similarly, f_d is increased with decreased pile height since the stress level is decreased resulting in higher elastic moduli. For very low values in either the blade speed or the pile height, there is not enough energy to excite the column vibrations in the vibration shear band and the sound emission is not possible.

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